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III.—*Experimental Researches into the Laws of Certain Hydrodynamical Phenomena that accompany the Motion of Floating Bodies, and have not previously been reduced into conformity with the known Laws of the Resistance of Fluids.*

By John Scott Russell, Esq. M. A., F. R. S. Ed.

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INTRODUCTION.

In the summer of 1834, I was led to examine with considerable interest some of the phenomena of fluids, from the circumstance of having been consulted upon the means of improving a system of navigation to be conducted at unusually high velocities. Being well aware, however, of the very imperfect state of that part of Theoretical Hydrodynamics which relates to the Resistance of Fluids to the Motion of Floating Bodies, and that there had been found in its application to the solution of practical questions, discrepancies so wide between the predicted results and the observed phenomena, as to render the principles of the theory exceedingly false guides, when followed as maxims of art, I felt it impossible to recommend conscientiously any mode of procedure founded on defective principles, and I therefore determined to undertake a series of investigations concerning the laws of the resistance of fluids, and the means of applying them to the formation of rules for the arts of practical navigation and naval architecture. In this investigation, I have now been engaged during the leisure of two summers, and I am still continuing to prosecute the investigation.

The following papers contain the experiments of the two summers 1834 and 1835, with the resolution of certain anomalous phenomena, and the illustration and application of certain laws that have been developed. The experiments were conducted on a very large scale, and the forms given to the floating bodies were analogous to those which are most highly approved in the practical construction of ships, as well as those of certain theoretical solids. The vessels used were from 31 to 75 feet in length. Accurate Chronometers and Dynamometers of various descriptions used by a number of highly educated and scientific assistant observers, render the experiments worthy of great confidence. In 1834 the power used to overcome the resistance was the force of horses directly applied to the vessels; but although out of a multitude of experiments, some were obtained that were distinguished by uniformity in the application of the force, yet in general the action of that species of moving force was found too desultory and discontinuous to furnish a measure of resistance sufficiently accurate to be used in comparisons of a delicate description, and therefore in 1835, means were provided for rendering the action of the moving force more nearly continuous by using a peculiar apparatus. With this apparatus experiments were made on the resistance of four vessels of about 70 feet in length, at different depths of immersion, so as to give comparative measures of resistance in reference to sixteen forms, at velocities from three to fifteen miles an hour.

The results of the investigations directed to the determination of the law which connects the resistance of the fluid with the velocity of the motion of the floating body, appear to establish the following conclusions,—that the resistance does not follow the ratio of the squares of the velocities, excepting in those cases where the velocity of the body is low, and the depth of the fluid considerable—that the increments of resistance are greater than those due to the squares of the velocities, as the velocity approaches a certain quantity, which is determined by the depth of the fluid—that at this point the resistances attain a first maximum, and that here, by certain elements of the form of the body, and of the dimensions of the fluid, they may become infinite—that immediately after this, there occurs a point of minimum where the resistance becomes much less than that due to the square of the velocity, and after which it continues to receive increments, of which the ratio is less than that due to the increment of the square of the velocity—that according to the law of progression which has been established, the resistance will reach a second point of maximum, when a velocity shall be attained of about 29 miles an hour, after which it will be rapidly diminished with every increase of velocity.

Extracts from the Experiments, shewing the connection between Resistance and Velocity.

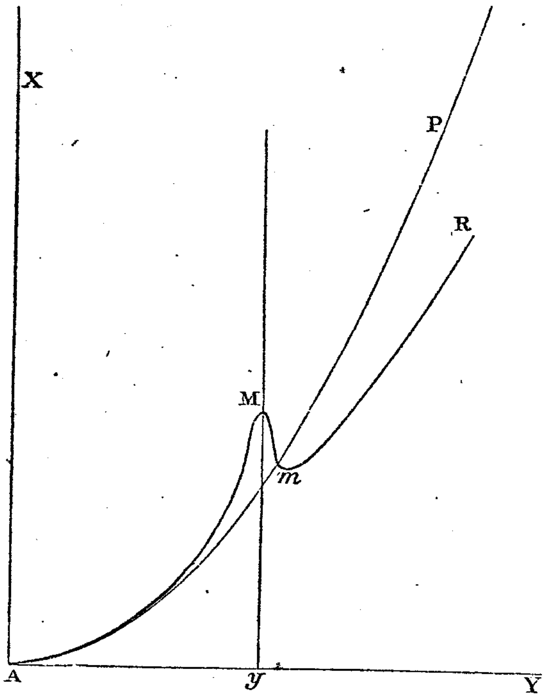
Example I.

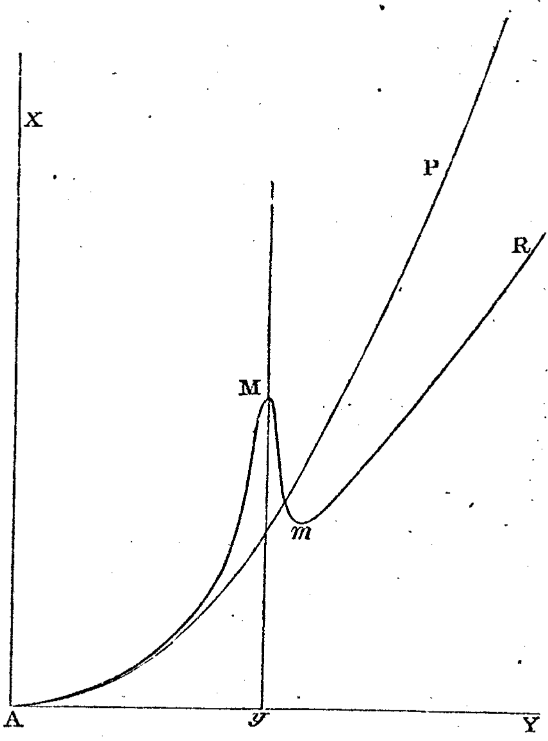
Velocity.	Resistance.
3.7	28.0
4.0	33.75
5.0	51.0
6.1	91.0
7.1	217
7.5	265.
..	Point of First Maximum and Minimum.
8.5	215.
9.0	234.
11.3	246.
12.3	
15.1	

Example II.

Resistance.	Squares of Velocities.
	14.3
39.	16.
	25.
111.	38.4
255.	51.5
330.	57.3
210.	72.6
235.	81.8
	129.
352.	153.6
444.	229.5

The curve of resistance derived from these examples instead of being a parabola, will be of the following species. Fig. 1 and 2.





AX and AY rectangular co-ordinates.
Velocity measured on AY, and resistance on AX.
AP the parabola resulting from the squares of the velocities.
AM*m* R the line of resistance, M the point of first maximum, and *m* the succeeding point of minimum.

The causes of these deviations from the law of the squares of the velocities, are fully investigated in the course of observations forming Part I. of this paper, Part II. being filled with the details of the experiments of 1834, and Part III. with those of 1835.

The first element of deviation which presented itself, was a phenomenon of *Emersion* of the solid from the fluid, due to the velocity of the motion, and by which the dynamical immersion of the floating body is rendered less than its statical immersion in the fluid. The law connecting this emersion with the velocity of the solid is deduced in Sect. (1.) from elementary considerations, and coincides with the experiments.

Having determined the effect of motion upon the floating body itself in relation to the fluid, I have next examined the effect produced on the particles of the fluid itself by the motion of the floating body. At this part of the inquiry I discovered phenomena of a most singular character, by means of which the resolution of the anomalies in resistance has been most successfully effected, and which gives to many of the facts of practical experience a satisfactory explanation, and points the way to many important improvements in the construction of vessels, the navigation of rivers and shallows, inland navigation, and other departments of hydrodynamical engineering. These phenomena arise from the *generation and propagation of Waves* of the fluid by the motion of the floating body.

It has appeared, in the course of these investigations, that the restoration of equilibrium among the particles of the fluid when it has been deranged by the motion of a floating body, is effected, not so much by means of the generation of currents in the fluid, as has hitherto been generally assumed, as by means of the generation of waves of the fluid, in which form the elevations of the fluid raised on the front of the moving body are propagated with an ascertained velocity in the direction of the motion of the disturbing body. It appears that these waves move with a velocity that is nearly uniform,—that they travel to very great distances,—that their velocity is not in any degree connected with the form of the vessel,—that their velocity is not at all dependent on the velocity of the body which generates them,—that their velocity is due alone to the depth of the fluid, being equal to the velocity acquired by a body falling in vacuo through a space equal to half the depth of the given fluid,—and that the height of the wave itself above the fluid will only increase its velocity by so much as it increases the depth of the fluid at that point, reckoning from the summit of the wave.

I next proceeded to examine the nature of the interference of such waves, so as to determine their effect in modifying the resistance of the fluid to the motion of the body giving rise to them. I found immediately that the point of first maximum of resistance coincides accurately with the point at which the velocity of the motion of the floating body becomes *equal* to the velocity of the motion of the propagated waves. It appeared further, that the effect of the formation of these waves, when the velocity of the solid was *less* than the velocity of the waves, was to send forward towards the anterior part of the solid an accumulation of successive waves (to which accumulation I have given the name of the anterior wave), and to create a posterior depression in that part of the fluid from which these waves had been sent out, and thus to change the form of the surface of the fluid in such a manner that the axis of the floating body, formerly horizontal, no longer remained so, but was elevated anteriorly and depressed posteriorly, so as to form a considerable angle of inclination, and greatly increase the anterior section of displacement of the solid, whereby, at velocities less than the velocity of the wave, a very rapid increase of resistance was experienced in approximating to that velocity. It appeared, on the other hand, that at velocities greater than that of the waves, the effect of their generation by the floating body was to diminish the resistance given by the fluid, because the elevations of those waves falling behind those points of the body by which they had been raised, constituted an accumulated wave towards the middle of the body, upon the crest of which wave, poised in a position of stable equilibrium, it was borne along in a horizontal position, with a diminished section of immersion at the stem and stern, and consequently with a diminished section of resistance.

Besides the diminution of the resistance to the floating body experienced at velocities greater than that of the wave, it is also rendered apparent why there should be likewise experienced a great diminution of that commotion which takes place in the fluid at velocities less than that of the wave, and how it is that the phenomenon of stern surge, so destructive to the banks of the channel, and so dangerous in the practical navigation of shallow water, is found to disappear entirely at velocities greater than that of the wave in the given depth of fluid.

The effect of the motion of the floating body in changing the form of the fluid is least when the velocity is least and greatest; its effect is greatest when its velocity most nearly approximates to the velocity of the wave.

From the investigations of 1834, a form of great resistance suggested itself to me. A vessel, named in the tables "The Wave," was constructed of this form. This vessel was made the subject of experiment in 1835. It appears from the tables, that the resistance of this vessel was much less than that of other beautifully shaped vessels with which it was compared; and one phenomenon which I observed seems to me to establish as true, that the form of this vessel does not deviate widely from the form of least resistance. This tentative phenomenon seems to me to demonstrate, that the motion communicated to the particles of the fluid is the smallest that is consistent with the translation of the moving body, and it is this,—that at all velocities extending up to seventeen miles an hour, no spray, no heaping up of water at the stem, no lateral currents extending beyond the precincts occupied by the body itself, were ever sensible, but the body entering the water having a smooth and glassy surface, left it unchanged and unruffled. In the motion of all the other forms it was observed, that the water was thrown aside at the stem of the vessel in the form of a "head and feather" of spray, and that "broken water" extended to a distance even among the particles considerably removed from the line of the vessel's motion. The equation to the curve of least resistance was found by supposing the lateral motion given to a particle of the fluid, to receive equal increments in equal times from zero to a given maximum of velocity, after which, by equal decrements in equal times, it should again be brought to rest at the required distance from its original position in the place necessary to permit the transit of the greatest diameter of the immersed body. The curve thus obtained is concave outwards at the stem, and becomes convex towards the maximum breadth, having an intermediate point of contrary flexure. Delineations of this form, and tables of comparative resistances, are given in Part III.

I have given, at the end of the first part of this paper, some illustrations of the subject, drawn from facts and observations in practical experience, which have either been communicated to me or recorded by myself. The navigation of shallow rivers, lakes, seas, and canals, affords many illustrations of the principles I have developed. The canals of Holland and the rivers of America, as well as those of our own country, are navigated on a practical system, which is fully explained by the interference of the wave,—and the improvements of which those species of navigation may be capable, can only be effected in conformity with the knowledge of these laws that has now been obtained. By the propagation of waves, and propelling vessels upon those waves, there is a prospect opened up of attaining velocities upon the surface of the water, that have been hitherto held to be impracticable. The length, however, to which this communication had previously extended compelled me to shorten this part of the paper, and I have been contented rather to shew the applicability of the principles to practical improvement than to carry out this application.

Such are the results of that portion of the investigations I have undertaken, which has been completed; and I feel it to be my duty thus publicly to acknowledge, that if any benefit shall be conferred, either upon theoretical hydrodynamics or the practical arts connected with it, by these inquiries, it is not to myself alone that they owe their value. To so large and extensive a series of experiments my own exertions and my own pecuniary resources would have proved inadequate, had I not been placed in circumstances peculiarly fortunate. Two scientific friends, ALEXANDER GORDON and ANDREW CRAWFORD, Esqrs. afforded me invaluable and long continued assistance, and to their labours and those of Dr GEORGE GLOVER, Mr WILKINSON, and Mr MUIR, with about a dozen of hired assistants, the experiments owe much of their extent and accuracy. To the Committees of Management of the Edinburgh and Glasgow Canal Companies, who have permitted the use of their public works, and of their servants, and of their moving power, and of their vessels, and defrayed a large expenditure of money, and to ROBERT ELLIS, Esq. W. S., through whose influence principally these privileges were obtained, under the enlightened conviction, that, from the improvement of that science with the applications of which they are so nearly connected, these mercantile companies would be the first to derive important benefit; to them and to him, for his devotion to the interests of science, and for the unwearied kindness, and judgment and skill with which he has assisted me in this undertaking, I consider it my privilege to offer my thanks, as the means of accomplishing a task which might otherwise have proved to be impracticable. To J. W. SMITH, Esq. of Philadelphia, I am indebted for much valuable information regarding the state of practical navigation in America, of part of which I have availed myself in illustrating the subject. Whatever is still wanting to complete this investigation, I hope, in the course of a few years, if I enjoy life so long, to be able to accomplish; but, in the mean time, one series of the observations has appeared sufficiently entire to be presented by itself, and I have been induced to give them publicity, by the kind recommendation of Professor WHEWELL, who has taken a generous interest in their progress, by which I have been encouraged to pursue the investigation through the many annoyances and disappointments and dangers that necessarily accompany an undertaking of this nature.

PART I.

General Observations on the Phenomena that accompany the Motion of a Floating Body on the surface of a Quiescent Fluid.

Every instance of the want of perfect agreement between the predictions of theoretical mechanics and the results of practical experience, may be traced almost invariably to the existence of certain latent conditions that have been omitted from the fundamental hypotheses. Discrepancies of this nature are sufficiently numerous in the subject of hydrodynamics; so much so, indeed, that in relation to it, the appellations "practical" and "theoretical" are continually applied as terms of antithesis. The various hypothetical constitutions assumed for fluids by NEWTON, BERNOULLI, EULER, D'ALEMBERT, and their followers, have enabled them to obtain one law regarding the resistance of fluids to the motion of solids, which accords very closely with the phenomena of certain solids in certain circumstances and at certain velocities, but that law has not been found adequate to the solution of the case of a solid partly immersed, as when a floating body moves along the surface of a quiescent fluid. This law, which connects the resistance of the fluid with the second power of the velocity, is in very close accordance with the motion of bodies that are wholly immersed, and with the motion of floating bodies that have certain velocities and are placed in certain circumstances; but it has proved widely erroneous in its direct application to the motion of floating bodies in different circumstances and at higher velocities. So far, indeed, does the resistance actually obtained in these cases differ from the theoretical resistance, that examples may be found in every large collection of experiments, and are to be met with in almost every page of those which I have given at the end of this paper, where the resistance, instead of following the law of the squares of the velocities directly, has been found to vary, not only with every different power of the velocities from the first to the fourth power, but also in the inverse ratio of some of those powers. In addition to the examples at the end of this paper, I may here adduce two very obvious illustrations, the one shewing an increase of resistance corresponding to a very high power of the velocities, and the other exhibiting a diminution of resistance with an increase of velocity greater than the former. The experiments were made on the 18th of October 1834 with the floating body, whose form is given in Plate III. Fig. 4, having a mass of 12,579 lbs. All the circumstances attending the performance of the two experiments were alike, and the last column shews the comparative resistances as obtained by a dynamometer.

Example I.

	Space Described.	Time.	Velocity in Feet.	Resistance in lbs.
Experiment I.	1000 feet	$117^{s}.5$	8.51	233.
Experiment II.	1000 feet	93.5	10.69	425.

Example II.

	Space Described.	Time.	Velocity in Feet.	Resistance in lbs.
Experiment III.	2640 feet	$302^{s}.$	8.76	261.
Experiment IV.	500 feet	35.	14.28	251.

In the first of these examples, the velocities being in the ratio nearly of 85. to 106., the resistances are nearly as the third powers of the velocities; and in the second case, the velocity being increased from about 5.9 miles an hour to 9.6 miles an hour, the resistance is found to diminish in a ratio of 26.1 to 25.1.

To the imperfection of this branch of science, I may also adduce the testimony of two eminent individuals, to whose exertions we owe much that is now being accomplished for its improvement. In the Essay towards an approximation to a Map of Cotidal Lines, in the Philosophical Transactions of 1833, Professor Whewell remarks, that " the phenomena of waves, the motion of water in tubes and canals, in rivers, the motion of winds, and the resistance of fluids to bodies in motion, are all cases in which we are yet far from having drawn our analytical mechanics into a coincidence with experiment, or even a close approximation to it;" and Mr Challis has made the same admission at the end of his Report on the state of the Theory of Hydrodynamics, made for the British Association, where he says, that his " review may serve to shew that this department of science is in an extremely imperfect state, and that possibly it may on that account be the more likely to receive improvement;" and he adds, that " a singular fact relating to the resistance of bodies partly immersed in water has been observed, viz. that a boat drawn on a canal with a velocity of more than four or five miles an hour, rises perceptibly out of the water, making the resistance less than if no such effect took place;" and he further observes, that " theory, although it has never predicted any thing of this nature, now that the fact is proposed for solution, will probably soon be able to account for it on known mechanical principles."

To observe with accuracy the conditions of some of these discrepant phenomena, and reduce to the dominion of known laws certain anomalous facts, so as to obtain a closer approximation to a correct system of theoretical and practical hydrodynamics in those points in which they have hitherto been furthest apart, has been the object of this series of investigations. If the reasoning I have used in the sequel follow accurately from the experiments I have adduced, it will be shewn that there have hitherto been neglected in the calculation from theory of the resistance of fluids to the motion of floating bodies, two important elements of that resistance which affect low velocities by very small quantities, and have therefore escaped observation until certain practical results gave their effects more prominent importance; and that these two elements are, (1.) An emersion of the floating body developing itself as a function of the velocity of the motion, and of the measure of gravitation; and, (2.) The generation of waves by the motion of the floating body which are propagated in the fluid, and which affect the form of the surface of the fluid, the position of the floating body, and the resistance.

It appears to me probable that I shall most readily and simply communicate to others the information I have acquired on this subject, by following the order in which I was myself led to the acquisition of it. My examination was first of all directed to the effects produced by motion upon the floating body itself, and afterwards to the motions of the particles of the fluid in which the body is moved.

SECTION I.—*The Effect produced by Motion on the Immersion of a Floating Body.*

It has been suggested as an explanation of the cases in which the motion of a floating body is observed to be facilitated at high velocities, that the moving power by drawing a vessel partially out of the water, so as to diminish its immersion, may lessen the sectional area of resistance of the solid; and further, that if the moving force be supposed to be applied to the anterior part of a vessel, so as to elevate the prow above the surface of the fluid, the diminished immersion of that part would sufficiently account for the diminished resistance. These suggestions are not confirmed by observation. The amount of force required to produce the said effect by either of these methods, is found to be more than equivalent to the diminution of resistance produced by such force, and it has been observed on the contrary, as will be apparent in the sequel, that great and marked facilitation of the motion is observed when the line of effect of the moving force has a downward, instead of an upward, direction; and that any elevation of the prow or anterior part of a vessel, instead of facilitating its motion, increases the resistance to it.

To determine the real condition of the immersion of the floating body at various velocities, and trace the phenomena to some known mechanical principle, was the object of the first series of my experiments in 1834. For this purpose there was constructed an experimental skiff, a very light vessel of a very small draft of water, and furnished with apparatus for determining resistance and immersion. The skiff and its apparatus are described and delineated in that part of this paper which contains the details of the experiments of 1834. Chronometers, dynamometers, and two modifications of PITOT'S tube were observed. Twelve openings in the bottom of the vessel allowed the water to rise in glass-tubes carefully graduated, to the level of the fluid without; and furnished measures of the statical and dynamical immersion of the floating body. The vessel thus furnished, was made the subject of careful experiment at velocities of from 3 to 20 miles an hour.

These experiments give a decided and consistent result. It was found that in every case the statical immersion of the floating body was less than its dynamical immersion. The following are taken from the experiments of 1834, given in Part II. The statical immersion being 2.7 inches, the dynamical immersions observed at given velocities in miles an hour were as follows—

Velocity,	0.	3.016,	4.00,	5.165,	6.431,	7.253,	8.11,	9.164,	10.237,	20. +
Immer.	2.7,	2.6	2.5	2.2	1.9	1.8	2.2	2.3	2.0	1.5

After having determined the existence of a dynamical emersion, I endeavoured to discover the law of connection between the diminished immersion and the velocity of the motion. A singular change in the immersion at the velocity 8.11, and those immediately following it, gave me much trouble in my attempts to do this. I at first imagined the experiments might have been erroneous, but obtained the same results on each repetition. It afterwards turned out that these very anomalies shewed the continuity of the law; for it will soon be apparent in following out the subject, that at that very velocity of 8.11, the fluid undergoes a very extraordinary change in its form, which increases the immersion at the middle of the floating body when these immersions had been observed, and diminishes it at other parts of the body. Leaving, therefore, indications 8.11, and those which succeed it, to have the reductions made upon them, which subsequent investigations render necessary, and taking those below that point, and far above it, we may now proceed to examine whether any known principle will lead us to assign a law accordant with these phenomena.

Extensive series of experiments with the tube of Pitot, conducted by the most eminent experimentalists, and confirmed by the accordance of collateral phenomena, have established the doctrine as an axiom in hydrodynamics, That the resistance of a small unit of surface to a fluid, when either the fluid is in motion, or the surface itself, is equal to the statical pressure of a column of fluid having for its height the height due by gravity to that velocity. Had not this been satisfactorily established by previous experiments, and universally received as an unquestionable truth, my own experiments with the tube of Pitot would have been sufficient to shew the truth of the doctrine, which is merely the converse of the theorem, That the statical pressure of a column of fluid generates a velocity in the effluent jet equal to that which is required by a heavy body falling freely by gravity through a height equal to the depth of the fluid. This statical quantity being the measure of the pressure of the fluid upon the anterior surface of the immersed solid, will also be the measure of the quâquaversus pressure of the fluid in every direction, and therefore will measure the pressure of the water upon the vessel causing its emersion. Opposed to this we have the downward pressure arising from the gravity of the solid. Now, the measure of this pressure is the weight of the column of water displaced by the body, the depth of which is equal to the depth of the statical immersion of the solid, and each of these pressures is at every velocity equal to the other, and in the opposite direction to it. Whence,

Let $s =$ Transverse section of Statical Immersion.
$v =$ Velocity of Motion.
$g =$ Measure of Gravitation.
$s' =$ Section of Dynamical Immersion.
$\therefore vs =$ Volume of Fluid displaced by Statical Section; and
$vs' =$ Volume of Fluid displaced by Dynamical Section; and
$\frac{v^2}{2g} =$ Height due to the velocity v.

If ρ be the density of the fluid,

$$s'v\rho = sv\rho - \frac{v^2\rho}{2g}$$

$$\therefore s'v = s\left(v - \frac{v^2}{2g}\right) \text{ and}$$

$$s' = s\left\{1 - \frac{v}{2g}\right\}$$

Proceeding from this equation of the dynamical section, to determine the variation of total resistance, on the condition of proportionality to the law of the squares of the velocities, as regards that portion of the section of the solid which remains immersed, from the general equation

$$R = s v^2 \frac{\rho}{2g}$$

we deduce in this case of diminished section by substitution,

$$R' = s v^2 \left\{ 1 - \frac{v}{2g} \right\} \frac{\rho}{2g}$$

of which the successive differential equations in regard to v are,

$$\frac{dR'}{dv} = \left\{ 2v - \frac{3v^2}{2g} \right\} \frac{\rho}{2g} \quad . \quad . \quad . \quad . \quad (1.)$$

$$\frac{d^2R'}{dv^2} = \left\{ 2 - \frac{3v}{g} \right\} \frac{\rho}{2g} \quad . \quad . \quad . \quad . \quad (2.)$$

$$\frac{d^3R'}{dv^3} = \left\{ \quad - \frac{3}{g} \right\} \frac{\rho}{2g} \quad . \quad . \quad . \quad . \quad (3.)$$

From eq. (1.) if we make

$$\frac{dR'}{dv} = \left\{ 2v - \frac{3v^2}{2g} \right\} \frac{\rho}{2g} = 0,$$

we obtain, in the case of a maximum or minimum,

$$2 - \frac{3v}{2g} = 0 \quad \text{and} \quad v = \frac{4g}{3}.$$

By substituting this value in eq. (2.) we get

$$\frac{dR}{dv} = \left\{ 2 - \frac{8}{2} \right\} \frac{\rho}{2g},$$

being a negative quantity, whence it follows, that

$$R' \text{ is a maximum, when } v = \frac{4g}{3};$$

$$s' = 0, \quad \text{———} \quad \text{when } v = 2g.$$

These expressions may be converted into the following laws.

Laws of Dynamical Emersion and Diminished Resistance.

1. If a floating body be put in motion with a given velocity, the pressure which it exerts downwards upon the fluid in virtue of gravity, is diminished by a quantity equal to the pressure of a column of the fluid having the height due to the velocity of the motion.
2. The Section of Dynamical Immersion is less than the Section of Dynamical Emersion, in the same proportion in which the difference between the velocity of the motion and the height due to it is less than the velocity of the floating body.
3. The Resistance being taken in the ratio of the square of the velocity upon that part of the section only which remains immersed, the aggregate resistance will increase in the ratio of the squares of the velocities, very nearly

only at low velocities, and at higher velocities it will increase very slowly, and will even diminish as the velocity is increased.

4. The Resistance increases very slowly from about 25 to 29 miles an hour, at which point the velocity being $\frac{4}{3}$ of that which is the measure of the force of gravity for a given point of the earth's surface, or about 43 feet per second, and 29 miles an hour; the resistance has attained a maximum, and rapidly decreases, and continues to do so.
5. At 43.8 miles an hour (when $v = 2g$), the floating body emerges wholly from the fluid, and skims its surface.

It should be observed, that the phenomena corresponding with these results will be modified when the depth of the fluid is small, by the wave and other elements of resistance, upon the consideration of which we are to enter in another part of this paper.

It is also to be observed, that the form of the floating body is no element in the formula of emersion—that the law is a general one. This caution is the more necessary, because Mr CHALLIS has given a formula of emersion for a sphere, derived from the summation of all the elementary forces acting upwardly upon the sphere, which are obtained from resolving the oblique forces on each point of the sphere into co-ordinates of vertical and horizontal action. The particular case treated by Mr CHALLIS, although true for a sphere, does not apply to an elongated body, so as to diminish its emersion, but merely changes its position, and in such a manner as to increase, instead of diminishing, the resistance of the fluid. The effect he refers to is a great evil incident to a certain form of vessel, which otherwise possesses considerable advantages. The law of Diminished Resistance and Immersion which I have developed, is perfectly general in its application, and wholly independent of casual form. It has for its foundation merely the simple principle, That gravity, acting on a solid body during a given unit of time, is a constant quantity, and that the displacement of the fluid by the weight of the body, being a quantity that increases both with the velocity and the quantity of that displacement, must ultimately be equal in quantity, as it is opposite in direction, to the pressure of the solid downwards by gravity.

SECTION II.—*On the Motions that are communicated to the Particles of a Fluid by the Motion of a Floating Body.*

Many of the attempts which have been made to verify by experiment, or to discover empirically, the laws of the motion of floating bodies, have been defeated by the untoward circumstance of the *disturbance* which is caused by the protrusion of a solid into the space occupied by the fluid. The particles which are thus displaced are thrown aside by the anterior part of the body, and then collapse upon other parts of it; or they are thrown forward before the body, which is afterwards protruded on them a second time; or they are thrown up in heaps in certain forms of equilibrium, and the accumulations and irregularities of pressure which are thus occasioned give rise to currents of the fluid and mutual collisions amongst divided masses of it, and surges and other phenomena, all of which entering at once as elements of the resistance into the production of the resulting phenomena, do so modify them, as to give results that are totally inconsistent with theory, and are apparently at variance with each other, That theory, therefore, which will venture to assign the measure of the resistance of a fluid to a solid, upon the supposition that the surface of the fluid remains horizontal, and that the anterior part of the solid finds the surface of the liquid a level plane, will proceed upon imperfect data.

The only one of these disturbing causes which has hitherto been investigated in theories of hydrodynamics, is the lateral current proceeding from the stem towards the stern of the moving body.

The elements which I have added to those previously investigated are the Anterior, Posterior, and Central Waves.

I was first led to investigate the disturbances produced by the entrance of a floating solid into a quiescent fluid, by encountering a series of anomalous irregularities in my attempts to measure the immersion and resistance of the floating body at different velocities. There were certain velocities at which the body appeared to be almost buried in the water, and was so much impeded, that any force employed to accelerate the velocity of the body seemed only to accumulate resistance upon it, while at other velocities greater or less than these, the body would suddenly change its position, and instantly emerge out of the trough of the fluid to a considerable height above its statical elevation. But what happened in one portion of fluid did not occur in a different portion of the same fluid even at the same velocity. The resistance would sometimes exceed the third power, and again in another portion of fluid fall below the first power, for the very same velocity. These were disruptions of the law of continuity which the gradual law of an emersion as a function of velocity and gravity alone could not solve. I therefore entered upon a series of inquiries, directed solely to the subject of discovering and determining the unknown constituents of the disturbance of the equilibrium of the fluid occasioned by the presence of the floating body. The results of my inquiries I am now to state as briefly as clearness will allow, premising, at the same time, that the facts which presented themselves appeared at first to myself as extraordinary as they may now probably seem to those who may learn them for the first time; but

I may add, that, in the same degree in which they appeared wonderful to me at first, do they now appear to me the necessary and most satisfactory results of elementary and axiomatic principles.

In directing my attention to the phenomena of the motion communicated to a fluid by the floating body, I early observed one very singular and beautiful phenomenon, which is so important, that I shall describe minutely the aspect under which it first presented itself. I happened to be engaged in observing the motion of a vessel at a high velocity, when it was suddenly stopped, and a violent and tumultuous agitation among the little undulations which the vessel had formed around it, attracted my notice. The water in various masses was observed gathering in a heap of a well-defined form around the centre of the length of the vessel. This accumulated mass, raising at last a pointed crest, began to rush forward with considerable velocity towards the prow of the boat, and then passed away before it altogether, and retaining its form, appeared to roll forward alone along the surface of the quiescent fluid, a *large, solitary, progressive wave.* I immediately left the vessel, and attempted to follow this wave on foot, but finding its motion too rapid, I got instantly on horseback and overtook it in a few minutes, when I found it pursuing its solitary path with a uniform velocity along the surface of the fluid. After having followed it for more than a mile, I found it subside gradually, until at length it was lost among the windings of the channel. This phenomenon I observed again and again as often as the vessel, after having been put in rapid motion, was suddenly stopped; and the accompanying circumstances of the phenomenon were so uniform, and some consequences of its existence so obvious and important, that I was induced to make *The Wave* the subject of numerous experiments.

It very soon began to appear probable, that the existence of this phenomenon of *the solitary wave would exercise very great influence on the quantity and nature of the resistance of the fluid to a body moving with a given velocity, according as that velocity was equal to, or greater, or less than, the velocity of the wave.* And on making this the subject of a series of experimenta crucis, the correctness of the anticipation was established, and it appeared that *the velocity of the motion of the solitary wave had a peculiar relation to a certain well-defined point of transition in the resistance of the fluid.*

In prosecuting the inquiries to which this discovery gave rise, I found that, *in every instance of progressive motion of a solid in a fluid, the displaced fluid generated waves of the fluid that were sent in the direction of the motion of the body, and propagated with a constant velocity, which was quite independent of the velocity of the motion of the body, and that the magnitude, disposition, and velocity of these waves formed very important elements in the resistance of the fluid to the floating body.* I therefore directed my investigations to the discovery of the law of the genesis and motion of such waves, and the nature of their interference with the resistance of the fluid.

SECTION III.—*On the Laws which Regulate the Genesis and Propagation of* The Progressive Wave *which is created by the Motion of a Floating Body.*

It is very necessary that *The Wave* be carefully distinguished from certain elevations on the surface of a fluid which may likewise be included under the generic title of Wave, as observers who do not make this discrimination will be led into great confusion. I have observed at least four species of Wave,—the Ripple or Dentate Wave,—the Oscillatory Wave,—the Surge Wave,—and "The Wave" "par excellence," the solitary, progressive, great wave of equilibrium of the fluid. In regard also to the vessel, I have observed several waves. The Great Primary Wave of Displacement,—the Secondary Wave of Unequal Displacement,—the Great Posterior Wave of Replacement,—and the Secondary Waves of Replacement. It is the Great Primary Wave of Displacement which alone belongs to the species of the wave which I am now to examine.

The wave has been generated in two ways. By the addition of a solid to a limited portion of quiescent fluid, and by the addition of a given quantity of fluid. A loaded vessel being suddenly drawn with considerable force towards the mouth of a narrow channel, sends forward the displaced water into it in the form of the wave. A vessel being in the course of its motion made to vary suddenly, either made to move more rapidly or more slowly, or suddenly stopped, will send forward a sensible wave; and at all times, in smooth water, when moving with a velocity less than that of the wave, there will be perceived a series of waves preceding the vessel. If, also, there be made, by means of a sluice or otherwise, a sudden and considerable addition to the waters of a limited channel, the elevation will be transferred along the surface in the form of the wave.

It was found that the mode of the genesis of the wave, whether by a large or small vessel, by a long or short vessel, by a sharp or obtuse vessel, by a deep or a shallow vessel, whether by the addition of a quantity of water in one manner or in another manner, that the mode of genesis did not in any way, except in magnitude, as a great or small wave, produce any modification of form or velocity in the resulting wave. It was remarkable, also, that the velocity of the motion of the generating body did not in any way affect the velocity of the resulting wave, a wave, for example, of 8 miles an hour being produced alike from bodies moved at the rate of 2, 5, 6, and 12 miles an hour.

A very simple and early observation convinced me that the velocity of the propagation of the wave was owing chiefly to the depth of the fluid. After having propagated a given wave that had a velocity of 8 miles an hour, it was traced to a point at which the channel became deeper, and here its velocity was suddenly accelerated. The channel was also constructed as to become alternately narrower and wider, but no sensible effect was produced by the change; and when the wave once more reached that part of the channel which was of the original depth, its velocity returned to the original quantity.

Another observation equally simple served to shew that a large or high wave had a greater velocity than a small one. When a small wave preceded a large one, the latter invariably overtook the other, and when the large wave was before the less, their mutual distance invariably became greater.

In channels of rectangular section, the velocity was found by numerous experiments not to differ sensibly from that which is acquired by a heavy body in falling freely by gravity through a space equal to half the depth of the fluid.

In channels of variable depth in the transverse section, the velocity was found to be diminished below that which was due to the maximum depth, and to be equal to the mean of the velocities due to the differential depths.

The experiments on the magnitude of the wave shewed, that the velocity of larger, that is, of higher waves, appears to be greater than that of smaller ones, nearly in the ratio which is obtained by supposing the depth of the channel to be increased by a quantity equal to the height of the wave above the level of the surface of the quiescent fluid.

Experiments on the age and history of the wave, that is, upon the time which has elapsed, and the distance which has been travelled, and the route which has been described by it from the time and place of generation to the time and place of observation, shew that, after having traversed spaces from 100 to 2500 feet long in a sinuous channel, the wave remains unchanged in form and in velocity.

As the full investigation of the laws of the Genesis and Propagation of Waves forms a very extensive subject, in which I am at present engaged as a separate investigation, I have not loaded this paper with such observations as belong more properly to *that* subject. But I have given in this paper those examples which have peculiar reference to those experiments on resistance which I have now occasion to discuss in connection with the wave of the channels in which they were made.—(See Parts II. and III.)

The experiments on resistance were made in a channel 5.5 feet deep in the middle, but of irregularly diminishing depth towards the sides. The velocity of the wave in these experiments is about 8 miles an hour, being from 11 to 12 feet per second, varying with the height of each wave according to the law already given.

Very small waves, whose height does not exceed 0.1 of the depth of the quiescent fluid, are considerably retarded below the velocity due to the length, and move slower than the larger waves in a less depth.

The following extracts from the tables of a separate series of investigations directed exclusively to the examination of the laws of the wave, will serve to shew the degree of correspondence of the phenomena with the law already mentioned, and the connection between the velocity of the wave and the depth of the fluid, the fourth column being formed by adding to the first the mean of the second and third.

THE WAVE.

In a rectangular channel 13 inches wide, 75 feet long.

Depth of the Fluid at Rest in inches.	Heights of the Wave above the level of the Fluid in inches.		Total Depth reckoned from the top of the Wave.	Time for 70 feet. s.	Velocity in Feet per sec.
3.25	1.2	0.6	4.15	23.0	3.04
4.0	1.3	0.8	5.1	21.5	3.26
4.5	1.0	0.5	5.25	20.5	3.47
5.5	1.5	1.3	6.9	18.	3.9
6.25	2.5	1.5	8.25	16.5	4.49
6.25	3.5	2.5	9.25	15.5	4.52
9.0	2.3	1.0	10.65	14.5	4.82
9.0	3.0	2.5	11.75	14.0	5.00
9.0	3.5	2.3	11.90	13.5	5.19
9.5	1.0	0.6	10.3	14.5	4.82
9.5	2.5	1.2	11.3	14.0	5.00
13.0	1.0	0.5	13.75	14.0	5.00
13.0	2.0	1.1	14.55	13.0	5.38
13.0	3.0	1.4	15.2	12.0	5.83
37.0	9.0	5.0	44.0	*	10.598
66.0	4.0	4.0	70.0	†	14.087
66.0	6.0	6.0	71.0	†	14.284
66.0	9.0	9.0	75.0	†	14.727

SECTION IV.—*On the Form which is given to the Surface of a Fluid by the Motion of a Floating Body.*

It is only in a state of perfect rest that the surface of a limited reservoir of liquid can be considered as a horizontal plane. The displacement of any portion of that fluid deranges the equilibrium of all the particles in the vicinity of the disturbing cause, and it is only after the lapse of a considerable interval of time, and by means of an extensive series of interchanges of motion and position that the equilibrium is readjusted and the horizontal plane restored.

When a floating body is made to pass from one point in a fluid to another, it communicates motion to all the particles in the vicinity of its path. Such particles of the fluid as lie directly in that path are removed from it by immediate contact; these impart motion to those upon which they are protruded, and the

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* This experiment was in a channel 12.3 feet wide.

† These three examples were in a channel 12.3 feet wide.

agitation is thus extended to particles remote from the body. In certain cases motion is thus communicated to particles before the body, so that when it reaches them it neither finds them in a state of rest nor terminated by a horizontal plane. This change of form must constitute an important element in the resistance experienced by the floating body.

The form which a fluid assumes when disturbed by a body moving with a velocity less than that of the wave, is very different from that which it takes when the velocity of the body is greater than that of the wave.

The phenomena attending velocities less than that of the wave, which are most general and important, are the *Great Anterior Wave of Displacement,* the *Posterior Wave of Replacement,* and the *Lateral Current.* The secondary wave of excessive displacement and the secondary wave of replacement, are phenomena of a peculiar and accidental nature, resulting from the form of the disturbing body.

The great anterior wave of displacement is produced by the translation of the fluid from the path of the solid—the mass of displaced fluid forms an elevation towards the anterior parts of the vessel, which is propagated continually forwards in the direction of the motion in the form of the wave, and with the velocity due to half the depth of the fluid. This anterior accumulation is constantly maintained by the continual displacement of the moving body, and forms a smooth well defined wave, extending many feet forward from the bow of the vessel, and across the whole width of the channel. The rounded summit of this wave is placed at low velocities considerably anterior to the stem of the vessel. At low velocities also the wave is small, but the wave increases with the increase of the velocity of the vessel, and at the same time the vessel is brought forward towards the highest part of the wave.

The lateral current of the fluid around the vessel from the stem towards the stern is a phenomenon that always accompanies the anterior wave. The elevation of the fluid anterior to the solid by its introduction into the space occupied by the anterior fluid, and the removal of the posterior part of the solid from the space previously occupied by it, form an elevation and depression, of which the inequality of the pressure determines a current with a given velocity in a direction opposite to that of the motion of the solid.

The great posterior wave of replacement is totally different in the nature of its generation and the law of its propagation from the anterior wave of displacement, and ought not in any way to be confounded with it. It is of the nature of an oscillatory wave, and frequently degenerates into a surge or breaking wave. It is formed in the following way: The motion of the solid having sent forward the particles of the fluid before it in the form of the anterior wave, there remains, when the posterior part of the body is withdrawn from a given part of the channel, a vacancy and corresponding depression of the surface of the fluid,—into this vacancy two currents are determined in opposite directions, the lateral current from the stem towards the stern sent backwards by the pressure of the anterior fluid, encounters near the stern a current in the opposite direction, sent forward by the pressure of that portion of the fluid behind the vessel which has regained its original altitude. The collision of these opposite currents generates around the point where they unite an accumulation of fluid, which performs a series of successive oscillations, until the equilibrium of the fluid under a horizontal plane is at last restored. At each velocity this posterior wave maintains a constant position in regard to the stern of the vessel. The velocity of this wave is equal to that of the vessel, but its position varies with the velocity, approaching nearer to the middle of the vessel at the slower velocities, and falling further behind as the velocity increases, so as to be frequently at a considerable distance behind the stern of the vessel.

While the velocity of the floating body continues to be small, the stem surge may be recognised in a gentle short undulation following in the wake of the vessel at the stern or near it, and followed at short intervals by a series of smaller waves of the same species. With an increase of velocity the crest of the surge rises in a sharper line, elevated to a greater height above the surrounding fluid, until it forms, at an increased distance, behind the stern, a high crested breaker, which foams and dashes along after the vessel with a loud roaring noise, tearing up the sides of the channel.

The form given to a fluid in which the velocity of the wave was found to be $8\frac{1}{4}$ miles nearly, is represented in the sections below, which represent the phenomena as observed at velocities of 4, 6, and $7\frac{3}{4}$ miles an hour, and compared with the fluid in a state of rest.

Fig. 3.
(at rest.)

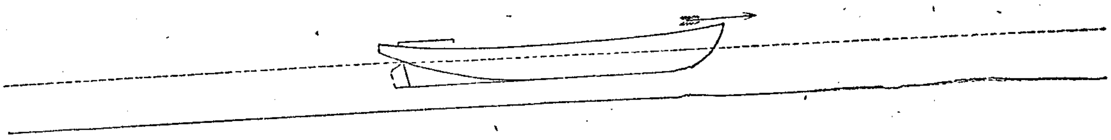


Fig. 4.
(at 4 miles an hour.)

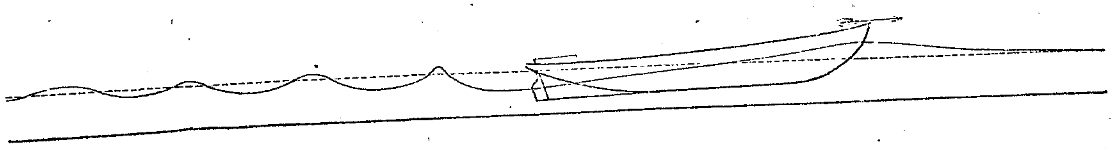


Fig. 5.
(at 6 miles an hour.)

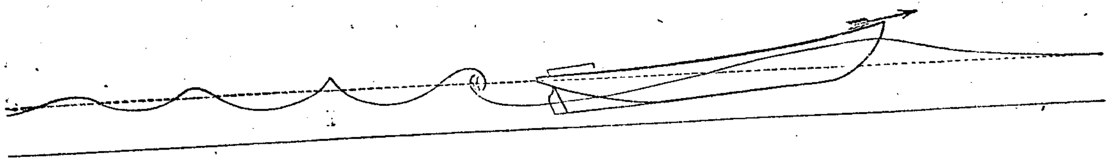
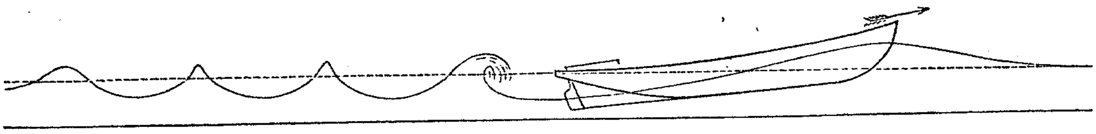


Fig. 6.
(at 7¾ miles an hour.)



The form given to the fluid in a channel about 8 feet deep, having a wave of the velocity of 10 to 11 miles an hour, is given in Plate II. fig. 1, the velocity of the vessel being about 7 miles an hour.

When the velocity of the solid is *greater than* the velocity of the wave of the fluid, the nature of the motion communicated to the fluid is totally different from that which is given to it by a lower velocity. The anterior wave no longer presses forward before the vessel, but the prow enters water that is smooth and undisturbed. The displaced fluid does not now accumulate at the prow, but is left on either side, forming a lateral elevation of fluid, which has the effect of increasing the depth of the fluid around the sides of the vessel, and forming a wave, on the summit of which the vessel may be poised in a position of equilibrium. This is in fact the wave formed by the displaced fluid, but moving with a less velocity than the vessel, and therefore posterior to the prow of the vessel instead of anterior to it, as in the former case, where the velocity of the vessel was less than that of the wave. It constitutes, therefore, a great central wave of displacement.

At velocities greater than that of the wave the stem surge has now disappeared. The wave of displaced fluid, instead of being sent forward, was to leave a vacancy in that part of the channel from which it was displaced, remains heaped up on the sides of the vessel until it has passed, and then collapses into the space which it had previously filled. The channel is therefore merely rendered fuller for the time being than it had formerly been.

Since, therefore, it appears that the form of the fluid is changed by the protrusion of a solid floating upon its surface, and is no longer bounded by horizontal plane; since, also, the form of the fluid is different when the velocity is less than that of the wave of the fluid, from its form when induced by a velocity in the solid greater than that of the wave; since, also, the mode of displacement and replacement are different, it may be expected that the law of resistance will exhibit a very important change at the point of transition. This will form the subject of the ensuing section.

SECTION V.—*On the Nature of the Increased Resistance experienced at Velocities less than that of the Wave.*

From the great change that is effected on the form of the fluid by the motion of a floating body with a velocity less than that of the wave, it is now very obvious that a vessel placed behind the wave, is in circumstances exceedingly different from the hypothetical condition of being drawn in a horizontal position along the surface of a level quiescent fluid. The prow of the vessel is pressed into the anterior wave, the stern is depressed into the hollow of the wave, the keel is inclined upwards in the direction of motion, at an angle amounting in some cases to 20°, an additional surface of horizontal displacement is presented, which increases as the sine of the angle of elevation of the keel. On attempting still farther to accelerate the velocity of the vessel in the vicinity of that of the wave, the variations which are thus produced in the condition of the vessel increase still further the causes of these variations, the increased immersion of the bow in the wave, augments the anterior wave formed by the displacement of the fluid, and the enlarged oblique surface now presented in the bottom of the vessel presses forward with increased velocity the wave on the slope of which it is elevated, and increases the elevation of that slope, becoming more depressed also at the stern, and giving rise to more rapid currents, and a higher stern surge. In short, it appears, that increased force applied gradually to the vessel for the purpose of rendering the velocity of the body equal to, or greater than that of the wave, has the effect at the same time of increasing at a more rapid rate the retarding forces, and a limit is soon reached, which it has in many cases been found impossible to pass. It is the circumstance of the very rapid increase of the resistance in approximating to the velocity of the wave, that has led to the false idea that there is a final and low limit to velocity on shallow water. There are circumstances in which this limit is final, the channel being very shallow, and the boat very bluff in its formation, I have seen in such an extreme case, when the depth of the channel was about five feet, the channel laid bare in the stern hollow behind the wave, so that the stern of the vessel no longer floated but rested on the bottom, while the bow was elevated and buried in a large anterior wave, rising more than two feet above the level, and overflowing the banks, and the posterior wave rushed on furiously behind, roaring and foaming, tearing up the banks of the channel, and threatening the destruction of the vessel, which, indeed, on stopping, it nearly accomplished. In such a case the persons in the vessel were not visible from the shore, being sunk in the hollow between the great anterior and posterior waves.

Any increase of velocity behind the wave is therefore accompanied by the following

Elements of Increased Resistance.

1. Increased Immersion of the bow in the anterior wave.
2. Inclination of the longitudinal axis of the floating body, so as to change the form of the displacing body.
3. Increased vertical section opposed to resistance ÷ the sin of the inclination.
4. Increased velocity of the lateral current.

The following Table, extracted from the experiments of 1835, will serve to shew the rapid increase of resistance which is experienced in approaching the velocity of the wave, which in these cases was 8 miles an hour.

Example I.

Velocity in Miles.	Resistance in Pounds.
5.05	52.25
5.45	78.5
5.68	82.5
6.49	111.0
6.81	125.0
7.57	255.0
7.5 to 8	330.0

8 miles an hour = Wave's vel.

Example II.

Velocity in Miles.	Resistance in Pounds.
5.05	95
5.45	100.5
6.19	152.0
6.49	312.0
6.81	386.0
6.81 to 7	392.0

8 miles an hour = vel. Wave.

The following examples will shew the very slight increase of velocity in the vicinity of the wave, even when the increments of force are considerable. (See Experiments XLIII. and XXXIX., 1835.)

Space. Feet.	Time. Secs.	Force. lb.
100	10	124.7
100	10	127.5
100	10	150.5
100	10	157.5
100	10	197.7
100	10	207.0

Space. Feet.	Time. Secs.	Force. lb.
100	9.5	172.2
100	9.25	200
100	9.25	212.2
100	9.0	227.7
100	9.0	239.7

Velocity of the Wave being 100 feet in 8.5 seconds nearly:

SECTION VI.—*On the Nature of the Diminished Resistance which is experienced at Velocities greater than that of the Wave.*

Having now understood the manner in which a floating body moving behind the anterior wave deranges the equilibrium, and alters the form of the fluid, so as to cause a rapid accumulation of the elements of excessive resistance, it will be readily perceived that the annihilation of these elements, which takes place at velocities greater than that of the wave, will prevent the continued increase of the resistance derived from them, and it will also appear that the new arrangement of the particles of the displaced fluid renders the wave an element of diminished resistance. By how much, in fact, the wave was at a lower velocity, a + element of resistance will it now act as a — element of resistance.

Let it now be supposed that the vessel had created by its motion an anterior wave, and let it be supposed possible to lift the vessel entirely out of the water, and place its centre on the top of the wave, the stem being anterior to the wave, and the stern behind it, and suppose the vessel to be of such a form as to remain in a position of stable equilibrium on the surface of a fluid having the form of the wave, and suppose such a velocity to be given to the vessel as to keep it in the same relative position to the wave, then the following results would be obtained.

(1.) The vessel would be permitted to recover the horizontal position, and would present the minimum transverse section of resistance.

(2.) The immersion of the vessel being increased by the height of the crest of the wave around its centre of gravity, the anterior and stern displacements would be diminished, the total immersion being a constant quantity, by the amount of excessive central displacement.

(3.) The velocity of the vessel being now increased beyond that of the wave, the waves of displaced fluid falling continually behind the points where they were raised, would form a continued series of great central waves, bearing the vessel up upon their summit.

Such are precisely the circumstances of a vessel moving with a velocity greater than that of the wave, as shewn in section in the following illustration.

Fig. 7.—Behind the Wave.

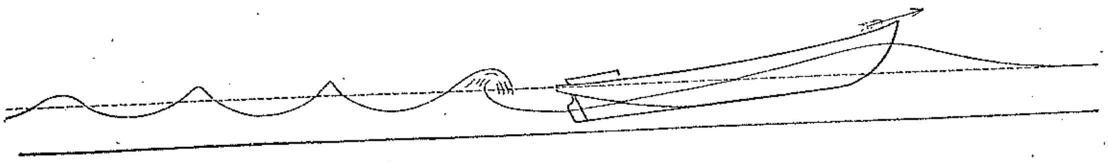
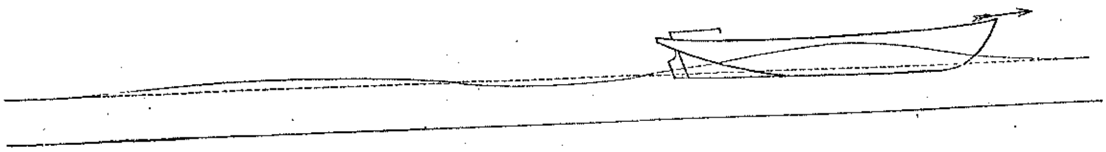


Fig. 8.—Upon the Wave.



But it will be inquired, how is a vessel to be placed in such circumstances? How is the extreme resistance of the anterior wave to be vanquished, and the vessel planted on its summit? This is admitted to be a practical problem, often of extreme difficulty; sometimes it is impracticable. There are some forms of vessel that do not admit of a position of stable equilibrium on the top of a wave. Still, however, it is a practical problem practically solved every day on all canals navigated on the Scotch system. Vessels of a greater length than the wave, having a fine entrance, built of light materials, and drawn by well trained highly bred horses, and guided by experienced postillions, are raised by a sudden and powerful jerk to the top of the wave (at from 6 to 8 miles an hour), and are drawn along on the summit of the wave with greater ease at 10 and 12 miles an hour, than at 6 or 7. (See Section IX.)

The progression of the resistance from 0, up to a velocity greater than that of the wave, follows therefore a very intelligible order. Suppose the velocity of the wave to be about 8 miles an hour, at the lower velocities of 2 and 3 miles an hour, the resistances bear to one another nearly the ratio of the squares of the velocities. But with the increase of velocity the excess of the resistance above that due to the velocity also increases, and nearly in the inverse ratio of the difference between the velocity of the vessel and the velocity of the wave, so that the ratio compounded of these two ratios accumulates very rapidly to a very high limit in the vicinity of the wave, which limit may in certain cases be infinity, but where it is not infinite, the resistance will suddenly diminish to a less quantity than the slower velocities under the wave, and will only increase in a ratio which will be less than that of the square of the velocity from two causes, from the diminished immersion due to the velocity (as in Sec. I.), and from the diminished anterior immersion explained in this section as the effect of the central wave; the resistance will then obtain a maximum and minimum as given in Sec. I.

The following experiment made with a simple dynamometer, giving only round numbers, will shew the manner in which horse-power may be exerted at velocities greater and less than the wave, and the exertion required to place the vessel on the wave. The velocity of the wave being 8 miles an hour, and the weight of the vessel and its load = 12,579 lbs. Two horses were used.

	Space. Feet.	Time. Secs.	Resistance. lb.	Velocity. In miles an hour.
Behind the Wave.	100	11.5	180	5.92
	200	11.0	200	6.19
	300	11.0	250	6.19
	400	10.0	300	6.81
	500	9.0	300	7.57
	600	9.0	350	7.57
	700	9.0	400	7.57
	800	9.0	500	7.57
Upon the Wave.	900	8.0	400	8.52
	1000	7.5	300	9.04
	1100	7.0	270	9.04
	1200	7.0	280	9.04
	1300	7.0	270	9.04
	1400	7.0	280	9.04
	1500	7.0	270	9.04

Although this experiment does not give accurate measures of force due to various velocities, it shews simply what was intended, the manner in which the force of horses is exerted to "overcome the wave" (as it is called). The following Table is made up of very correct experiments, continued through considerable spaces, upon the same basin of fluid, and with the vessel which is named the Raith in Part III., the weight of the vessel and load being = 10,239 lbs. 17th October 1834.

		Space described.	Time.	Velocity in Feet per second.	Velocity in Miles per hour.	Moving Force.
		Feet.	Secs.			Lbs.
Behind the Wave,	Experiment I.	2640	387	6.8	4.72	112
	Experiment II.	2640	302.5	8.6	5.92	261
	Experiment III.	2640	295.5	8.9	6.19	275
Upon the Wave,	Experiment IV.	1000	74.0	13.5	9.04	250
	Experiment V.	1000	65.0	15.3	10.48	268.5

The resistance here is greater at 6 miles an hour behind the wave, than at 9 miles an hour upon it; and the resistance at $10\frac{2}{5}$ miles, is little more than at $5\frac{9}{10}$ miles an hour.

It is easy to see how the wave influences the resistance in the cases where the vessel has been raised upon it, and is drawn along at precisely the same velocity; but it is perhaps not quite so clear at first sight what are the phenomena which accompany velocities that are greater than that due to the wave, because, in that case, the vessel would leave the wave behind. But it should be observed, that a new wave is formed at every successive instant by the motion of the vessel through the water, whatever be the velocity of its motion; for the displaced fluid thrown aside at the bow, generates a series of waves, which move with a less velocity than the vessel, and fall back to a position behind the bow. The displaced fluid, which, in the case of motion with a less velocity than that of the wave, passed forward before the vessel, causing an extensive accumulation, cannot now pass forward with a velocity greater than that due to the depth and to the wave, and is therefore left behind, to fill up the vacuity which will remain when the stern of the vessel shall have passed on. The displaced fluid is therefore pushed aside by the bow of the vessel, and forms lateral accumulations on both sides of it, in the form of a continuous wave, upon the ridge of which the centre of the vessel is sustained in a position of station of stable equilibrium. The buoyant force of this ridge is the cause of the diminished anterior section of resistance.

It is always found that the commotion produced in the fluid is much greater at velocities less than the wave, than at velocities which are greater than it. The stem of the vessel, in the latter case, enters water which is perfectly smooth and undisturbed, because no wave has previously passed forward before the vessel to produce any anterior derangement; the water which is pushed aside by the bow of the vessel, forms a lateral accumulation proportioned to the increase of volume arising from the sudden entrance of the solid; and when the vessel has passed forward, the subsequent collapse of the lateral ridge restores the equilibrium. The disturbance of an anterior wave is thus rendered impossible, and the cause of the destructive stern surge is removed; for the displaced water remains to fill up that vacuity into which a stern surge would otherwise have been driven.

It is evident, therefore, that the nature of the motions communicated to a fluid at velocities greater than the velocity of the wave, are radically different in their nature from those of the less velocities. Lateral currents, breaking surges, can no longer exist. The fluid is simply divided by the entrance of the vessel, stands aside until it has passed, and gently subsides to the original level when the separating body has passed away.

The practical applications of these facts and phenomena are of great value in the navigation of canals and shallow rivers. (See Sec. VIII and IX.)

Tables of resistance at various velocities are given from seventeen different forms of the immersed portion of the floating body, at velocities from 3 to 15 miles an hour, in Parts II. and III.

SECTION VII.—*On a General Expression of the Law of Resistance of a given Solid in a given Limited Fluid.*

If the immersion of a floating body were like that of a solid wholly immersed in the fluid, a constant quantity, and if the surface of the fluid remained horizontal and plane, and if the particles of the fluid remained at rest until immediately acted on by the solid, and were the motion given by the solid to the displaced fluid horizontal without vertical excursions, then the usual simple expression,

$$R = \frac{v^2}{2g} \cdot m s \rho \quad . \quad . \quad . \quad . \quad (1.)$$

would represent the resistance R, being the weight of a column of fluid having the height due to the velocity v, g being the measure of gravity, s the anterior transverse section of the immersed part of the solid when at rest, m being a constant representing the modified resistance derived from the form of the anterior part of the solid, and ρ the density of the fluid, to which might also have been added a constant quantity for adhesion, but we have omitted it, for the sake of simplifying the expression.

If, however, we include the element of diminished immersion, we shall have by (Sec. I.)

$$R' = \frac{v^2}{2g} \cdot \rho \cdot ms \left(1 - \frac{v}{2g}\right) \quad . \quad . \quad . \quad . \quad (2.)$$

If we now include the element of change in the position of the body, and of increased anterior immersion when behind the wave, we shall have, by using θ for the angle of elevation of the axis of the solid, δ for the difference of the anterior section or height of the wave forming on the solid, modified by the constant n, for the form of that part of the solid, and measured at a given unit of velocity in relation w the velocity due to the wave, we shall have

$$R'' = \frac{v^2}{2g} \cdot \rho \left\{ ms \left(1 - \frac{v}{2g}\right) \cdot (1 + \sin\theta) + \frac{n\delta v}{w - v} \right\} \quad . \quad . \quad . \quad (3.)$$

When the velocity is less than that of the wave, the quantity $\frac{n\delta v}{w-v}$ is positive, w being greater than v, and the effect of the wave is then to increase the resistance; as v increases, $w - v$ diminishes, still remaining positive. If the sides of the channel and of the vessel were infinitely high, and the increments of force uniform and very slow, the phenomena would give the case represented, when

$$\frac{n\delta v}{w - v} = \frac{n\delta v}{0} = \infty,$$

the resistance being infinitely great; and when the velocity v becomes greater than that of the wave w, $\sin\theta$ being $= 0$, the expression $\frac{n\delta v}{w - v}$ becomes negative, its denominator having become negative, and the expression is reduced to

$$R''' = \frac{v^2}{2g} \rho \cdot \left\{ ms \left(1 - \frac{v}{2g}\right) - \frac{n\delta v}{v - w} \right\} \quad . \quad . \quad . \quad (4.)$$

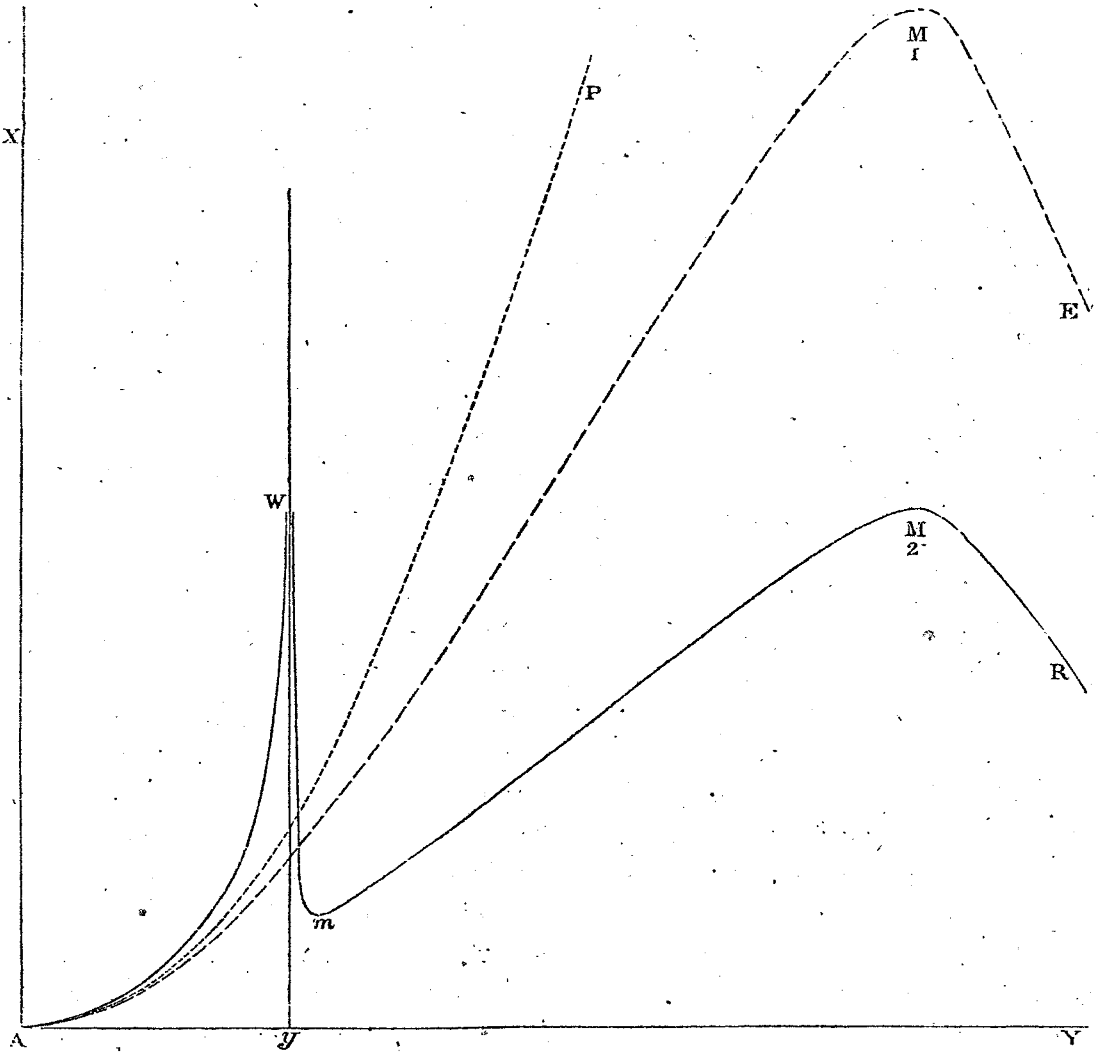
the expression of the case when the velocity is greater than that of the wave.

The line of resistance AP corresponding to Eq. (1.), is a parabola, AX being the axis of the parabola, and AY the tangent of the vertex, the velocities being represented by the ordinates parallel to AY, and the resistances being represented by the abscissæ parallel to AX; A being the origin.—(See Fig. 9.)

The line of resistance AM_2E corresponding to Eq. (2.), has all its abscissæ less than those of the former curve, and a point of maximum when $v = \frac{4}{3}g$, and of minimum when $v = 2g$.—(See Fig. 9.)

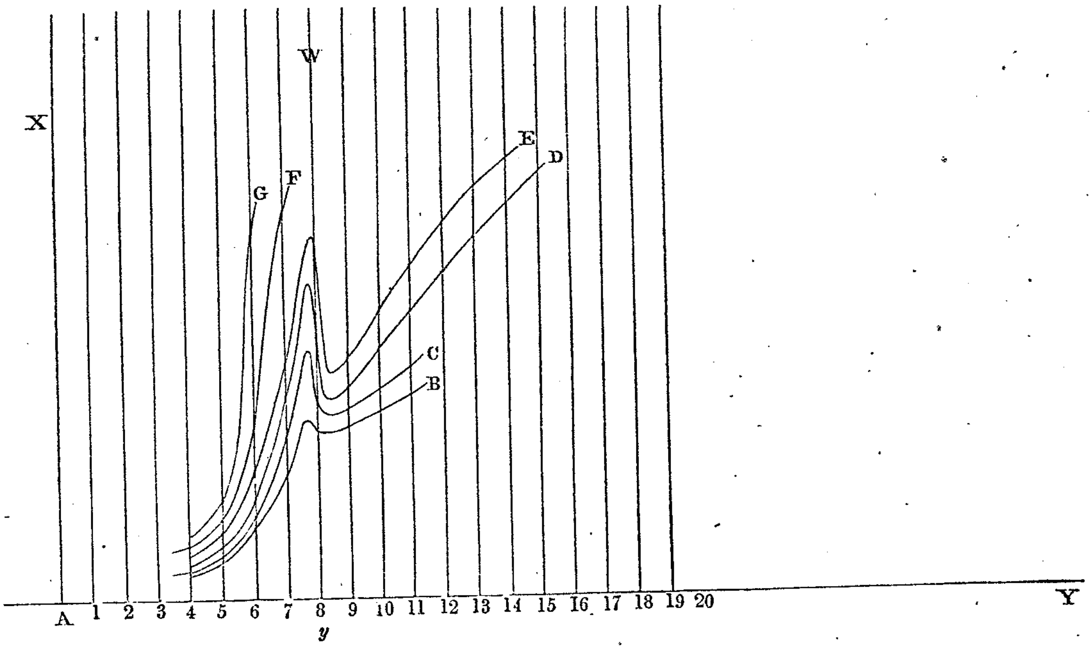
The line of resistance (AWm_1M_2R) corresponding to Eq. (3.), lies above the parabola, when $\frac{n\delta v}{w - v}$ is greater than $ms \left(\frac{v}{2g} - \sin \theta\right)$, becomes infinite when $v = w$ and falls below the parabola, when the velocity becomes greater than that of the wave, or when $v < w$.—(See Fig. 9.)

Fig. 9.



The lines of resistance derived from the experiments in Part III. are given in Fig. 10;—velocities being measured on AY, and resistances being taken parallel to AX, the lines of resistance AB, AC, AD, AE, AF, and AG, being formed from the Analysis of the Experiments of 1835, in pp. 100 and 101.

Fig. 10.



Section VIII.—*Practical Applications and Illustrations of the Law of the Wave in the Navigation of Rivers and Shallow Water.*

Experienced river boatmen are well aware of a fact which receives its explanation from Sect. III., that every large vessel moving with considerable velocity sends forward through the water an intimation of its approach while it is yet at a considerable distance, so much as several miles. This intimation consists of a wave or series of waves, propagated in the fluid, with a greater velocity than that of the vessel. A vessel that has ceased its motion, or suddenly varied its velocity, will send forward a very large and defined wave with the velocity due to the depth of the canal, and quite independent of its own velocity. These waves arrive at the place to which the vessel is moving long before she reaches it, and sensibly increase the depth of the water in the channel. I have in this way observed on the River Clyde the approach of a large steam-vessel, while yet at the distance of 2½ miles, the motion of the wave finding a beautiful index in the oscillations of the tall masts of vessels at anchor as it passed them in successive tiers. I have frequently been surprised by the appearance of such an indication, when I had no reason to suspect the approach of a vessel, and have invariably found it followed by the unexpected vessel. A distant storm in the ocean frequently gives a similar announcement, the waves moving with a velocity of 50 or 60 miles an hour, breaking in a heavy ground-swell upon a remote beach.

The effect of the formation of waves with a greater velocity than that of the vessel in forming an anterior accumulation, a posterior depression, and a stern surge, as shewn in Sects. IV.—VI., furnishes a satisfactory explanation of the phenomena of shallow navigation.

It is well known that it is extremely difficult to row or sail well in shallow water; this is the consequence of increased section of resistance from being behind the wave. But if by strong impulse the vessel were placed on the wave, it would then become easier than in deeper water at the same velocity. In water two feet deep, it is very difficult to row 4 and 5 miles an hour, and comparatively easy at six, the one less, the other greater, than the velocity of the wave. It is also found that in shallow water the stern of the vessel invariably takes the ground while the bow remains free, although drawing at least equal depth of water; this is the direct result of the depression between the anterior wave and the stern surge, as in Sects. IV. & V.

The difference of the immersion of a vessel below the surface of the fluid when on the summit of its wave, or in the depression behind it, Sect. IV., accounts satisfactorily for a long list of phenomena otherwise inexplicable. It has long been observed, that a vessel in motion will take the ground in water that is more than sufficient to float her when at rest, and it is equally well known that there are circumstances in which a vessel when in motion, will pass over a shallow without touching the ground, while it is not covered with the depth of water necessary for her statical floatation. Now, it is obvious, in the first instance, that if the vessel move with a velocity less than that of the wave, the prow being on the anterior wave, and the stern in the succeeding depression, the vessel will take the ground, and probably carry away her helm; while, if the vessel were poised on the summit of the wave in a horizontal position of equilibrium, and with the diminished immersion due to the velocity, much less depth of water would be sufficient, than with the slower motion, or even than is required to float the vessel in a state of rest. I have seen a vessel in five feet water, and drawing only two feet, take the ground in the hollow of a wave, having a velocity of about 8 miles an hour, whereas at 9 miles an hour, the keel was not within four feet of the bottom.

A highly scientific friend of mine, Mr Smith of Philadelphia, member of the Franklin Institute, has frequently observed in the Dutch canals, boats carrying passengers, kept in floatation by communicating to them rapid motion in shallow parts of canals, where they would otherwise have taken the ground, thus taking the advantage of moving on the summit of the wave.

I have also been informed, on the best authority, of the following fact, observed by a gentleman, who was surprised by the phenomenon, although unable to account for it, " The steam-boat Trenton, on the Delaware, in the United States, by passing over shallow portions with a high velocity, carries with it a body of water sufficient to float her over portions on which she would not have been floated if at rest." The body of water was of course the wave, the velocity of the vessel being above 13 miles an hour.

The navigation of the River Clyde presents excellent examples of the effect of the motion of the wave, and affords ample opportunity for the application of the principles developed in the preceding portions of the paper.

The wealth of an enterprising commercial community has enabled engineers to convert one of the worst rivers for navigation into a good, although as yet only shallow, channel. When the tide leaves the river, there are not more than six or seven feet of water in many parts of the channel, the wave having a velocity of about 9 miles an hour. Any observer looking attentively from the deck of a vessel on the sloping bank of the river, will see delineated there very distinctly the anterior wave preceding the bow of the vessel, the posterior depression, called by sailors " the suction of the vessel," and the stern surge, as delineated in Plate II. Fig.1, rushing along the bank with fury into the vacuity. It is invariably necessary for vessels of considerable size to lower their velocity very much in such places, to prevent their grounding in the depression of the wave. When two vessels pass each other this effect is much more sensible, as at the instant when the waves coincide, their elevation is equal to the sum of both, and when the depressions coincide, the hollow is equal to the sum of both; hence, although both may have floated previously, at the instant of passing either may take the ground, or both. It has on this account been found necessary to enact, that at low-water vessels before passing each other shall diminish their velocity. But if a velocity greater than the wave, such as 11 or 12 miles an hour, could be attained, there would no longer be any danger of grounding, or of lessening the speed, and thus the navigation be materially improved. Another very curious fact I have also observed on the Clyde, namely, that a vessel of greater power and velocity passing one of less power and velocity, will take the less powerful along with her in the depression, the more rapid sending forward the wave before the bow of the slower vessel, so as to obviate the immersion of the bow, and give her the impetus of the stern surge. It is further evident, that in a river so shallow as the Clyde, velocities of 7, 8, or 9 miles an hour behind the wave are very disadvantageous; whereas, if a vessel could be started over the wave, her progress would be so greatly facilitated, as to enable her with the same force to reach velocities of 12 or 13 miles an hour, when the stern surge would cease, and the danger of taking the ground cease along with it.

In shallow rivers, where the water is in motion, very singular phenomena result from the wave, from which it will be apparent, that a given velocity against the stream may, in certain circumstances, require less force to produce it, than the same velocity in the direction of the stream. Thus I have seen the current moved at the rate of about 1 mile an hour, and the wave about 4 miles an hour, on the surface of the water; when the velocity of the vessel, drawn against the stream, was 4 miles an hour in regard to the land, it was before the wave with diminished resistance, and when it was drawn with the stream also at the rate of 4 miles an hour, the vessel being then behind the wave, experienced the direct resistance arising from that cause, the velocity of the wave in regard to the land being in the one case 3, and in the other 5 miles an hour. Analogous phenomena to this, of a very curious nature, are to be recognised in the motion of a wave against a current. I have seen a wave move in the opposite direction to a stream, until it reached a rapid in which there existed a part of the stream where the current had a velocity equal to that of the wave, and in the opposite direction, and there, in consequence of the equality of velocities in opposite directions, I have seen the wave come to rest, and retaining its form unchanged, remain as a stationary heap of fluid, until, by the adhesion of the successive portions of water, it was at last rendered insensible. From these remarks it will be apparent, that the navigation of rivers may, in certain cases, be much facilitated by the action of the wave.

SECTION IX.—*Applications and Illustrations of the Law of the Wave in the Practical Navigation of Canals.*

Canal navigation furnishes at once the most interesting illustrations of the interference of the wave, and most important opportunities for the application of its principles to an improved system of practice.

It is to the diminished anterior section of displacement, produced by raising a vessel with a sudden impulse to the summit of the progressive wave, that a very great improvement recently introduced into Canal transports owes its existence. As far as I am able to learn, the isolated fact was discovered accidentally on the Glasgow and Ardrossan Canal of small dimensions. A spirited horse in the boat of WILLIAM HOUSTON, Esq., one of the proprietors of the works, took fright and ran off, dragging the boat with it, and it was then observed, to Mr HOUSTON's astonishment, that the foaming stern surge which used to devastate the banks had ceased, and the vessel was carried on through water comparatively smooth, with a resistance very greatly diminished. Mr HOUSTON had the tact to perceive the mercantile value of this fact to the Canal Company with which he was connected, and devoted himself to introducing on that canal vessels moving with this high velocity. The result of this improvement was so valuable in a mercantile point of view, as to bring, from the conveyance of passengers at a high velocity, a large increase of revenue to the Canal Proprietors. The passengers and luggage are conveyed in light boats, about sixty feet long, and 6 feet wide, made of thin sheet-iron, and drawn by a pair of horses. The boat starts at a slow velocity behind the wave, and at a given signal it is by a sudden jerk of the horses drawn up on the top of the wave, where it moves with diminished resistance, at the rate of 7, 8, or 9 miles an hour.

It was a natural consequence of this successful mode of transport on this one canal, that it should be immediately attempted on others, and numerous experiments were accordingly made with varying results. In some canals, and with certain vessels, similar phenomena were observed, and the like favourable results obtained. But in others the experiment totally failed, as it was not found that the tumult of the water subsided as in former cases, or that the resistance experienced any similar diminution. The cause of these variations was not then known. Many experiments were made, which failed in eliciting any solution of the difficulty. Many scientific and practical men, unable to account for such discrepancies, satisfied themselves with an unqualified denial of their existence, while those who were eye-witnesses of the fact could not assign any satisfactory cause.

It will not be difficult for us to account for these discrepancies, by what we have brought to light regarding the law of wave. In the canal where the fact was originally observed, having a depth of 3 or 4 feet, and a wave moving at about 6 miles an hour, it is obvious that the resistance of the anterior wave would only be encountered at velocities less than that of the wave, and the diminished resistance would be obtained by moving upon the wave, at a velocity of more than 6 miles an hour. Now, in making the same attempt in canals that were 5 or 6 feet deep, with a wave moving at the rate of eight miles an hour, the resistance would not be observed to suffer any diminution, till the velocity exceeded that of the wave; but would accumulate rapidly up to that point. While in canals that had a depth of 8 or 9 feet, and a wave moving at eleven miles an hour, no diminution could be observed till a velocity above that of the wave had been obtained, after which, the advantage of diminished anterior section could be acquired. Since the discovery of the law of the wave, I have had the experiments tried in such cases, the wave being passed, and the boat carried along on its summit at the rate of thirteen miles an hour.

When once the summit of the wave is attained, or its velocity exceeded, a comparatively small force may sustain the motion. But the resistances increase so rapidly in the vicinity of the wave, that this may become impracticable. If the increase of the velocity up to that of the wave be very slow and continuous, the waves will be closely crowded, and deeply accumulated around the bow of the vessel, so that an additional force will only increase the magnitude of the wave, and thus adding to its velocity, prevent the vessel from penetrating through or rising upon it. What I have stated accords perfectly with the experiments I have given, and with the experience of practical men. In these experiments it will be seen, that immediately behind the wave large increments of force are not accompanied with similar increase of velocity, while at the instant of passing the wave, the velocity makes with a given force a sudden transition to a higher velocity. And so in experience it is found very difficult, or quite impracticable, to pass the wave with a slowly accelerating motion. A sudden impulse from a low to a high velocity is found to be the easiest mode of effecting the change, and the method used is not to make the change immediately from a very high velocity behind the wave, to a very high velocity before it; but when it is intended to start a vessel over the wave, the speed must first be allowed to diminish, until it become nearly half of that of the wave, by which means the anterior wave is allowed to pass away with its proper velocity from before the bow of the boat, the stern surge is permitted to overtake it, and fill up the cavity behind the wave, and the surface of the water is reduced more nearly to a plane; and if now, in these circumstances, a sudden impulse be communicated to the vessel, it will easily attain a velocity greater than that of the wave.

A change in the depth of a canal produces a very marked change in the resistance in the vicinity of the wave. Certain portions of the Glasgow and Ardrossan Canal have their depth suddenly increased, and when a vessel that has been moving on the summit of the wave reaches these points, it finds its velocity diminished, in consequence of the wave having acquired a greater rapidity due to the increased depth.

The wind acting on the surface of a long canal has a force sufficient to send away so much of the fluid from one of its extremities, and accumulate it towards the other, that in a canal running about twenty-five miles in a direction east and west, a strong westerly wind will occasion a difference in depth of two feet, being at the east end one foot more, and at the west end one foot less than five feet, the average depth of the canal. It is observed in this case, that to maintain the vessel over the wave, a greater force is required at the deeper end, and a lessened force at the other.

In canals where the power of horses is applied to vessels navigating at high velocities, much inconvenience will be experienced, and much loss incurred, by giving to the water a depth which will produce a wave of so high a velocity, as to approach the limit of the available speed of horses. When the depth exceeds seven or eight feet, the struggle to conquer the wave will take place at or above nine miles an hour, being a velocity at which horses cannot advantageously exert much force above what is required for the transport of their own bodies;

and in such a case, in order to prevent any irregularity in the application of the force from permitting a wave to pass on before the vessel, the velocity will require to be maintained at twelve or thirteen miles an hour. Now, when the depth is so much less as to comprise the velocity of the wave within the limits of moderate exertion on the part of the horse, the higher velocities are gained without injury to the animal, and a rate of nine or ten miles an hour is maintained with certainty.

Two or three years ago, it happened that a large canal in England was closed against general trade by want of water, drought having reduced the depth from twelve to five feet. It was now found that the motion of the light boats was rendered more easy than before; the cause is obvious. The velocity of the wave was so much reduced by the diminished depth, that instead of remaining behind the wave, the vessels rode on its summit. I am also informed by Mr Smith of Philadelphia, that he remembers the circumstance of having travelled on the Pennsylvanian canal in 1833, when one of the levels was not fully supplied with water, the works having been recently executed, and not being yet perfectly finished. This canal was intended for five feet of water, but near Silversford the depth did not exceed two feet, and Mr Smith distinctly recollects having observed to his astonishment, that, on entering this portion, the vessel ceased to ground at the stern, and was drawn along with much greater apparent ease than on the deeper portions of the canal.

In a canal where the velocity due to the wave is nearly coincident with that velocity of transport which is found to be most desirable for the species of traffic, (for example, ten or eleven miles an hour, as has been the case recently on the Forth and Clyde Canal, whose maximum depth is about nine feet), in such a case this velocity is either impracticable or very disadvantageous, giving rise to a constant struggle with the wave. To solve the problem, however, the following mode has been found efficient: one mile is performed at the rate of eight miles an hour, being so far behind the wave as to suffer little from its accumulation on the prow, and at the end of that mile the boat is brought to the bank where the canal is shallow, and by starting the horses to a gallop of 13 or 14 miles an hour for another mile, being in advance of the wave, and this process being continued in alternate miles, a mean velocity of ten and a-half or eleven miles is attained in the transport, at a resistance whose mean is less than the resistance of the mean of the two velocities intermediate.

In every canal there must be two velocities, at which principally the transport is conducted, one sufficiently far behind the wave to render its interference inconsiderable, and another sufficiently in advance to give security against its passing in small changes of moving power; at a velocity one-half of that of the wave, and at another one-fourth part greater than the said velocity, both of these objects will be attained.

When a canal is to be constructed for a given kind of transport, such a depth ought to be selected as will admit of those velocities above and below the wave, which are required for the trade of the canal, the velocity of the wave being as far removed as possible from the velocities below it and above it.

When vessels only of a small draught of water are required for the trade, the canal should be as shallow as possible, and when larger vessels are desirable, the depth should be increased as much as possible, so as to remove the wave to a distance beyond the velocity of the motion of the vessels, and prevent anterior accumulation.

The breadth of the canal materially affects the resistance produced by the wave, although it does not directly affect its velocity. By preventing the diffusion of the wave, the narrowness of the canal increases the height of it, in consequence of which the resistance to the lower velocities is augmented, and facilitation in the higher velocities increased. But in general the depth is of much more consequence than the breadth of the canal, as the retardation or facilitation produced by the vicinity of the wave, is a quantity which may be made to bear an almost infinite ratio to the other elements of resistance.

For slow velocities alone, a broad and deep canal, but especially deep, should be made; and for high velocities, a narrow and shallow one, especially shallow, that the range of velocities may be extensive, and the velocity at which the wave is to overcome small.

There are also certain relations to be observed between the velocity of the wave, and the dimensions of the vessels of easiest transport, also between the form of the vessel and that of the wave; but this is an inquiry which I have not yet completed, but hope soon to terminate successfully. Relations have been distinctly indicated, but not accurately defined.

It is perhaps worthy of remark, that a vessel on the summit of the wave is more easily directed by the helm, than when behind it. In the latter case, the vessel by her anterior immersion is prevented from answering the helm, while in the former case, this obstruction being diminished, and the displaced fluid collected around the centre of gravity, horizontal rotation on the vertical axis passing through the centre of gravity is less resisted by the fluid than formerly, in proportion as the third power of their present distance from the particles of the wave is less than the third power of their former distance from the centre of rotation.

Another circumstance still more curious than the foregoing is, that at the instant of passing one another at high velocities, vessels are much more delicately poised than at any other time; the waves coinciding form a wave equal to their sum, on which the centres of gravity receive an additional elevation.

It appears from the experiments of 1835, that a vessel has conveyed on a canal given weights with the following forces:—

Moving Force.	Weight Moved.	Velocity.
71.5 lbs.	19,222 lbs.	4 miles an hour.
86	19,222	4.5
112.7	19,222	5.2
243	8,022	11.3
264	19,262	13.6
331	10,262	15.1

The examples are taken from the experiments made with the vessel named "The Wave," which was constructed according to the form which I have assigned as the solid of least resistance.

PART II.

THE EXPERIMENTS OF 1834.

The experiments of 1834 were directed chiefly to the determination of the velocity of the wave, the emersion due to the velocity, and the amount of animal force required to overcome the resistance of the water at various velocities. The experiments of 1835 were the result of the experience of 1834, in consequence of which a vessel of a peculiar form had been constructed, and a mode of estimating the absolute and comparative resistance of the fluid at various velocities, with different vessels, and at several degrees of immersion, had been obtained, giving results more accurate, more uniform, and more worthy of confidence than those of the former year.

On the Velocity of the Wave.—The Progressive Wave, which forms the subject of these experiments, differs entirely in its nature and laws from the small undulations or oscillations of a fluid which are occasioned by the sudden elevation or depression of a small portion of the fluid, in which case we have a series of successive small undulations and depressions succeeding each other at nearly equal intervals. The progressive wave, sent forward by a floating body in rapid motion, is not necessarily preceded nor followed either by a depression, or an elevation, or any series of such depressions or elevations, but is a single elevation, of a well defined form, moving with an uniform velocity along the surface of the fluid; the forms of the fluid vary, but maintain an obvious relation to one another; they are of the same family of waves, or may be resolved into compounds of the members of the same family. A few of those that have been carefully and frequently observed, are given in Plate I. figs. 2, 3, 3 and 4.

The three first examples appear to be simple examples of the trochoid, a curve that appears to comprehend all the elementary forms of the wave. Other forms which make their appearance, seem to be compounds of these, into which they may be resolved by a very simple analysis, as is done in the succeeding figures. When one portion of such a compound wave is higher than another, I have invariably observed the higher portion move more rapidly than the rest, and finally separate itself, leaving the rest behind, and assuming a definite elementary form. Figs. 5, 6, and 7, shew the outline and analysis of some compound waves, which afterwards resolved themselves into simple ones of the forms given in Figs. 2, 3, or 4.

The first series of experiments on the wave, were directed to the determination of the relation between its velocity and the form and dimensions of the channel. A sheltered situation and calm day were selected, so that the form of the waves might be sufficiently perfect to enable the observers to mark with precision the place of the summit of the wave. At the termination and the commencement of distances that had been accurately measured, graduated rods were placed in a vertical position, and careful observers, furnished with assistants and accurate chronometers, were stationed opposite to each of them. A wave was generated by giving rapid motion to a vessel, and then depriving it of motion at a given distance from station A; and at the instant of the coincidence of the summit of the wave with the rod at A, a signal was communicated by sound to station B, the time of the transit being recorded at A, and the time of the sound at B. The wave now passed on towards B, and at the instant of its arrival time was observed at B, and the time of the signal of arrival communicated to A was also registered by chronometer A. Thus, without calculating the velocity of sound or δ, the time of describing the space, or s, was determined; for

$$s - \delta = \text{difference of times at B.}$$
$$s + \delta = \text{difference of times at A.}$$
$$\frac{\therefore s + s + \delta - \delta}{2} = \text{true time corrected for the velocity of sound.}$$

The following observations were made at the experimental station at Hermiston, where also almost all the experiments on resistance were subsequently carried on.

Experimental Station.—Union Canal.

Breadth at top = 40 feet,
Breadth at bottom = 30 feet,
Maximum depth = 5.5 feet,
Clayey bottom.
See Fig. E, Pl. II.

Exper.		Secs.	
1.	Space = 1000 feet,	85	Vel. = 11.730; 7.8473 miles.
2.		85	
3.		85	
4.		86	
5.	Space = 700 feet,	61.5	Vel. = 11.352; 78473 miles.
6.		62	
7.		61.5	
8.		61.5	
9.		61.5	
10.		62	
11.	Space = 800 feet,	63	Vel. = 11.7713; 7.8359 miles.
12.		69	
13.		68	

Paisley and Ardrossan Canal.—Dumbreck Bridge.

Breadth at top = 23.27 feet,
Breadth at bottom = irregular,
Mean depth = 3.3 feet,
Muddy bottom.
See Fig. D, Pl. II.

Exper.		Secs.	
14.	Space = 556 feet,	61	Ve = 9.114; 6.0972 miles.
15.		61	
16.	Space = 820 feet,	90	Vel. = 9.111; 6.0952 miles.

Slateford Aqueduct.—Union Canal.

Breadth at top = 12.33 feet,
Breadth at bottom = 12.0 ...
Maximum depth = 5.6 ...
Smooth iron bottom.
See Fig. A, Pl. II.

Exper.		Secs.	
17.	Space = 486 feet,	34	Vel. = 14.352; 7.5944 miles.
18.		34	
19.		33	
20.		33	
21.		35	
22.		34	
23.		36	

Same station.

Depth diminished until = 3.4 feet.

Exper.		Secs.	
24.	Space = 150 feet,	14	Vel. = 10.593; 7.0867 miles.
25.		14	
26.		15	
27.		14	
28.		14	
29.		14	This experiment was made under the superintendence of Mr Ellis.
30.		14	
31.		14	
32.		14.5	

Glasgow and Ardrossan Canal.—Port Eglinton.

Breadth variable, with vertical sides.
Depth 5.5 feet.

Exper.		Secs.	
33.	Space = 501 feet,	40	Vel. = 17.431; 8.3163 miles.
34.		41	
35.		40	

Union Canal.—Tunnel.

Breadth at top = 17.75,
Breadth at bottom = 11.00,
Depth = 5.5 nearly
Rocky bottom, irregular.
See Figs. B & C, Pl. II.

Exper.		m s	
36.	Space = 2038 feet,	2 35	Vel. = 13.208; 8.8361 miles.
37.		2 35	
38.		2 35	
39.		2 35	
40.		2 33	
41.		2 33	

On the velocity of waves in regard to their height above the surface of the water, the following experiment was made.

Experimental Station.—Hermiston.

Exper		Height.	Secs.
42.	Space = 700 feet,	6 in.	61.5
43.		5 ...	61.75
44.		3.5 ...	62.5
45.		2 ...	63.5

The following series of experiments were made with the view of determining whether the velocity of the wave remained unchanged during the whole of its progress, or varied with the distance over which it had travelled. I may observe as a matter of some interest, that when the wave had to traverse 1000 feet before arriving at the first station of observation it had to encounter a change in the direction of the canal, equal to a curved deflexion of 90°; and where it passed over

2500 feet, it had been deflected through double that quantity. The spaces marked as the distances of generation are exclusive of the distance between the stations A and B = 700 feet.

Experimental Station.—Hermiston,

Space traversed by the wave from A to B = 700 feet.

Exper.		Height A. Inches.	Height B. Inches.	Time. Secs.	
46.	Wave generated close to A.	7	5	61.5	Mean Velocity = 11.359 feet per second = 7.59315 miles.
47.		6	5	61.5	
48.		6	5	61.5	
49.		5	5	62	
50.	Wave generated 500 feet from A,	6	4.5	62	Mean Velocity = 11.290 feet per second = 7.553010 miles.
51.		3	2	62	
52.	Wave generated 1000 feet from A,	3	2	62.5	Mean Velocity = 11.200 feet per second = 7.49280 miles.
53.		4	2	62.5	
54.		4	2	62.5	
55.		0	2	62.5	
56.	Wave 1500 feet from A,	2	2	63.5	Mean Vel. = 11.023 ft. per sec. = 7.37438 miles.
57.	Wave 2500 feet from A,	2	2	64.5	Mean Vel. = 10.852 ft. per sec. = 7.259988 m.

In these examples no particular velocity was employed for generating the wave. A vessel was put in pretty rapid motion by a couple of horses, over a space of about 500 feet, and was then suddenly stopped, so as to allow the water it had set in motion to move forward before the vessel in the form of a wave, and the velocity of the wave was then measured from a mark at a station of observation to that of another station whose distance was known. These examples which have been given comprehend the waves of a considerable variety of velocities of motion. The following observations were made with this view alone, of determining whether the velocity of the vessel had any influence on that of the wave, from which the influence appears to be insensible.

Exper.		Velocity of Boat. Miles per hour.	Time. Secs.
58.	Space 700 feet,	5	62
59.		3	61
60.		10	61
61.		7	62
62.		7	62
63.		4	61.5

From these experiments it appears that the velocity of the wave, is that acquired by a heavy body falling through a space equal to half the depth of the fluid, and that the velocity appears to vary with the magnitude of the wave very nearly in the ratio which is obtained by supposing the depth of the fluid increased by a quantity equal to the height of the wave, so that the variations of velocity in a given depth may be traced to the varying height of the wave, the mean height in these experiments having been three or four inches. When the depth of the cross-section of the channel varies, the velocity is nearly the mean of the velocities due to the depths. For the more perfect determination of the laws of the motion of waves, I have begun a series of experiments extending through a much more extensive range of dimensions; those made in 1834 having been almost exclusively made in reference alone to their connection with the law of resistance.

On Resistance and Immersion.—For the purpose of conducting the inquiries regarding the immersion of bodies moving at high velocities, and the resistance of the fluid at these velocities, an experimental vessel was constructed, a very light skiff, capable of containing four or six observers, with the apparatus of experiment. The "skiff" was constructed of iron plates, extremely thin, and only weighed 430·lbs. The length of the skiff was 31.25 feet, breadth 4 feet, and her figure as given in Plate III. This vessel has been drawn by a highly-bred horse, at a rate of more than 20 miles an hour. The skiff carried the following apparatus.

(1.) Two forms of the tube of PITOT P′ P″ P‴, Plate III. Fig. 5. P′ being the aperture exposed to the water, P′ P″ a long tube, separating into two branches communicating by stop-cocks with P″ P‴, two vertical glass tubes carrying graduated scales, one connected with the open tubes being graduated in inches and decimals, zero being at the level of the fluid, to be used for low velocities, and the other for higher velocities, graduating so as to indicate, by the compression of air in a ball on the top of the tube, the height to which the water would have been sustained in an open tube of unlimited length. The observations made with PITOT's tube do not in any respect vary from those given by others, and generally received as correct. The tubes of PITOT were only useful as giving an index of velocity of considerable extent, and giving variable indices of velocity cotemporaneous with the variations of moving force. The observations with the tubes served to confirm those of the chronometers as indices of velocity.

(2.) *Gauges of Immersion.*—Many modes have been attempted of determining whether the immersion of a floating body in motion be variable or constant. Rods have been applied vertically between the gunwale of the vessel and the surface of the water, but the change of form caused by the currents of the fluid and its waves have interfered with this method. Lines stretched above the vessel, so as to measure the distance between the summit of the vessel and the fixed string in the two states of motion and rest have indicated an elevation, but have not given the means of distinguishing whether this elevation consisted of the fluid rising along with the vessel, or the vessel emerging from the fluid, or both of these causes united or modifying each other. The method I have used is this: into apertures pierced in the bottom of the vessel glass-tubes, open at both ends, and graduated in decimals of an inch, were inserted, as T_1, T_2, T_3, T_4, T_5, T_6, Plate III. Fig. 5, into which the external fluid was pressed up to the level of the surface of the quiescent fluid. The action of these gauges was found very delicate, a slight variation in the position of the tube, or a trivial error in the formation of its aperture at the bottom, giving irregular results. The value of the indications of the tubes was determined by drawing the vessel in opposite directions, which immediately shewed which of the tubes were affected by errors of position:—those which were free from such errors were selected for observation, and their indications are given in the following experiments.

(3.) *Dynamometers.*—Much has been written on the subject of dynamometers, and much in praise of a species of that instrument, in which the minor oscillations of the moving force, or the variations of the resistance, are suppressed, and only some unknown function of these variations supposed to approximate to their sum, or rather their mean is exhibited, this effect being produced by the application of the well-known principle of the retardation of a fluid passed through a very small aperture. I made trial of a very simple dynamometer formed on this principle, which was very accurately constructed for me by Mr John Adie. A helical spring contained in a cylinder, was compressed by a piston, which communicated through the piston-rod with the moving power. The cylinder being closed was filled with oil, and a communication between opposite ends of the cylinder effected through an external tube, governed by a stopcock. The stopcock gave the means of retarding or facilitating the passage of the fluid, and enabled the observer to render the position of the index more or less stable, by turning the stopcock in such a way as to facilitate or retard the motion of the fluid in the variations of force. I had at first considerable faith in this species of instrument. It certainly accomplished the purpose of giving a stable instead of an oscillatory indication, an indication easily observed. But it may be questioned, whether it be really a desideratum to obtain indications which have not the variations of the subjects themselves that are to be measured. The indications of the instrument are in truth false, or at least they only shew what effect the action of a desultory force produces on the motion of the fluid of the instrument itself. In applying this instrument to the measurement of the resistance of fluids, when the resistance is by no means very desultory, it is most desirable that the variations of the power should be apparent, instead of being rendered latent. It is obvious that the force communicated by jolts to a body in motion, produces effects that are so widely different from those of uniform pressure, that the sum of the impulses due to a given velocity is very different in its effect from a uniform pressure equal to that sum.

The disadvantages of using a desultory power like that of horses in producing motion, to which the resistance is like that of a fluid continuous, are very great. The following examples at velocities almost precisely equal, made with the same bodies, and in the same circumstances, as indicated by the compensating dynamometers, will shew the comparative effects of a variation in the action of the power.

Example I.

	Force indicated.	
Trotting,	95 lb. 100 ... 108 ...	Mean = 101 lb.
	Same velocity.	
Cantering,	138 ... 152 ... 155 ...	Mean = 101 lb.

Example II.

	Force indicated.		
Trotting,	107 lb. 108 ... 108 ... 108 ... 108 ... 108 ...	67^{s}.	Space = 1000 feet.
Cantering,	130 ... 118 ... 130 ... 128 ... 128 ... 135 ...	66^{s}.	Space = 1000 feet.

The specimen of the dynamical effect of trotting which I have given in Example II., is the most perfect specimen I have ever been able to obtain, and was obtained by a very powerful well-bred, well-trained horse, which was ridden in a very superior manner. Out of an immense number of experiments made with horse-power, I have been able to obtain comparatively few in which the differences of the successive impulses were sufficiently small to admit of an arithmetical mean being used to represent a constant force. All the others were of course comparatively valueless, except as illustrative of the manner in which the power of horses was applied in overcoming the peculiar mode of resistance of the fluid.

Although, therefore, during 1834, I made a very great number of experiments on the resistance of various vessels, in various conditions of immersion, and at many different velocities, in which the direct power of horses was applied, and measured by the action of the dynamometer I have described as the fluid dynamometer, and with the ordinary dynamometer, I am now disposed to place little faith in those where the application of the force deviated widely from uniformity, especially when absolute measures of the resistances are required, or delicate comparisons instituted. For observations on which we may rely implicitly as measures of resistance, I refer with perfect confidence to the experiments of 1835, which were made with continuous power, and under the improved arrangements which the experience of 1834 had dictated. I give here, however, a set of experiments of 1834, which were obtained after long experience had enabled us to render the variations of our desultory power as small as possible, assigning to them that degree of value only which the approximation to uniformity may appear to entitle them.

Experiments of 1834, with the Skiff;

The motion being produced by the direct application of the power of horses.

Weight of the Skiff,	810 lb.
Weight of observers, ballast, &c.	430 ...
Total Weight moved,	1240 ...
Depth of immersion by the gauges at the centre P″ when at rest,	2.7 inches.
Length of the Skiff on the gunwale,	31.25 feet.
Length of the Skiff on the keel,	30.75 ...
Maximum breadth,	4.21 ...

For the form of the Skiff, see Plate III. Fig. 5.

The first column contains the space described during the experiment; the second column consists of the resistances as registered at the commencement of the space, at the end of the space, and at equidistant points of division; the third column consists of the time in which the space was described; the fourth the velocity in miles per hour; and the fifth contains the indications of the gauges of immersion.

Space. Feet.	Moving Force. Lbs.	Time. Secs.	Velocity. Miles per hour.	Immersion. Inches.
500	10 10 10	113	3.0168	2.6
500	17 18.5 17.5	85	4.0096	2.5
500	16 19 18.5 18.5	80	4.2613	2.4
1000	25 24 27 26 31 27 27	132	5.1652	2.2
500	35 45 35 40	60	5.6816	2.2
1000	45 48 45 53 55 55	116	5.9288	2.0

Space. Feet.	Moving Force. Lbs.	Time. Secs.	Velocity. Miles per hour.	Immersion. Inches.
500	55 55 65 65 65	53	6.4318	1.9
500	78 80 84 84	47	7.2534	1.8
1000	60 70 73 74 70 79	85	8.11	2.2
1000	85 87 87 87 87 87 85 87	75	9.164	2.3
1000	85 85 85 86 86 87 88	74	9.16	2.0
1000	107 108 108 108 108 108	67	10.237	2.0
1000	100 115 118 95 100 108	58	11.755	1.9
500	Not observed.	17	20	1.5

In the last experiment the vessel had been lightened, so as to draw only 2 inches, and retained only one individual, who executed the duties both of directing the vessel, and observing the gauges of immersion. It was not found practicable to observe the oscillations of a moving force so impetuous.

The experiments which have been given, afford the means of estimating the value of the following Table. To remove the appearance of affectation of superior accuracy, where the nature of the indications, and the subject of measurement, would not bear out the degree of apparent precision given by such figures, fractions of a pound have been omitted, as the experience of making these experiments has shewn me that errors of directing the vessel, and the resistance of the helm, must have affected the result to an amount greater than any such fraction. In general, I have endeavoured to render the apparent precision of number indicated a measure of the precision of the observation, having never allowed the indications to be read off with greater minuteness than the total of the probable error.

The following Table; formed from the Experiments of 1834, with the Skiff, regards only the resistance of the vessel. The vessel was steered within three feet of the line of motion of the horse, the line was sixty feet long, reducing the correction for the deviation of the line of traction from the line of the keel to a very small quantity; this correction has not been applied in the following table, which contains the experiments exactly as made. The first column gives the time of describing 500 feet; columns second and third giving the velocity; the fourth column contains the mean motive force applied at these velocities; the fifth column shews the number of feet described from which the observations have been drawn; and the sixth column is a table of squares of velocities, and affords the means of comparing the law of the squares with the real law of resistance. From the small immersion of the Skiff, her wave was small, and the velocity of a wave equal to that generated by her motion is given in the table as observed by experiment.

The Skiff.

Time to 500 Feet.	Velocity in Feet.	Miles per Hour.	Motive Power in lbs.	No. of Experiments made.	Ratio of Squares of Velocities.	Remarks.
s.	s					
113	4.42	3.0168	10.1	Five	9.1011	Weight of Skiff 3 cwt. 94 lbs.; load in Skiff 7 cwt. 26 lbs. Total 1240 lbs. Length of Skiff 31 feet 3 inches, gunwale 30 feet 3 inches keel; maximum breadth 4 feet 2½ inches.
85	5.88	4.0096	17.6	Three	16.095	
80	6.25	4.2613	18.6	Three	18.158	
66	7.58	5.1652	26.7	Seven	26.669	
64	7.81	5.3248	27.5	Four	28.353	
60	8.33	5.6816	39.0	Four	32.280	
59	8.45	5.7777	48.0	Two	33.382	
57.5	8.62	5.8789	50.2	Four	34.561	
58	8.69	5.9288	51.2	Six	35.151	
56.5	8.85	6.0335	55.	Nine	36.403	
53	9.43	6.4318	61.	Five	41.368	
48.5	10.30	7.0290	79.		49.407	Velocity of Wave 2 inches high, = 10.853 feet, = 7.3997 miles.
47	10.64	7.2534	82.5	Seven	52.612	
42	11.90	8.1168	82.5	Three	65.882	
40	12.50	8.5228	81.1	Nine	72.368	
37.2	13.44	9.1642	86.3	Twenty-one	83.982	
35	14.28	9.7403	89.2	Twelve	94.873	
33.3	15.01	10.2370	107.	Nine	107.86	
29	16.90	11.7555	111.	Six	138.19	

PART III.

The investigations of 1834 had established the principal points in the relation between the resistance of a fluid, the diminished immersion, and the velocity of the wave. The prominent features in the representation of the law had been traced, but the outline being in many parts faintly and ambiguously given, required to be retouched, corrected, and filled in. The power of horses, which had been used as the moving force, was desultory in its action, so that the measure of the force employed did not always afford the means of obtaining even a tolerable approximation to an accurate measure of resistance at a uniform velocity. Yet the power of horses had this advantage, that it could be continued for a much greater length of time, and over a much longer space, than that obtained by the action of a falling weight, or any other convenient mechanical means. For small models, indeed, it would have been sufficiently simple to provide, as has frequently been done, the means of applying a continuous moving force; but I was not, in 1834, in possession of any plan by which this object could be accomplished, so as to obtain a continuous moving power, acting through a great space, to generate high velocities in vessels of large size carrying considerable weights. In 1835, I had, however, attained this desideratum.

The means of obtaining the continuous action of a moving force with great power and through a great space, were very simply and conveniently obtained; and as the method may be useful to other inquirers, I shall on that account describe it more particularly than might perhaps be necessary for the mere purpose of appreciating the experiments conducted with it. The method which has been previously used for obtaining the power by means of a weight, has been by suspending that weight from an elevated structure by strands of rope passing over pulleys, by which a given weight, in falling through a given space, acts through one of the strands so as to move the end of the rope through a space greater than the space through which it falls by as much as the number of strands exceeds unity. In this case the weight to be raised, in order to obtain a given power, increases so rapidly with the increase of the space and the friction of the pulleys, and the effect of rigidity increases so rapidly along with it, that the limit of practicability, and, at all events of inconvenience, is very soon attained. Further, after one experiment has been obtained by an apparatus of this kind, considerable time must elapse before the weight is again elevated, and the rope drawn out to its former station for commencing another experiment. In the method which I have adopted, the weight never requires to exceed twice the moving force required, plus friction and rigidity, for five pulleys; the weight requires no increase for the space moved over, except for the friction of the additional horizontal rope on its supporters; and when one experiment has been completed, the arrangements are thereby made for instantly beginning the succeeding experiment. The mode was this: A pyramidal framing (see Plate III. fig. 6.), was raised to a height of 75 feet, formed by four logs of pine rising from the corners of a square of 45 feet, and firmly united at the apex, so as to give attachment to two fixed pulleys, and the structure was made rigid by an oblique framing of spars and ropes. This structure was placed on the bank of the sheet of water at the Experimental Station close to the Bridge of Hermiston, from which there extends a line of bank in a straight line of more than 1500 feet in length, which was accurately divided by painted rods into equal portions. Through the two pulleys (C) at the top of the framing were passed the two ends of the rope, and from the intermediate part of the rope, by means of a moveable pulley (D), was suspended the moving weight. The two ends of the rope that had been passed over the pulleys at the vertex of the pyramid were thus brought down to a point (B), raised 6 feet above the level of the water, where they were passed through two other pulleys fixed in the pile of masonry forming one of the piers of the bridge. This forms the whole of the apparatus for the application of the moving force.

The pyramid being placed at one end of the station, the vessel subjected to experiment was brought to the other end, and one end of the rope of the pyramid was brought along the whole length of the station and attached to (E) the bow of the vessel. Horses were attached (A) to the other end of the rope, which was cut short after leaving the pulleys fixed in the masonry. The horses now started, and having first tightened the rope, began to elevate the weight towards the top of the pyramid. But the other end of the rope fixed to the bow of the vessel had to sustain a tension in raising the weight equal to the part acted on by the horses, and, in consequence of this action, the vessel would have begun to move at the same time at which the horses began to raise the weight, but the vessel had been previously fixed by the stern-post to the bank, and thus a reaction was obtained to sustain the weight. When, however, the observers in the vessel observed the weight rise to a given height in the pyramid, they withdrew a small catch in the stern fastenings of the vessel, and she immediately proceeded towards the pyramid. In the mean time, however, the motion of the vessel allowed the weight to fall towards the ground, which it would have reached when the vessel had moved through a space equal to twice its original elevation, had the horses been allowed, after having raised the weight, to stand still; but as they were urged to a motion at their end of the rope with the same velocity which the vessel acquired at the other end of the rope, the weight was kept at rest in the air; or if the horses moved either a little slower or a little faster than the boat, the effect was merely to allow the weight to ascend or descend in the pyramid with a velocity equal to half the difference of the velocities of the horses and the vessel, and thus the difference of the action of the horses was not sensible in the force acting on the vessel. When the horses had arrived at that end of the station from which the vessel had commenced its motion, the vessel had reached the point from which they had started.

The apparatus was now ready for the succeeding experiment, one end of the rope being at the pyramid and the other at the starting-point. The vessel was immediately drawn back to the starting-point while the horses were returning to the pyramid, and being again attached to the extremities of the rope, the next experiment was begun.

It was found that considerable time elapsed before the vessel attained the uniform velocity due to the moving force, and therefore the vessel was put in motion through a considerable space previous to making the observations. Where this proved inconvenient, a very simple mode was used of attaining a higher velocity, which was by the attachment of an additional weight (F) by a rope about 50 feet long to the former weight in the pyramid, so that this weight should rest on the ground, unless the principal weight were raised to a height greater than 5 feet, in which case alone the additional weight would be called into action. By this means it was easy, on commencing the experiment, to keep the principal weight so high as to raise the additional weight to produce the required acceleration, and afterwards, before commencing the observations, to allow it, by resting on the ground, to cease from acting on the vessel. The velocity due to the moving force was thus attained in a shorter time than would otherwise have been necessary.

The observations were made in the vessel upon time and resistance, the rope through which the propelling force was applied being attached to the vessel by the hook of an accurate spring dynamometer, indicated the resistance in pounds, and accurate chronometers gave the time. One observer being placed so as to have a line of sight at right angles to the line of motion, communicated by sound the instant of passing the rods placed at equal distances along the bank, and at the same instant the time was read off by a second observer, and written down on paper by a third; a fourth observer read off the indications of the dynamometer at the same instant, and they were registered opposite to the instant of time to which their observation corresponded; and, for the sake of accuracy, two copies of this register were kept. The indications of this register form the body of experiments of 1835.

The experiments of 1835 were conducted on the following vessels:—

The Wave, . .	No. I.
The Dirleton, .	No. II.
The Raith, . .	No. III.
The Houston, .	No. IV.

The first of these having been made the subject of experiment at seven different degrees of immersion, and each of the others at three, are equivalent to experiments upon sixteen vessels of sixteen different forms.

The forms of the vessels are shewn in Plate III., being projected at an angle of the line of vision $\sin.^{-1} = \frac{1}{3}$. The "water lines of the entrance and of the run" are shewn below the projections of each vessel, as taken at successive heights of 6 inches. The comparisons of form may thus be easily made.

The principal dimensions of the vessels were nearly identical. The maximum breadth at the gunwale being about 6 feet, and the length, exclusive of the helm, 69 feet, there being added to this in the case of the "Wave" a cutwater or very sharp part of the bow 6 feet long, and of very small capacity.

The Wave is a vessel of very peculiar form. My observations on the nature of the resistance of fluids in 1834 suggested a form of least resistance. The Wave was built of that form, and answered fully, and even surpassed the expectations I had formed of the facility of her motion. The lines of entrance are parabolic tangent arcs, having a point of contrary flexure between the maximum transverse section and the stem. The run is formed of elliptical arches, and is by no means so fine as runs usually are. It has long been matter of observation with me, that the maximum resistance to a vessel of ordinary form is experienced in the immediate vicinity of the stem,—that the water there is thrown aside with a velocity much greater than is requisite to remove the particles from the portions of space to be passed over by the succeeding points of the bow. This "head of water" at the bow, instead of being merely thrown aside, is also thrown upward and forward, so as very much to increase the resistance beyond what appears necessary for the transit of the vessel. It occurred to me as probable that a form of vessel might perhaps be obtained, which would not at any given velocity raise a head of water above the level, but merely give to the particles displaced the minimum possible of lateral motion required to permit the transit of the vessel. The theoretical law of least displacement, which I imagined gave me the equation of a curve, which appeared to me to be a curve of minimum resistance. That this curve would be the curve of least resistance I could not a priori determine; but it appeared to me that an experimentum crucis might decide the question after the vessel was built. The experiment was simply to give the vessel a very high velocity, such as 17 miles an hour, and if it should then be found that no particle of water had any motion communicated to it except simply what was necessary for the passage of the vessel, if no spray were thrown up before the vessel or dashed aside by the prow; if, in fact, the vessel, on entering smooth water, should pass into it leaving the surface still unruffled, and producing no motion among the particles but what was the necessary result of mere repletion, by the presence of an additional body, then I should be warranted in denominating such a body the *solid of least resistance.* This experiment was actually tried. The vessel was built of this form (as given in Plate III. fig. 1), and was named "The Wave;" and it is a remarkable fact that, even when deeply laden, and when urged to a velocity of 17 miles an hour, there is no spray, no foam, no surge, no head of water at the prow, but the water is parted smoothly and evenly asunder, and quietly unites after the passage of the vessel, without having changed the natural relations of its particles to one another. Adhesion alone to the surface of the vessel drags forward a film of adjacent fluid, all else remains quiet and smooth.

The three other vessels, the Dirleton, the Raith, and the Houston, were built on the models of Mr John Wood of Greenock, a gentleman of much scientific knowledge and great practical skill; they are much more nearly analogous to the ordinary forms given to sea-going vessels. The Dirleton is the most recent and the best vessel; the Raith and the Houston are inferior and older.

It is worthy of remark, that the Wave is the sharpest vessel, the Dirleton next to her, the Raith third, and the Houston the most bluff in the entrance; that the Wave is fullest, the Dirleton next to her, the Raith next, and the Houston most fine in the run. From the experiments it would seem, that a fine entrance is of much more importance to velocity than has been hitherto supposed, and that a fine run is by no means entitled to the importance that has been attached to it. It should also be observed, that the increase of immersion causes a very great increase of resistance in the three last vessels, and comparatively little in the Wave; and that the water-lines become bluff as they descend, but retain the original curve in the Wave.

Table I. contains the Results of the Experiments of 1835, arranged in reference to the Velocity of the Wave of the Fluid, and is deduced from Tables II, III, IV, and V.

Table II. contains the Original Experiments of 1835 on the Wave, the form of vessel given in Plate III. fig. 1.

Table III. contains the Original Experiments of 1835 on the Houston. The form of the vessel is given in Plate III. fig. 3.

Table IV. contains the Original Experiments of 1835 on the Dirleton. The form of this vessel is given in Plate III. fig. 2.

Table V. contains the Original Experiments of 1835 on the Raith. The form of this vessel is given in Plate III. fig. 4.

ANALYTICAL TABLE of the RESULTS of the Experiments of 1835, upon the Resistance of four Vessels

TABLE I.		WAVE.—PL. III. FIG. 1.							
		LIGHT.	I. TON.	II. TONS.	III. TONS.	IV. TONS.	V. TONS.	VI. TONS.	
Total mass moved in lbs.		5,782	8,022	10,262	12,502	14,742	16,982	19,222	Total mass.
Immersion in the fluid,		11.0 in.	13.5 in.	15.0 in.	16.5 in.	18.0 in.	19.0 in.	20.0 in.	Immersion.
	Velocity in miles an hour.	Resistance in pounds.	Resistance in pounds.	Resistance in pounds.	Resistance in pounds.	Resistance in pounds.	Resistance in pounds.	Resistance in pounds.	Velocity in feet per second.
Velocities less than the Velocity of the wave of the fluid,	3.7879	...	...	...	...	...	...	...	5.55
	3.8961	...	...	...	...	40.2	58.5	...	5.71
	4.0107	30.0	32.0	...	...	...	...	...	5.88
	4.1322	...	...	39.0	...	...	...	...	6.06
	4.2613	32.0	33.7	...	42.7	...	64.6	...	6.25
	4.3988	...	...	...	...	...	...	74.0	6.45
	4.5454	34.4	35.5	...	...	52.3	79.0	81.3	6.60
	4.7229	...	...	...	...	...	...	...	6.89
	4.8702	40.6	42.8	47.5	...	...	90.0	...	7.14
	5.0508	...	...	...	...	...	95.0	95.5	7.40
	5.2448	44.0	51.0	...	76.0	...	...	...	7.69
	5.4545	...	...	...	...	83.3	100.5	121.5 155.0	7.99
	5.6818	53.8	56.0	76.0	...	84.0	...	188.0	8.33
	5.9289	...	...	82.5 108.0	111.0 127.5	...	112.0	312.0 384.0	8.69
	6.1983	73.3	76.0	91.0 165.0	186.0 214.0	...	142.0 332.0	...	9.09
	6.4935	88.5	95.7 110.0	103.0 193.0	193.0 266.0	...	300.0 372.0	...	9.52
	6.8182	93.5 119.0	120.0 197.0	133.5	232.0 298.0	...	386.0 392.0	...	10.00
	7.1770	100.0 131.7	166.0 199.3	206.0 240.0	...	...	...	...	10.52
	7.5758	131.7 174.0	217.0 265.0	225.0 320.0	329.0 336.0	...	...	...	111.1
The Velocity of the Wave of the Fluid in these Experiments was about Eight miles an hour,									
Velocities greater than the velocity of the Wave of the fluid,	8.5227	168.0	214.0	...	...	...	...	...	12.50
	9.0491	...	210.0	...	...	...	...	...	13.33
	9.6955	189.5	227.3	235.0	...	...	...	...	14.28
	10.4895	...	...	...	...	...	...	...	15.38
	11.3634	212.3	245.5	333.0	...	344.0	...	...	16.16
	12.3967	222.0	...	352.0	...	408.0	...	...	18.18
	13.6364	...	...	...	...	...	...	...	20.00
	15.1515	...	...	444.0	...	...	...	...	22.22

NOTE.—The double observations shew a variation in the resistance at the same velocity of which the cause is to be found in the *Hist* given velocity its height is small, and the resistance is also small; but when the velocity has been acquired by small increments du and the resistance arising from it is also increased; thus in the vicinity of the wave, and immediately behind it, the history of the w tions.

different Depths of Immersion, giving measures of resistance for sixteen forms of the floating body.

DIRLETON.—Pl. III. Fig. 2.			HOUSTON.—Pl. III. Fig. 3.			RAITH.—Pl. III. Fig. 4.			
Light.	II. Tons.	IV. Tons.	Light.	II. Tons.	IV. Tons.	Light.	II. Tons.	IV. Tons.	
5,859	10,339	14,819	6,076	10,556	15,036	5,859	10,339	14,819	Total mass.
8.7 in.	13.5 in.	16.0 in.	7.3 in.	11.0 in.	15.0 in.	7.5 in.	11.0 in.	15.0 in.	Immersion.
Resistance in pounds.	Resistance in pounds.	Resistance in pounds.	Resistance in pounds.	Resistance in pounds.	Resistance in pounds.	Resistance in pounds.	Resistance in pounds.	Resistance in pounds.	Squares of Velocity.
...	...	42.0	...	...	...	...	...	...	14.3482
...	...		...	...	...	...	...	...	15.1796
...	47.3	46,5	...	...	...	...	...	...	16.0857
...	...	...	...	...	...	...	...	...	17.0753
38.0	...	...	...	...	60.0	...	...	...	18.1592
...	52.5	...	...	...	...	...	...	...	19.3497
...	...	...	37.5	...	66.0	40.2	...	...	20.6612
...	54.0	...	...	...	...	...	...	...	22.1106
...	...	...	44.0	64.5	...	...	81.0	...	23.7182
...	...	109.5	53.0	...	...	...	...	...	25.5076
...	...	114.0	...	72.7	...	60.0	83.0	116.0	27.5075
...	...	117.0	...	...	...	...	...	...	29.7521
...	99.7	126.0 144.0	83.0	...	166.5	...	103.5	148.0 180.0	32.2838
...	114.7	...	...	...	172.5 255.0	...	...	...	35.1511
94.5	133.5 169.0	169.0 235.0	98.0	197.0	252.0 279.0	87.0 98.0	126.7 218.0	110.0 294.0	38.4195
...	177.0 210.0	...	...	195.0 258.0	288.0 324.0	100.0 156.0	...	306.0 357.0	42.1656
...	216.0 285.0	...	195.0	...	...	...	...	..	48.4876
147.3	...	...	181.0 210.0	...	...	...	...	...	51.5098
178.0	255.0 342.0	...	...	306.0 330.0	...	163.0 189.0	252.0	...	57.3921

or Twelve feet per second; the form and dimensions of the channel are given in Plate II. Fig. E.

204.0	328.0	...	...	...	...	...	...	...	72.6369
216.0	...	...	234.0	...	...	...	...	...	81.8869
225.0	...	...	...	...	...	225.0	...	...	94.0628
...	...	...	241.5	...	...	...	...	...	110.0290
228.0	...	...	...	...	...	...	...	...	129.1325
...	...	...	...	...	...	...	...	...	153.6784
...	...	...	...	...	...	...	...	...	185.9504
...	...	...	...	...	...	...	...	...	229.5683

of the Wave. The *magnitude* of the wave depends upon the *wave's age* in such a manner, that when the vessel has been rapidly brought to a a considerable interval of time, the anterior waves have accumulated in the direction of the motion, and the height of the wave is increased explains the variations in the resistance of the fluid to a given body moving with a given velocity, and is the source of the double observa-

TABLE II. THE WAVE.

No. of Experiments.		Weight of Ballast.	Total Weight Moved.	Depth of Immersion.	Weight on Pyramid.	Time of commencing Observation.	Resistance at 0.	100 Feet.		200 Feet.		300 Feet.	
								Time.	Force.	Time.	Force.	Time.	Force.
		Lbs.	Lbs.	Inches.		h m s	Lbs.	s		s		s	
7th July 1835,	III.	0000	5782	11	2 cwt.	8 6 53	30.0	21	30	20	30.0	19	29.3
	IV.	0000	5782	11	3 cwt.	8 26 46	32	16	32	16	35.5	13	39.0
	V.	0000	5782	11	3 cwt.	9 48 22	35.5	16	37.3	14	44.0	14	45.7
	VI.	0000	5782	11	3 cwt.	10 2 4	28.0	15	37.3	15	39.0	15	39.0
	VII.	0000	5782	11	4 cwt.	10 23 9	39.0	13	40.3 41.5	12	41.5 42.7	12	47.5 51.0
	VIII.	0000	5782	11	4 cwt.	10 40 16	52.3	13	53.5 54.7	12	54.7 70.0	10	70.0 73.0
	IX.	0000	5782	11	4 cwt.	11 0 30	47.5	12	53.5 56.0	13	56.0 56.0	11	56.0 63.0
	X.	0000	5782	11	5 cwt.	11 17 55	85.7	10	87.5	11	87.5	11	93.5
	XI.	0000	5782	11	5 cwt.	11 45 15	66.5	11	87.5	10	93.5	10	98.0
	XII.	0000	5782	11	5 cwt.	12 7 30	82.5	11	84.0	10	81.7	10	100.0
	XIII.	0000	5782	11	6 cwt.	12 21 1	93.5	10	108.5	10	116.3	10	131.7
	XIV.	0000	5782	11	6 cwt.	1 25 20	...	9	100.0	10	108.5	10	131.7
	XV.	0000	5782	11	6 cwt.	1 46 33	119.0	10	119.0	10	131.7	9	137.0
	XVI.	0000	5782	11	7 cwt.	2 13 1	131.7	9	133.3	9	133.3	10	147.0
	XVII.	0000	5782	11	7 cwt.	2 27 32	124.7	9	140.0	9	140.0	9	163.5
8th July 1835,	XVIII.	0000	5782	11	7 cwt.	10 1 20	* 157.5	10	161.0	10	163.5	8	†174.5
	XIX.	0000	5782	11	7 cwt.	11 7 15	* 122.5	9	168.0	9	168.0	9	168.0
	XX.	0000	5782	11	8 cwt.	11 23 20	* 154.0	9	200.0	9	186.0	9	195.0
	XXI.	0000	5782	11	8 cwt.	12 7 10	* 147.0	9	150.5	9	163.5	9	193.0
	XXII.	0000	5782	11	8 cwt.	12 18 36	* 147.0	9	147.0	9	159.3	†8	187.7
	XXIII.	0000	5782	11	8 cwt.	12 37 26	* 140.0	9	...	9	154.0	9	186.0
	XXIV.	0000	5782	11	8 cwt.	12 50 20	122.5	9	124.7	8	143.5	9	154.0
	XXV.	0000	5782	11	8 cwt.	1 19 34	131.7	9	137.0	9	159.3	9	163.5
	XXVI.	0000	5782	11	8 cwt.	1 36 23	...	11	161.0	8	...	6	200.0
9th July 1835,	XXVII.	0000	5782	11	10 cwt.	8 30 36	210.5	7	210.5	6	210.5	7	210.5
	XXVIII.	2240	8022	13.5	2 cwt.	10 26 13	41.5	...	...	...	40.3	15	41.5
	XXIX.	2240	8022	13 5	2 cwt.	11 5 49	32.0	17	33.7	16	35.5	15	39.0
	XXX.	2240	8022	13.5	3 cwt.	11 24 49	106.0	11	...	11	...	...	...
	XXXI.	2240	8022	13.5	3 cwt.	11 39 0	...	...	52.3	15	52.3	14	63.0
	XXXII.	2240	8022	13.5	3 cwt.	11 51 10	* 41.5	16	45.7	15	49.3	13	52.6
	XXXIII.	2240	8022	13.5	4 cwt.	12 2 15	* 53.5	49	...	...	...	...	...
	XXXIV.	2240	8022	13.5	3 cwt.	12 59 26	* 32.0	...	...	...	39.0	14	51.0
	XXXV.	2240	8022	13.5	4 cwt.	1 21 36	76	12	76	11	76	11	81
	XXXVI.	2240	8022	13.5	4 cwt.	1 31 8	82.5	...	82.5	...	82.5	10.5	82.5
	XXXVII.	2240	8022	13.5	5 cwt.	1 42 54	124.7	...	124.7	...	124.7	10.5	124.7
	XXXVIII.	2240	8022	13.5	3 cwt.	1 52 50	24.5	18	28.0	18	28.0	16	30.0
	XXXIX.	2240	8022	13.5	5 cwt.	2 10 28.5	* 131.7	11.5	119.0	10	122.5	10	124.7
	XL.	2240	8022	13.5	6 cwt.	2 20 2	* 131.7	10	131.7	9	141.7	9	145.3
	XLI.	2240	8022	13.3	6 cwt.	3 38 0.5	* 105.5	...	152.3	9.3	162.3	9.7	168.0
	XLII.	2240	8022	13.5	6 cwt.	3 46 49	111.0	10	135.5	9.5	140.0	10	167.0
	XLIII.	2240	8022	13.5	8 cwt.	3 57 40.5	* 170.0	9	170.0	9.5	172.3	9.3	200.0
	XLIV.	2240	8022	13.5	8 cwt.	4 0 2	174.5	9.3	176.7	9.2	191.3	9	210.3
	XLV.	2240	8022	13.5	8 cwt.	4 19 39	173.0	10	173.0	8.5	191.3	8.7	197.0
	XLVI.	2240	8022	13.5	9 cwt.	4 31 26	195.0	† 8.5	200.0	8	214.0	7.5	234.7
	XLVII.	2240	8022	13.5	9 cwt.	4 40 1.5	197.0	† 5.5	197.0	8.5	238.0	8.5	238.0
	XLVIII.	2240	8022	13.5	9 cwt.	4 59 41	173.0	9	207.0	9.5	223.7	8.5	231.3
	XLIX.	2240	8022	13.5	9 cwt.	5 11 38.5	214.0	7.5	214.0	8.3	216.3	7.2	217.0
	L.	2240	8022	13.5	10 cwt.	5 21 5	* 225.9	8	227.3	8	227.3	7	227.3
10th July,	LI.	4480	10262	15.0	2 cwt.	10 11 45	39.9	...	39.0	...	39.0	16	39.0
	LII.	4480	10262	15.0	2 cwt.	10 24 12.5	76.0	15.5	12.0	16	18.0	17	28.0
	LIII.	4480	10262	15.0	2 cwt.	10 35 44	39.0	18	32.0	17	39.0	16	39.0
	LIV.	4480	10262	15.0	3 cwt.	10 40 5	41.5	16	44.0	15	45.7	15	47.5
	LV.	4480	10262	15.0	3 cwt.	11 2 2.5	39.0	15.3	39.0	14	47.5	13.5	51.0

* In this experiment an accelerating weight was used to acquire velocity previous to the first observation.

† Point of transition from a velocity less than the wave to a velocity greater than it.

‡ These examples shew the variation of resistance at the same velocity, due to the history of the Wave.

THE WAVE.

400 Feet.		500 Feet.		600 Feet.		700 Feet.		800 Feet.		900 Feet.		1000 Feet.		REMARKS.
Time.	Force.	Time.	Force.	Time.	Force.	Time.	Force.	Time.	Force.	Time.	Force.	Time.	Force.	
13	30.0	17	30	16	30	16	31	15	35.5	13	42.3	14	56	The two first experiments were lost.
14	44.0	13	44	12	51	12	65.5	12	70.0	12	...	...	...	16^s = 32 lb.; 13^s = 44 lb.
13	51.0	12	52.6	12	59.5	11	64.7	11	76.0	11	91.0	11	100	14^s = 40.6 lb.; 12^s = 51.8 lb.
13	42.7	13	44.0	13	51.0	12	66.5	10	70.0	11	82.5	9	91	15^s = 34.4 lb.
11	54.3 56.0	11	59.5 66.5	10	70.0 78.5	12	84.0 91.0	9	100.0 103.0	11	119.0 122.5	9	131.3	The rope slightly entangled.
12	74.5 76.0	11	77.3 81.0	11	84.0 91.0	10	98.0 103.0	10	106.0 119.0	10	135.0 135.0	9	137.0	11^s = 73.3 lb.; 10^s = 98 lb.
11	63.0 70.0	11	74.3 78.5	11	79.7 91.0	10	98.0 98.0	10	98.0 103.0	10	119.0 131.3	10	140	12^s = 53.8 lb.; 11^s = 67.4 lb.
10	100.0 101.3	10	106.0 119.0	10	124.3 134.3	9	138.5 154.0	9	157.5 186.0	8	193 200	...	...	10^s.5' = 88.5 lb.; 10^s = 93.5...119 lb.‡
10	103.0	11	124.3	9	127.7	10	127.7	10	163.5	9	179.0	...	...	10^s = 93.5...127 lb.‡
10	114.5	10	131.7	10	137.0	10	137.5	9	174.5	8	193.0	8	114.0	10^s = 93.7...137 lb.‡
9	139.3	10	157.5	8	163.5	†8	166.0	8	193.0	7	232.7	...	...	10^s = 93.5...116.3 lb.‡
9	147.0	9	157.5	9	166.0	9	186.0	†8	203.5	8	232.7	7	256.6	9^s.5 = 100...131.7 lb.; 9^s = 131.7...
10	161.0	10	168.0	9	174.5	9	174.5	9	186.0	8	197.0	...	...	9^s = 168...174 lb.‡ [166 lb.‡
10	163.5	8	184.3	†7	186.0	6	186.0	7	114.0	6	114.0	...	...	9^s = 133.3...163.5 lb.;‡ 6^s.5 = 186 lb.
8	179.0	9	182.5	†7	182.5	7	182.5	6	217.0	...	...	...	...	7^s = 182.5 lb.
8	197.0	8	197.0	7	198.5	7	198.5	7	222.0	...	...	...	...	8^s = 169 lb; 7^s = 197.5
†8	168.0	8	182.5	7	197.0	7	200	6	...	6	...	6	...	9^s = 122.5...168 lb.;‡ 8^s = 168 lb.;
8	201.7	...	...	...	...	...	...	...	...	...	...	...	...	An accident. [7^s = 189.7 lb.
10	193.0	†8	194.0	7	195.0	6	207.0	5	238.0	6	...	6	...	7^s = 194 lb.
8	189.5	7	189.5	7	189.5	6	198.5	6	245.0	6	341.6	...	...	8^s = 173.5 lb.; 7^s = 189.5 lb.
9	186.0	†8	199.3	8	203.5	8	217.0	5	245.0	7	266	6	...	9^s = 140...186 lb.‡
9	179.0	†8	182.0	8	187.0	9	200.0	6	227.3	5	266	6	...	9^s = 122.5...179 lb.‡
9	186.0	8	194.0	†8	195	8	207.	6	224.5	...	...	...	...	The times in these two experiments
11	174.5	8	179.0	9	197	8	207	7	232.6	6	256	7	...	inaccurately observed.
5	...	6	236.0	7	...	6	161.0	7	193.0	7	193	7	...	See continuation in Exper. CIV.
14	41.5	14	42.7	14	51.0	12	56.0	12	56.0	12	56.0	...	...	14^s = 41.5 lb.; 12^s = 56 lb.
14	44.0	14	45.7	14	51.0	11	57.7	14	73.0	12	82.0	...	...	16^s = 33.7 lb. 14^s = 42.8 lb.
...	...	...	...	...	...	...	...	...	...	...	...	...	...	An accident.
13	66.5	12	66.5	11	70.0	11	91.0	11	91.0	12	106	12	...	
14	63.0	...	131.7	...	137.0	12	137	10	96.0	11	107.3	...	...	The accelerating weight accidental- [ly raised.
...	...	...	...	...	...	...	...	...	...	...	...	...	...	An accident.
14	51.0	13	53.5	11	63.0	11	76.0	12	...	...	...	...	...	
11.5	84	12	96.0	10	114.5	10	116.3	10.5	127.5	10	154.0	11	...	11^s = 76 lb.
13.5	84	12	84.0	10.5	93.5	11	98.0	10.5	119.0	10	150.5	...	...	10^s.5 = 95.7 lb.
10	137.0	10.5	147.0	9	163.5	11	168.0	9	198.5	10	207.0	...	...	
16	30.0	15	30.0	15	40.3	14	47.5	12	51.0	12	53.0	...	...	
10.5	127.5	9.5	150.5	10	157.5	10	197.7	10	207.0	...	...	...	...	10^s = 122.5...197 lb.‡
10	176.7	9	179.0	10.5	193.0	8.5	216.3	10	233.9	8	...	...	...	
10	179.0	10	188.3	9	217.0	9	217.0	†8	248.5	8	245.0	6	...	
9.5	189.3	9.5	199.3	9.5	243.3	9	245.0	†7.5	280.0	7.5	327.3	7	...	9^s.5 = 166...199.3 lb.‡
9.2	212.3	8.5	217.7	9	239.7	†8	265.0	7	269.0	6.5	352.0	5.5	...	
9.5	217.0	9	238.0	†8	253.5	8	268.6	7	308	6	308.0	...	...	
9.0	198.5	9	216.3	†8.7	240.3	8	266.6	7	268.6	6	295.6	6	...	
8	234.7	7	235.7	6	245.0	6	267.5	6	150.5	...	...	...	...	8^s = 214 lb.; 6^s = 245 lb.
8	238.4	8.5	239.7	7.5	256.0	7	266.0	5.5	268	5.5	...	...	...	
†8	245.0	8.7	280.0	7.8	282.0	7.3	332.5	5.2	336.0	6	...	...	...	
7	217.0	6.5	...	...	...	...	203.5	8	...	...	...	...	...	8^s = 226.5 lb.; 7^s = 227.3 lb.
7	227.3	5.5	223.6	6.5	137.0	7	222.0	6	186	7	140.0	...	...	See in continuation Exper. CXXII.
15.5	39.0	14.5	44.0	14.5	51.0	13.5	140.0	11	140.0	11.2	96	11.8	...	
16	39.0	16.5	39.0	14.5	39.0	13	124.7	12	106.0	12	76	12	...	
16.5	39.	14.5	40.0	15	47.5	14	53.5	14	63.0	12.5	66.5	12.5	...	16^s.5 = 39 lb.; 14^s = 47.5 lb.
13	53.5	11.5	56.0	13	63.0	12.5	124.7	11	96.0	10.5	...	...	...	14^s = 46.6 lb.
13.5	53.5	12.5	70.0	11.5	108.5	10	99.0	11.5	106.0	10.5	...	...	...	13^s.5 = 51.0 lb.

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* In this experiment an accelerating weight was used to acquire velocity, previous to the first observation.

† Point of transition from a velocity less than the wave to a velocity greater than it.

‡ These examples shew the variation of resistance at the same velocity, due to the history of the Wave.

TABLE II.—continued.

THE WAVE.

No. of Experiments.		Weight of Ballast.	Total Weight Moved.	Depth of Immersion.	Weight on Pyramid.	Time of commencing Observation.	Resistance at 0.	100 Feet.		200 Feet.		300 Feet.	
								Time.	Force.	Time.	Force.	Time.	Force.
		Lbs.	Lbs.	Inches.		h m s	Lbs.						
10th July 1835,	LVI.	4480	10262	15.0	3 cwt.	11 14 36.5	39.0	14.5	39	15.5	45.7	14.5	53.5
	LVII.	4480	10262	15.0	4 cwt.	11 28 24	78.5	13	78.5	12.5	78.5	11.5	85.7
	LVIII.	4480	10262	15.0	4 cwt.	11 38 21	76.0	13.5	76.0	12	76.0	12	82.5
	LIX.	4480	10262	15.0	4 cwt.	11 54 57	59.5	13	70.0	13	71.5	12	82.5
	LX.	4480	10262	15.0	5 cwt.	12 5 48	96.0	12	103.0	11	109.7	10	124.7
	LXI.	4480	10262	15.0	5 cwt.	12 15 4	91	11	102.0	11.5	107.3	10.5	111.0
	LXII.	4480	10262	15.0	5 cwt.	12 29 13	78.5	12	85.7	11	...	12	106
	LXIII.	4480	10262	15.0	6 cwt.	12 40 24	154.0	11	157.5	11	157.5	9	182.5
	LXIV.	4480	10262	15.0	6 cwt.	1 2 20	124.7	10	131.7	11	133.5	10	166.0
	LXV.	4480	10262	15.0	6 cwt.	1 13 14	137.0	11	138.5	10.5	140	10.9	161.0
	LXVI.	4480	10262	15.0	6 cwt.	1 23 25	137.0	11	163.5	10	174.5	10	182.5
	LXVII.	4480	10262	15.0	6 cwt.	1 36 13	238.0	7.5	245	9	200.0	10.5	193
	LXVIII.	4480	10262	15.0	8 cwt.	1 49 0	...	...	...	...	...	...	...
14th July 1835,	LXIX.	6720	12502	16.5	2 cwt.	10 13 0	...	...	53.5	17	51.0	16	44.0
	LXX.	6720	12502	16.5	2 cwt.	10 30 14.5	44	18.5	39	17	41.5	16.5	42.7
	LXXI.	6720	12502	16.5	2 cwt.	10 44 55	32	20	33.4	17	33.7	17	39.0
	LXXII.	6720	12502	16.5	3 cwt.	10 56 0	51	16	51.0	14.5	57.7	15	60.6
	LXXIII.	6720	12502	16.5	4 cwt.	11 9 37	56.0	14	70.0	13.5	82.5	12.5	...
	LXXIV.	6720	12502	16.5	4 cwt.	11 20 40.5	81	13.5	81.0	13	81.0	13	81.0
	LXXV.	6720	12502	16.5	5 cwt.	11 34 45	124.7	11	127.5	10.5	137	11.5	154
	LXXVI.	6720	12502	16.5	6 cwt.	11 53 56	* 258.0	9	154	10	189.5	11.5	268
	LXXVII.	6720	12502	16.5	8 cwt.	12 7 42.5	* 261.3	8.5	266.0	9	209.0	11	287
	LXXVIII.	6720	12502	16.5	8 cwt.	12 20 9	* 207.0	9.5	193.0	10	214	10.5	232.6
	LXXIX.	6720	12502	16.5	9 cwt.	12 28 28	* 200	9.5	214	...	...	...	...
	LXXX.	8960	14742	18.0	2 cwt.	2 10 17	37.2	19	39	17	40.2	17	47.5
	LXXXI.	8960	14742	18.0	3 cwt.	2 23 2	63.0	13.5	51	14.5	51	13.7	66.5
	LXXXII.	8960	14742	18.0	4 cwt.	2 34 14	* 170.0	12	81	12	84	12	94.7
	LXXXIII.	8960	14742	18.0	5 cwt.	2 23 39	* 140.0	11	137.0	17	138.5	11	104.7
15th July 1835,	LXXXIV.	11200	16982	19.0	2 cwt.	7 24 13	...	15	...	20	...	19	50
	LXXXV.	11200	16982	19.0	2 cwt.	7 41 14	58.3	16	68	16	64.6	16	65
	LXXXVI.	11200	16982	19.0	3 cwt.	7 54 55	80.0	16	79	15	79	16	85
	LXXXVII.	11200	16982	19.0	4 cwt.	8 10 6.5	108.0	14.5	112	13	113	12	{102.5 102.5}
	LXXXVIII.	11200	16982	19.0	4 cwt.	8 23 43	...	13	108	12	113.5	12.5	130
	LXXXIX.	11200	16982	19.0	5 cwt.	9 47 41.5	164.0	11.5	165	10.5	174	11.5	192
	XC.	11200	16982	19.0	5 cwt.	10 6 26	...	12	182	10	184	12	206
	XCI.	11200	16982	19.0	6 cwt.	11 23 27	204	10	228	10.5	234	11.5	243
	XCII.	11200	16982	19.0	8 cwt.	10 36 1	264	12	276	9.5	276	10.5	321
	XCIII.	11200	16982	19.0	8 cwt.	10 49 14	270	10	288	10	300	10.5	312
	XCIV.	11200	16982	19.0	8 cwt.	11 2 41.5	280	...	...	...	299.3	11	329
	XCV.	11200	16982	19.0	10 cwt.	11 15 1	124.7	10.3	347.3	9.2	347.3	10	369.6
	XCVI.	13440	19222	20	2 cwt.	12 56 0	...	...	...	...	80	16	60
	XCVII.	13440	19222	20	2 cwt.	1 10 28.5	76.3	18	76.3	16.5	73.5	17	71
	XCVIII.	13440	19222	20	3 cwt.	1 23 30	79.5	18	79.0	14	80.5	15	82
	XCIX.	13440	19222	20	3 cwt.	1 33 40	73	11	73	16	74	15.5	75
	C.	13440	19222	20	4 cwt.	1 43 0	119	14.5	120.5	13.5	121.5	12.5	125
	CI.	13440	19222	20	5 cwt.	1 53 7	188.5	12.5	188	12	188	11	198
	CII.	13400	19222	20	6 cwt.	2 4 54	...	11.5	312	11.5	328.5	11	339
	CIII.	13440	19222	20	8 cwt.	2 37 54	268	10	245	11.5	245	11.5	245

* In this experiment an accelerating weight was used to acquire velocity previous to the first observation.

† Point of transition from a velocity less than the wave to a velocity greater than it.

‡ These examples shew the variation of resistance at the same velocity due to the history of the wave.

THE WAVE.

400 Feet.		500 Feet.		600 Feet.		700 Feet.		800 Feet.		900 Feet.		1000 Feet.		REMARKS.
Time.	Force.	Time.	Force.	Time.	Force.	Time.	Force.	Time.	Force.	Time.	Force.	Time.	Force.	
13.5	56.5	12.5	56.0	12.5	101.0	10.5	101.0	11	124.7	10.0	140.0	...	...	11^s = 111 lb.
11	96.0	12	107.2	11	111.0	10	131.7	10	147.0	10.5	...	...	...	11^s = 107.2
11	91.0	12.5	99.0	10	101.5	10.5	106.0	10	137.0	10	150.5	...	...	12^s = 76 lb.; $11^s.5$ = 82.5
12	82.5	11	87.5	10	100.0	10	103.0	11	124.7	12	143.5	10.5	...	$11^s.5$ = 82.5 lb.
11	124.7	10	140.0	11	145.3	10	155.7	9.5	186.0	11.5	193.0	...	...	$10^s.5$ = 103...193 lb. ‡
11	124.7	10.5	149.3	10.5	150.5	10	162.6	10.5	189.5	10.5	207.0	...	...	11^s = 91...111 lb.;‡ $10^s.5$ = 124.7...
11	111.0	10	115	11	131.7	10	161	10	179	10.7	195	...	...	11^s = 85.7 lb. [189.5 lb. ‡
11	187.7	10	193.0	10.5	210.5	10.5	224.6	10	238.0	10	269.0	...	...	
10	168.0	10	195.0	10	207	11	217	10	248.5	10	...	...	...	10^s = 133.5...248.5 ‡
9	193.0	10	210.5	10.5	224.6	10.5	238	10	256.6	10	261.3	...	...	
10	194.5	10	203.5	10	235.3	10	245.0	9	268	9	280.0	...	...	10^s = 163.5...235.3 lb. ‡
10	200	10	203.5	11	217.0	10	229.5	10	250.2	9	266.0	10	...	See in continuation Exper. CXXX.
...	...	...	...	...	...	...	...	...	...	...	...	...	...	An accident.
16	51.0	13	53.5	14	59.5	12	63.0	13	70.0	12	54	13.5	96	
15.5	46.3	16	60.6	13	66.6	14	66.5	13.5	73	12.5	73	13	73	16^s = 42.7 lb.
15	46.3	15	47.5	14	56	14	70.0	13	76	12	87.5	13	...	17^s = 33.5 lb.
13.5	63.0	13	70	12	82.0	13	91.0	12	93.3	10.5	93.5	13.5	...	
13	96.0	12	...	...	...	...	...	...	...	...	...	...	...	
12.5	96.0	12.5	111.0	11	111.0	11	192.0	11	131.7	11.5	131.7	...	...	11^s = 111 lb.; 13^s = 81 lb.
10	186	11	193	10	200	10	222	10.5	...	...	...	...	...	11^s = 127.5 lb.; $10^s.5$ = 186.5 lb.
10	214	11	238	10.5	269.3	10.5	266	10.5	287	10	...	15.5	...	
10	232.6	10.5	238	11.5	261.3	10.5-	266	9.5	298.6	10	336	...	...	
10.5	238	10	266	10	287	9.5	329	9	352	9	336	...	...	10^s = 193...266 lb.; 9^s = 329...336 lb.
...	...	...	...	...	...	...	...	...	...	...	...	...	...	An accident.
17	51.0	15	53.5	15	63	14	70	13	78.5	13.5	96	...	...	17^s = 402 lb.; 15^s = 52.3 lb.
14	76.0	13.3	81.0	12.5	83.3	12	923	12.5	100.0	11.5	111.0	...	...	$12^s.5$ = 83.3 lb.
11	115.0	11	154.0	11	207.0	11	140.0	11.5	140.0	11.5	161	...	...	12^s = 84 lb.
10	179.0	11.5	154.0	10.5	154	11	215.5	10	227.3	11.0	...	...	...	See in continuation Exper. CXLVI.
18	50	19	48.0	16.5	58.5	17.5	58.3	16.5	59.7	15.5	69.0	13	...	$17^s.5$ = 58.5 lb.
16	70.9	15	76	15	83	13	90	14	90.0 90.0	13	96 100	15	...	16^s = 64.6 lb.
13	89	14	95	13.5	95	12.5	100.5 103.5	11.5	112 118.5	11.5	128 140	14	...	$16^s.5$ = 79 lb.; 13.5 = 95 lb.
12.5	124 140	12.5	153	11	152	12	155 176	12	180 186	20.5	...	...	...	$11^s.5$ = 112 lb.
12.5	141	11.5	142 148	11	156 158	11	162 163	11	182 183	11	196 196	...	...	11^s = 142...195 lb. ‡
12	202	11	220	11	232	11	140	11	60	15	...	...	...	11^s = 202...232 lb. ‡
11.5	220	11.5	236	11	240	11.5	240	11	240	10.5	...	...	...	
11	255	10	270	10.5	288	12	300	11	312	10.5	...	...	...	
11.5	324	10.5	336	11	357	10.5	360	10.5	360	11.5	...	...	...	11^s = 321...336 lb.
10.5	342	10.5	360	11	372	10.5	180	14	120	...	...	...	...	$10^s.5$ = 300...372 lb.
10.5	338.8	10.5	354.5	10.5	359	10.5	371.9	10.5	386.3	13.5	...	...	...	
10.5	390	9	...	10	...	10	...	10	238	...	...	...	...	
...	...	...	...	...	...	...	...	...	78	15	78	...	...	
17	71	17	72	...	...	...	716	17	86.5	16	90	14.5	102	
15	89	14	102	13	104	13	113	12	118	13	122 136	15	...	15^s = 81.3 lb.; 13^s = 103 lb.
14.5	79	15	91	18	95	13	96	14	112	13	122	13	...	$15^s.5$ = 74; $13^s.5$ = 95.5 lb.
12.5	133	11.5	156 152	13.5	164	13	168	11	180 188	12.5	188	...	...	$12^s.5$ = 121.5...156 lb.
11.5	208	11.5	214	10.5	2-0 228	11	226 228	13	236 240	...	...	...	...	12^s = 188 lb.
12	342	12	354	11	369	11.5	372	11	384	11.5	384	...	...	$11^s.5$ = 312...384 lb.
11	245	11	263.6	12	269	11	287	12	294	11	...	...	...	

* In this experiment an accelerating weight was used to acquire velocity previous to the first observation.

† Point of transition from a velocity less than the wave to a velocity greater than it.

‡ These examples shew the variation of resistance at the same velocity, due to the history of the Wave.

TABLE II.—continued. THE WAVE.

	No. of Experiments.	Weight of Ballast.	Total Weight Moved.	Depth of Immersion.	Weight on Pyramid.	Time of commencing Observation.	Resistance at 0.	100 Feet.		200 Feet.		300 Feet.		400 Feet.		500 Feet.	
								Time.	Force.	Time.	Force.	Time.	Force.	Time.	Force.	Time.	Force.
		Lbs.	Lbs.	Inches.		h m s	Lbs.										
26th July 1835,	CIV.	0000	5782	11.0	2 cwt.	10 4 11.5	* ...	14.5	...	13	51.5	11	74	11	85	10	106
	CV.	0000	5782	11.0	3 cwt.	10 15 55.5	* ...	...	...	...	...	...	...	...	...	...	...
	CVI.	0000	5782	11.0	3 cwt.	10 40 30.5	* 100	11	625	10.5	77	10.5	77	11.5	112	8	124
	CVII.	0000	5782	11.0	3 cwt.	10 53 24	54	11.5	100	11.5	100	10.5	92	10	116	9.5	92
	CVIII.	0000	5782	11.0	4 cwt.	11 1 40	96	9	108	10	134	11	156	9	166	10	200
	CIX.	0000	5782	11.0	4 cwt.	11 7 12	106	10	118	10	130	10.3	148	9.7	139	9	168
	CX.	0000	5782	11.0	4 cwt.	11 19 30	224	7.5	96	10	112	7.5	134	10	162	9	170
	CXI.	0000	5782	11.0	4 cwt.	11 27 59	62	11	72	11	78	10	94	11	112	10	132
	CXII.	0000	5782	11.0	4 cwt.	11 35 13	164	8	96	9	112	10	128	9	170	9	170
	CXIII.	0000	5782	11.0	5 cwt.	11 44 30	128	9.5	148	8.5	160	†8	180	7.5	192	7	...
	CXIV.	0000	5782	11.0	5 cwt.	11 53 11	132	9	148	10	152	8.5	176	†8	212	7.5	218
	CXV.	0000	5782	11.0	5 cwt.	12 1 53	132	9.5	150	8.5	160	9	180	†8	174	7.5	180
	CXVI.	0000	5782	11.0	5 cwt.	1 36 57	140	9	152	9	172	8.5	190	†7.5	216	8	222
	CXVII.	0000	5782	11.0	6 cwt.	1 45 27	204	†7	152	7	164	7	220	6	224	6	224
	CXVIII.	0000	5782	11.0	6 cwt.	1 53 1	208	†8	...	7	...	6	...	6	...	...	...
	CXIX.	0000	5782	11.0	6 cwt.	2 1 43.5	255	†6.5	255	7	234	7	252	6	300	6	348
	CXX.	0000	5782	11.0	8 cwt.	2 10 42	219	†7	219	6	223.5	6	270	6.3	282	5.7	...
	CXXI.	0000	5782	11.0	9 cwt.	2 38 0	...	...	...	...	...	...	...	...	...	...	...
27th July 1135,	CXXII.	2240	8022	13.5	2 cwt.	11 25 53	82	13	82	12.5	84	12	110	11	106	12	120
	CXXIII.	2240	8022	13.5	3 cwt.	11 38 51	100	12	103	11	110	10.5	116	11	120	10	...
	CXXIV.	2240	8022	13.5	3 cwt.	11 48 20	224	9.5	144	11.5	160	10	163	9.5	164	10	204
	CXXV.	2240	8022	13.5	5 cwt.	11 56 17	216	10	224	†8	240	6	...	9	...	...	...
	CXXVI.	2240	8022	13.5	5 cwt.	12 5 54	228	9	204	8.5	228	8.5	237	†8	243	7	288
	CXXVII.	2240	8022	13.5	6 cwt.	12 14 3	243	†7.5	210	6.5	240	7	246	6	318	6	300
	CXXVIII.	2240	8022	13.5	6 cwt.	12 21 8	210	†8	210	7	228	6.5	270	6	300	..	...
	CXXIX.	2240	8022	13.5	8 cwt.	1 6 20.5	240	7.	270	...	...	...	...	...	...	...	...
	CXXX.	4480	10262	15.0	2 cwt.	2 0 29	60	15	66	14	68	13	80	13.5	83	12	90
	CXXXI.	4480	10262	15.0	3 cwt.	2 9 57	106	12	106	11.5	110	11.5	118	11	120	11	120
	CXXXII.	4480	10262	15.0	4 cwt.	2 15 36	166	11	164	11	164	10.5	164	11	170	10.5	172
	CXXXIII.	4480	10262	15.0	5 cwt.	2 22 47.5	172	11	168	10.5	168	11	168	10.5	192	10.5	224
	CXXXIV.	4480	10262	15.0	6 cwt.	2 30 15	206	9.5	240	9.5	240	9.5	...	9.8	...	7	180
	CXXXV.	4480	10262	15.0	6 cwt.	2 40 34	225	9	225	9	270	9	300	9	306	9	330
	CXXXVI.	4480	10262	15.0	7 cwt.	2 0 0	...	...	...	...	...	...	...	...	...	...	...
	CXXXVII.	4480	10262	15.0	7 cwt.	3 22 43.5	246	9.5	303	10	350	...	...	...	...	...	...
31st July 1835.	CXXXVIII.	4480	10262	15.0	6 cwt.	10 43 6.3	177	11.7	192	8	203	11	220	10	240	...	...
	CXXXIX.	4480	10262	15.0	8 cwt.	10 55 18	216	9.7	223.5	10	253.5	9.8	283.5	7.7	297	9.5	330
	CXL.	4480	10262	15.0	8 cwt.	11 10 47	223.5	9	240	8.7	240.5	9.8	300	9.5	312	8	336
	CXLI.	4480	10262	15.0	10 cwt.	11 24 13.5	284	8.5	300	†7.5	312	7.5	312	6.7	332	5.3	...
	CXLII.	4480	10262	15.0	12 cwt.	11 39 21	352	6.5	404	6	440	...	480	...	160	...	..
	CXLIII.	4480	10262	15.0	14 cwt.	11 58 19	392	6	392	6	400	...	200	6	200	6	280
	CXLIV.	4480	10262	15.0	14 cwt.	...	...	...	...	...	...	...	...	...	...	...	...
	CXLV.	4480	10262	15.0	14 cwt.	12 30 48	428	...	...	...	428	5.5	456	5	480	...	...
	CXLVI.	8960	14742	19.0	10 cwt.	2 2 4.7	343.5	...	...	...	...	7.3	...	10	372	7.3	...
	CXLVII.	8960	14742	19.0	12 cwt.	2 41 24	472.0	7.5	484	6.7	392	6	392	5.3	408	5.5	192
	CXLVIII.	4480	10262	15.0	12 cwt.	3 32 19	400.0	7	360	6	344	6	344	5.5	400	6	400
	CXLIX.	4480	10262	15.0	14 cwt.	3 55 13	348	5.3	352	7.2	...	...	...	...	...	...	...
	CL.	4480	10262	15.0	14 cwt.	4 23 16.5	...	5.5	192	6	264	5	400	4.5	488	4.5	...

REMARKS.

Exp. CVI. $11^s = 77$ lb. Exp. CIX. $10^s = 106...148$ lb Exp. CXIII. $8^s = 162.6$ lb. Exp. CXIV. $8^s = 176$ lb. Exp. CVII. $7^s = 185$ lb.; $6^s = 224$ lb.

Exp. CXXIII. $10^s.5 = 110$ lb.; $10^s = 120$ lb. Exp. CXXIV. $10^s = 163$ lb. Exp. CXXVI. 8^s 237 lb. Exp. CXXVII. $7^s = 231$ lb.; $6^s = 246$. Exp. CXXVIII. $7^s.5 = 210$ lb.; $6^s.5 = 228$ lb.

Exp. CXXX. $13^s.5 = 75$ lb.; $13^s = 81.5$ lb.; $12^s = 86.5$ lb. Exp. CXXXI. $11^s.5 = 108$ lb.; $11^s = 119$ lb. Exp. CXXXII. $11^s = 165$ lb. Exp. CXXXIII. $10^s.5 = ‡168...192.‡$ Exp. CXXXIV. $9^s.5 = 206...240$. Exp. CXXXV. $9^s = ‡225...320$. Exp. CXLI. $7^s.5 = 306$ lb.; $6^s = 322$ lb. Exp. CXLV. $5^s.5 = 428$ lb.

Exp. CXLVII. $5^s.5 = 408$ lb.

Exp. CXLVIII. $6^s = 344$ lb. Exp. CL. $4^s.5 = 444$ lb.

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* In all the experiments from CIV. to CC. an accelerating weight was used to give velocity previous to the first observation.

† Point of transition from a velocity less than the wave to one greater than it.

‡ These examples shew a variation in the resistance at a given velocity which is due to the history of the Wave.

TABLE III. THE HOUSTON.

	No. of Experiments.	Weight of Ballast.	Total Weight Moved.	Depth of Immersion.	Weight on Pyramid.	Time of commencing Observation.	Resistance at 0.	100 Feet.		200 Feet.		300 Feet.		400 Feet.		500 Feet.	
								Time.	Forc e.	Time.	Force.	Time.	Force.	Time.	Force.	Time.	Forc
		Lbs.	Lbs.	Inches.		h m s	Lbs.										
28th July 1835.	CLI.	0000	6076	7.3	2 cwt.	11 40 37	46	14	122	10.5	54	11.5	68	1 2.5	72	11.5	100
	CLII.	0000	6076	7.3	2 cwt.	11 50 2	35	15	40	15	44	14	53	13.5	55	12.5	65
	CLIII.	0000	6076	7.3	3 cwt.	12 0 9	83	12	95	11	101	10.7	117	10	148	10.3	164
	CLIV.	0000	6076	7.3	4 cwt.	12 7 47	114	10	116	11	122	9.5	148	10.5	171	9.5	200
	CLV.	0000	6076	7.3	5 cwt.	12 14 0	174	9	177	10	195	10	201	8.5	211.5	8.5	219
	CLVI.	0000	6076	7.3	6 cwt.	12 20 7	181	9.5	210	9.5	222	7.5	234	8.5	246	5	276
	CLVII.	0000	6076	7.3	7 cwt.	12 26 31.5	222	8	234	6.5	192	6	241.5	6	241.5	7	270
	CLVIII.	0000	6076	7.3	8 cwt.	12 35 59	228	7	18	7	60	8	300	7	180	6	198
	CLIX.	4480	10556	11.0	2 cwt.	1 46 23	60	16	63	14	64.5	14	66	14	69	13	76
	CLX.	4480	10556	11.0	4 cwt.	1 56 36	189	11	195	11	225	10	234	11	252	10.5	258
	CXI.	4480	10556	11.0	7 cwt.	2 4 27	267	10	288	9.3	306	9.2	330	9	336	8.5	348
	CLXII.	8960	15036	15.0	2 cwt.	2 20 0	57	...	60	15.3	60	16	63	15	69	15	78
	CLXIII.	8960	15036	15.0	4 cwt.	2 28 51	165	11.7	168	12.3	172.5	11	192	11	246	12	255
	CLXIV.	8960	15036	15.0	6 cwt.	2 37 17.5	249	12	252	11	279	11	288	10.5	309	10.5	324

REMARKS.

Exp. CLII. 15^{s} = 37.5 lb.; 14^{s} = 44 lb.; $13^{s}.5$ = 53 lb. Exp. CLIII. 12^{s} = 83 lb.; 11^{s} = 98 lb. Exp. CLV. 10^{s} = 195 lb.; $8^{s}.5$ = 211.5 l

Exp. CLVI. $9^{s}.5$ = ‡181...210; $7^{s}.5$ = 234 lb. Exp. CLVII. $6^{s}.5$ = 241.5 lb.

Exp. CLIX. 14^{s} = $64^{s}.5$ lb.; 13^{s} = 72.7 lb. Exp. CLX. 11^{s} = 197 lb.; $10^{s}.5$ = ‡195...258 lb. Exp. CLXI. 9^{s} = ‡306...330 lb.

Exp. CLXII. 16^{s} = 60 lb.; 15^{s} = 66 lb. Exp. CLIII. 12^{s} = 166.5 lb.; $11^{s}.5$ = ‡172.5...255 lb. Exp. CLXIV. 11^{s} = ‡252...279 lb.; $10^{s}.5$ = 288...324 lb.

TABLE IV. THE DIRLETON.

No. of Experiment.	Weight of Ballast.	Total Weight Moved.	Depth of Immersion.	Weight on Pyramid.	Time of commencing Observation.	Resistance at 0,	100 Feet.		200 Feet.		300 Feet.		400 Feet.		500 Feet.	
							Time.	Force.	Time.	Force.	Time.	Force.	Time.	Force.	Time.	For
	Lbs.	Lbs.	Inches.		h m s	Lbs.										
CLXV.	0000	5859	8.7	2 cwt.	9 47 54.7	33	17.3	38	16.5	41	16.3	44	13.2	50	14.5	5
CLXVI.	0000	5859	8.7	4 cwt.	10 1 53	91	10.7	98	11.3	101	10	103	10	96	11	13
CLXVII.	0000	5859	8.7	6 cwt.	10 10 52	132	9.5	150	9.5	162	9	178	9.3	178	10.2	16
CLXVIII.	0000	5859	8.7	8 cwt.	10 20 9.5	204	8	216	7.5	216	7	234	7	150	7	13
CLXIX.	0000	5859	8.7	9 cwt.	10 42 30	210	8	210	8	213	6	228	6	150	7	15
CLXX.	4480	10339	13.5	2 cwt.	11 11 44	40.5	19	46.5	17	48	17	52.5	15.3	54	14.7	5
CLXXI.	4480	10339	13.5	4 cwt.	11 22 39	99	12	100	12	108	11.5	121.5	11.5	133.5	11	16
CLXXII.	4480	10339	13.5	6 cwt.	11 36 22	153	11	162	11	177	10	183	11	210	10	22
CLXXIII.	4480	10339	13.5	8 cwt.	11 47 53	211.5	9	228	10.5	247.5	9.7	276.5	10.3	301.5	9.3	32
CLXXIV.	4480	10339	13.5	6 cwt.	11 55 46	222	...	...	...	...	...	...	...	...	...	..
CLXXV.	4480	10339	13.5	8 cwt.	12 12 40	223.5	10	235.5	10	243	10	285.5	9	300	9.5	31
CLXXVI.	4480	10339	15.5	10 cwt.	12 21 42	255	9	270	9	342	9.7	345	9.3	348	8.3	36
CLXXVII.	4480	10339	13.5	10 cwt.	12 39 12	300	8	306	8.7	330	8.7	360	7.5	240	7.5	18
CLXXVIII.	8960	14819	16.0	2 cwt.	1 40 51.7	31.5	18.8	33	18.7	34.5	18.8	42	18	46.5	17	4
CLXXIX.	8960	14819	16.0	4 cwt.	1 51 24.5	109.5	13.5	114	13	114	12.5	120	12.3	126	12	14
CLXXX.	8960	14819	16.0	6 cwt.	2 1 5	172.5	12	172.5	12	189	...	...	...	...	...	.
CLXXXI.	8960	14819	16.0	6 cwt.	2 26 34.5	169.5	11	175.5	10.7	193.5	11	216	12.3	235.5	10.7	24

REMARKS.

Exp. CLXV. $16^{s}.5$ = 38 lb, Exp. CLXVI. 11^{s} = 94.5 lb.; 10^{s} = 102 lb. Exp. CLXVII. $9^{s}.5$ = ‡132...178 lb. Exp. CLXVIII. 8^{s} = 20

$7^{s}.5$ = 216; 7^{s} = 225 lb. Exp. CLXIX. 8^{s} = 210 lb; 6^{s} = 228 lb.

Exp. CLXX. 17^{s} = 47.3; $15^{s}.5$ = 52.5; $14^{s}.5$ = 54 lb. Exp. CLXXI. 12^{s} = 99.7. lb; $11^{s}.5$ = 114.7 lb.; 11^{s} = 133.5 lb. Exp. CLXXII. 11^{s} ‡153...162 lb.; $10^{s}.5$ = ‡177^{s}...210; 10^{s} = 216 lb. Exp. CLXXIII. 10^{s} = ‡228...276.5. Exp. CLXXV. 10^{s} = ‡223.5...285.5. E CLXXVI. 9^{s} = ‡255...342.

Exp. CLXXVIII. 18^{s} = 42 lb.; 17^{s} = 46.5 lb. Exp. CLXXIX. $13^{s}5$ = 109.5; 13^{s} = 114; $12^{s}.5$ = 117 lb; 12^{s} = ‡126...144 lb. Exp. CLXX 12^{s} = ‡172.5...189 lb. Exp. CLXXXI. 11^{s} = ‡169.5...235 lb.

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* In all the experiments from CIV. to CC. an accelerating weight was used to give velocity previous to the first observation.

† Point of transition from a velocity less than the wave to one greater than it.

‡ These examples shew a variation in the resistance at a given velocity which is due to the history of the Wave.

TABLE V.

THE RAITH.

No. of Experiments.	Weight of Ballast.	Total Weight Moved.	Depth of Immersion.	Weight on Pyramid.	Time of commencing Observation.	Resistance at 0.	100 Feet.		200 Feet.		300 Feet.		400 Feet.		500 Feet.	
							Time.	Force.	Time.	Force.	Time.	Force.	Time.	Force.	Time.	Force.
	Lbs.	Lbs.	Inches.		h m s											
CLXXXII.	0000	5859	7.5	2 cwt.	9 41 0	...	...	37	15	43.5	15	49.0	16	60	12	60
CLXXXIII.	0000	5859	7.5	4 cwt.	9 50 44	100	11	108	10	120	10.7	...	10.3	...	10	...
CLXXXIV.	0000	5859	7.5	4 cwt.	10 5 11	87	11	92	11	98	10.7	116	10.3	138	10.5	156
CLXXXV.	0000	5859	7.5	6 cwt.	·10 13 9	163.5	9	169.5	9.5	189	8.5	202.5	8	276	8	288
CLXXXVI.	0000	5859	7.5	8 cwt.	10 34 17	270	6	261	7	228	...	...	...	222	6.3	222
CLXXXVII.	0000	5859	7.5	11 cwt.	10 0 0	...	...	...	...	...	...	...	...	...	...	...
CLXXXVIII.	0000	5859	7.5	10 cwt.	10 49 56	225.5	6	225	7	232	7	267	6.3	336	6.7	120
CLXXXIX.	4480	10339	11	2 cwt.	11 18 10	81	14	81	14	82.5	14	84	12	108	12	72
CXC.	4480	10339	11	4 cwt.	11 29 11	97.	12	103.5	12.5	109.5	12	121.5	10.5	132	11	163.5
CXCI.	4480	10339	11	6 cwt.	11 43 16.5	187	11	218	11	240	...	...	...	255	10.3	270
CXCII.	4480	10339	11	8 cwt.	11 55 0	...	...	...	...	252	9	255	9	294	10	324
CXCIII.	4480	10339	11	10 cwt.	12 6 8	319.5	8	334.5	8.7	339.5	7.3	360	7	...	6	...
CXCIV.	4480	10339	11	10 cwt.	12 21 4.3	207	8.7	212	7	224	7.5	226	6.5	162	7	255
CXCV.	8960	14819	15	2 cwt.	1 40 12.5	68	17	42	18	72	13	72	14.5	70	14.5	88
CXCVI.	8960	14819	15	4 cwt.	1 51 9.5	100	10.5	124	13.5	80	13.5	108	13	52	15	88
CXCVII.	8960	14819	15	4 cwt.	1 59 17	116	13	116	13	148	11.5	139	11.5	180	13	180
CXCVIII.	8960	14819	15	6 cwt.	2 0 2	98	13.5	110	11	126	11.5	228	11	236	11	...
CXCIX.	8960	14819	15	8 cwt.	2 16 13.5	222	11	240	10.5	294	11.5	282	11.5	294	11	342
CC.	8960	14819	15	10 cwt.	2 0 41.5	306	10	354	11	357	11	...	10.5	...	11	...

REMARKS.

Exp. CLXXXII. 15^s = 40.2 lb.; 13^s = 60 lb. Exp. CLXXXIII. $10.^s5$ = ‡ 100...120 lb. Exp. CLXXXIV. 11^s = ‡ 87...98 lb.; 10^s5 = ‡ 68 ...156 lb. Exp. CLXXXV. 9^s = ‡ 163.5...189. Exp. CLXXXVIII. $6.^s5$ = ‡ 225 ... 267.

Exp. CLXXXIX. 14^s = 81 lb.; 13^s = 83 lb. Exp. CXC. 12^s = 103.5; 11^s. = 126.7 lb. Exp. CXCI. 11^s. = ‡ 187.5 ... 218 lb. Exp. CXCII. 9^s = ‡ 252 ... 294. Exp. CXCIII. 8^s. = 334.; 7^s. = 224.

Exp. CXCV. 14^s. = 72 lb. Exp. CXCVII. 13^s. = 116 lb.; 12^s. = ‡ 148 ... 180 lb. Exp. CXCVIII. 11^s. = ‡ 110 ... 236. Exp. CXCIX. 11^s. = ‡ 222... 294. Ex. CC. 10^s5 = ‡ 306. ... 357 lb.

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* In this experiment an accelerating weight was used to acquire velocity previous to the first observation.

† Point of transition from a velocity less than the wave to a velocity greater than it.

‡ These examples shew the variation of resistance at the same velocity due to the history of the Wave.

DESCRIPTION OF PLATES II. AND III.

PLATE II.

Fig. (1.) Represents the form given to the surface of a fluid by the motion of a floating body. The bed of the channel was nearly of the form given in Fig. F, which is a transverse section taken at right angles to the direction of motion of the floating body. The arrow at the stem of the vessel indicates the direction of the moving body, and on each side a dotted line shews the place of the fluid when at rest. The anterior wave at the bow of the vessel swells above and beyond the line of rest, the stern depression falls below and within it, the summits of the stern waves of replacement also protrude beyond and above it. The summits of the waves of unequal displacement, due to the improper form of the vessel, extend from it towards the banks, and give rise to undulations of the second order on the terminal line of the fluid.

Figs. (2.) (3.) and (4.) are the observed forms of the great primary wave of the fluid, in the channel of which the form is given in Fig. E, and in which the mean velocity of the wave is 8 miles an hour.

Figs. (5.) (6.) and (7.) are observed forms of compound waves which were afterwards analyzed, and gave the elementary and simple forms of Figs. (2.) (3.) and (4.) The outline represents the compound wave, the inner lines indicate the analysis.

Figs. A, B, C, D, E, and F. are sections of channels in which waves were propagated and other experiments made, and to which reference is made in the paper.

PLATE III.

Figs. (1.) (2.) (3.) and (4.) are projections of the forms of vessels made the subject of experiment They are simply fore-shortened, so as to diminish their length in the ratio of 3. : 1, or they are projected on an angle $\sin^{-1} = \frac{1}{3}$, so that the transverse sections are diminished in the ratio of the cosine of that angle, the dimension of depth remaining unchanged. The dotted lines at the sides are drawn for each six inches of immersion, so that a line may be drawn across the whole of each vessel at the depth of immersion given in the tables, for the purpose of shewing the parts of the vessel below and above the surface of the fluid. Below the projection are given the unprojected water lines for each six inches of immersion; the lines of the bow are placed above those of the stern.

Fig. (5.) consists of the transverse sections, longitudinal section, water lines, and elevation of the Experimental Skiff of 1834. P', P'' and P''', the position of the tube of PITOT. T_1, T_2, T_3, P_2, T_5, T_6, glass gauges of immersion.

Fig. (6.) Shews the improved mode of obtaining a continuous moving force as used in 1835. The power of horses is not applied directly to the object to be moved, but acts on the end of a rope at A, which rope extends directly from A to a fixed pulley at B, whence it passes to the summit of a pyramidal structure 75 feet high round another fixed pulley C, descends to a pulley at the weight D, and passing round it returns to a second pulley at C, descends once more to B, and is finally attached to a dynamometer at E, the bow of the vessel. The power of the horses is therefore used to sustain the weight, while its gravity overcomes the resistance of the fluid. An assistant at the foot of the pyramid prevents the weight from turning round, and a second weight F may be used for acceleration previous to the commencement of the observations.

PLATE II.

Fig. 1.

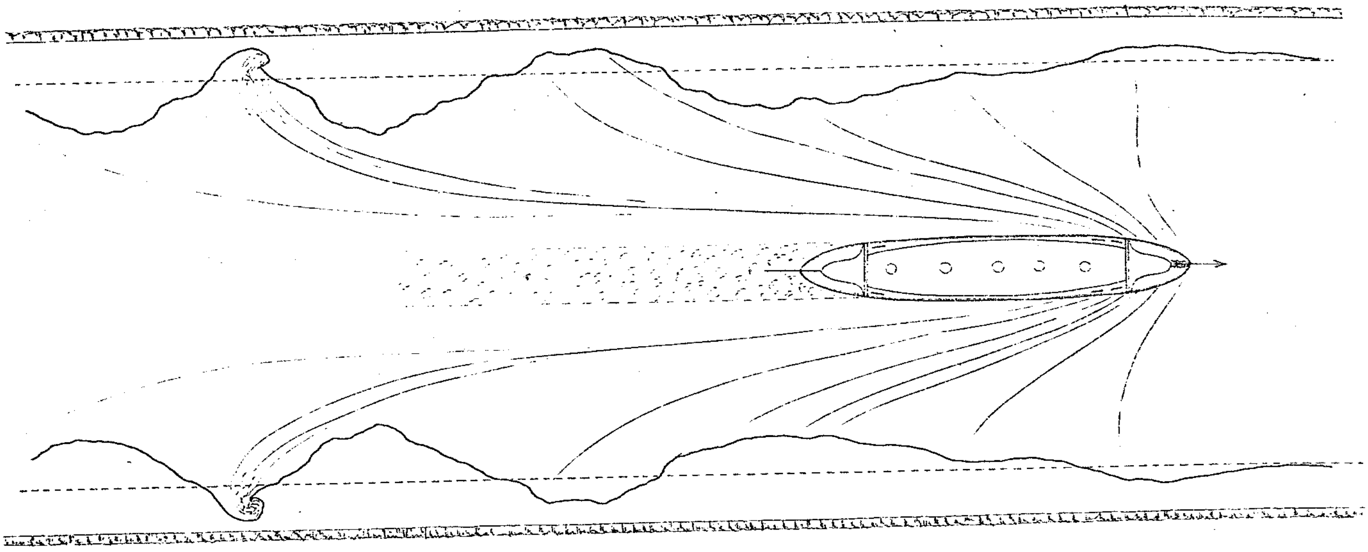


Fig. 2.

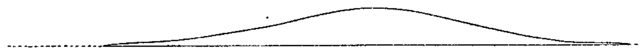


Fig. 3.

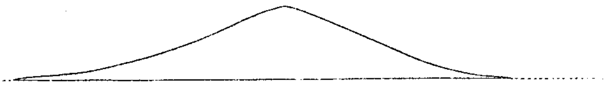


Fig. 4.

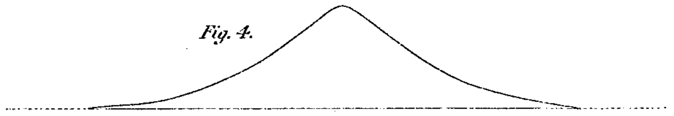


Fig. 5.

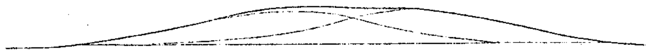
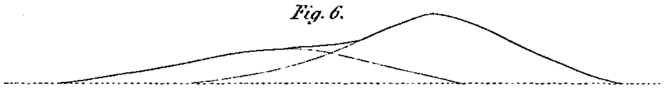


Fig. 6.



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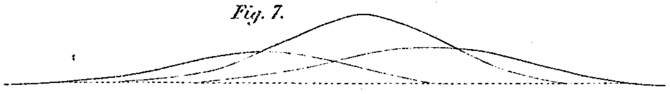

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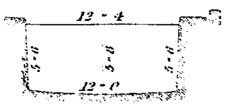

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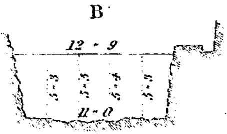

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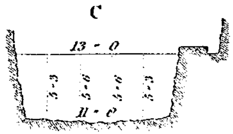

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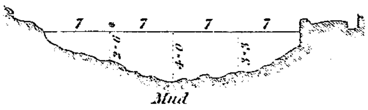
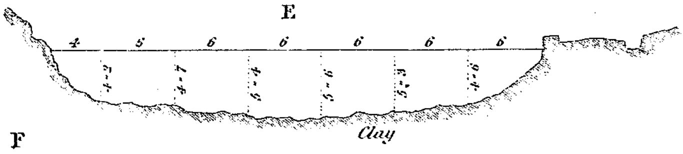
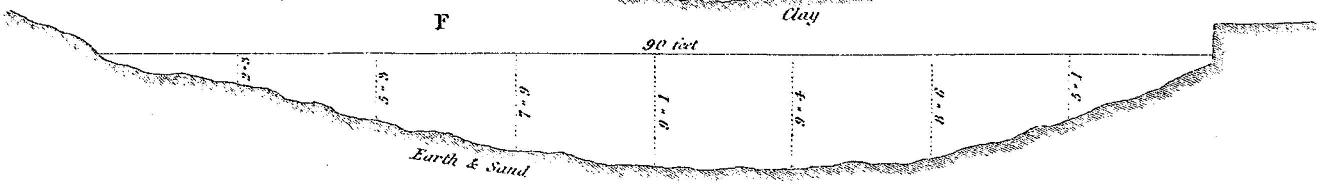


D

E



F



Mr. CONNELL'S PAPER.

Fig. 1.

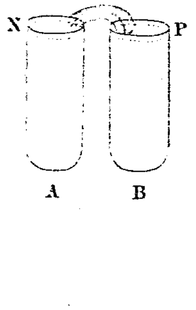


Fig. 2.

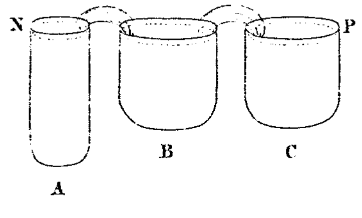


Fig. 3.

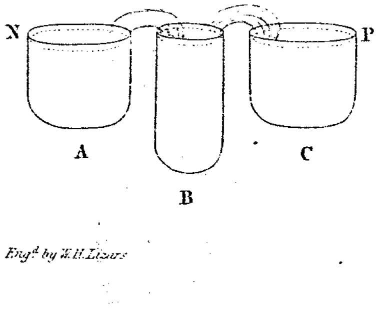
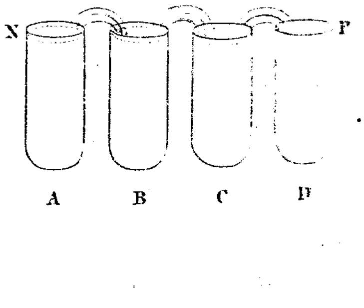


Fig. 4.



Engd. by W. H. Lizars

Fig. 2.
THE DIRLETON.

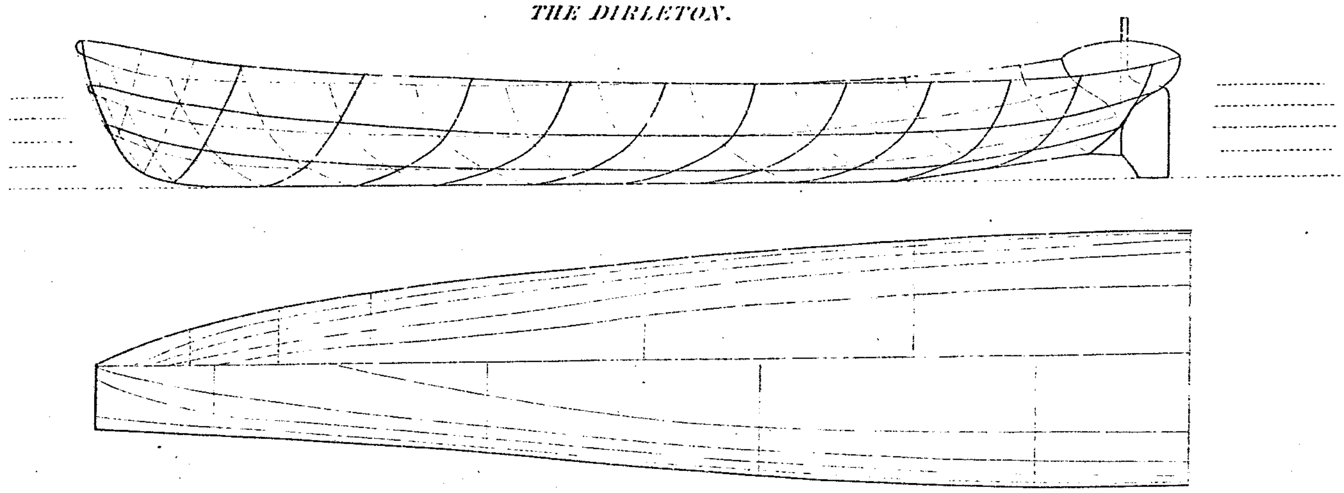


Fig. 4.
THE RAITH.

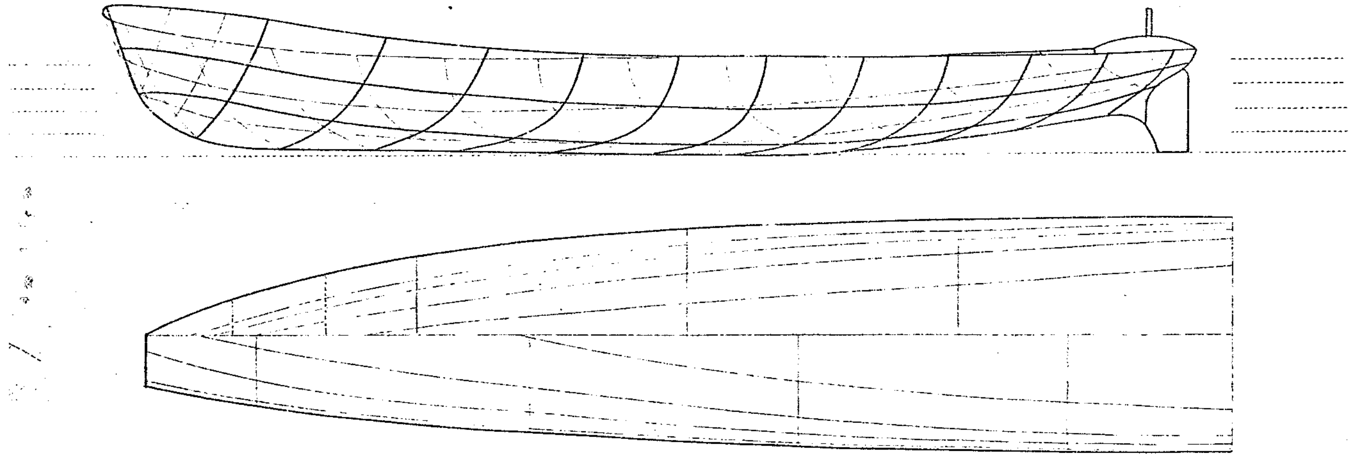


Fig. 5.
THE SKIFF.

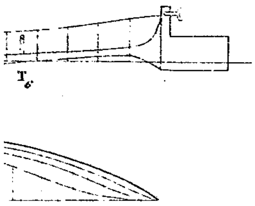
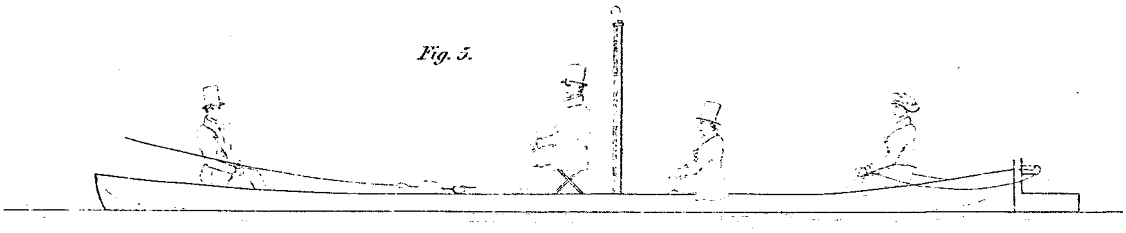


Fig. 5.



Eng^{d} by W.H. Lizars

Fig. 1.

THE WAVE.

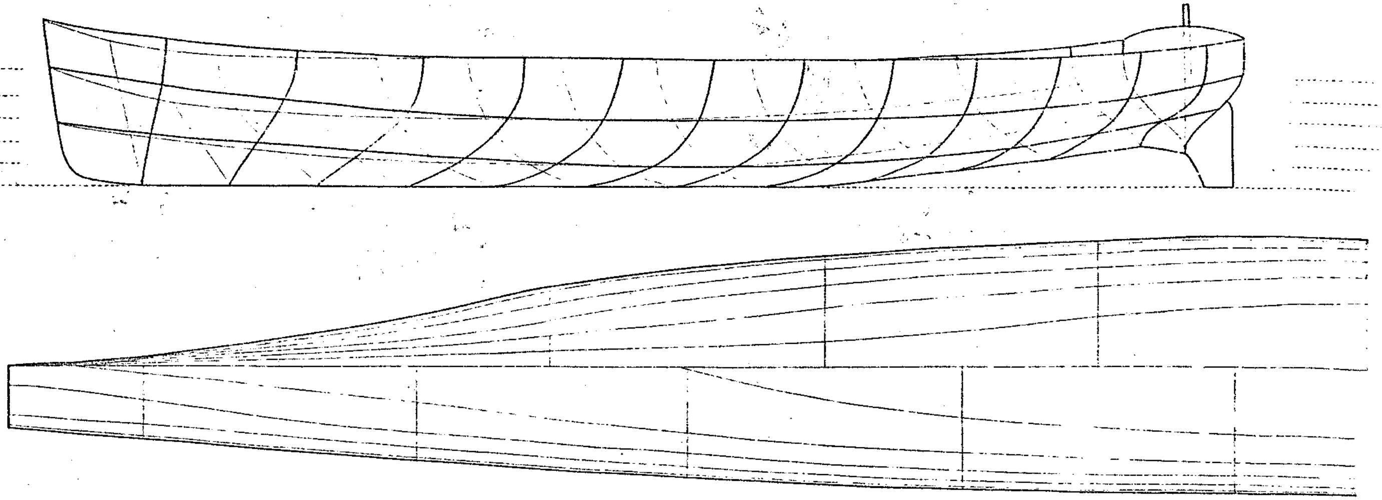
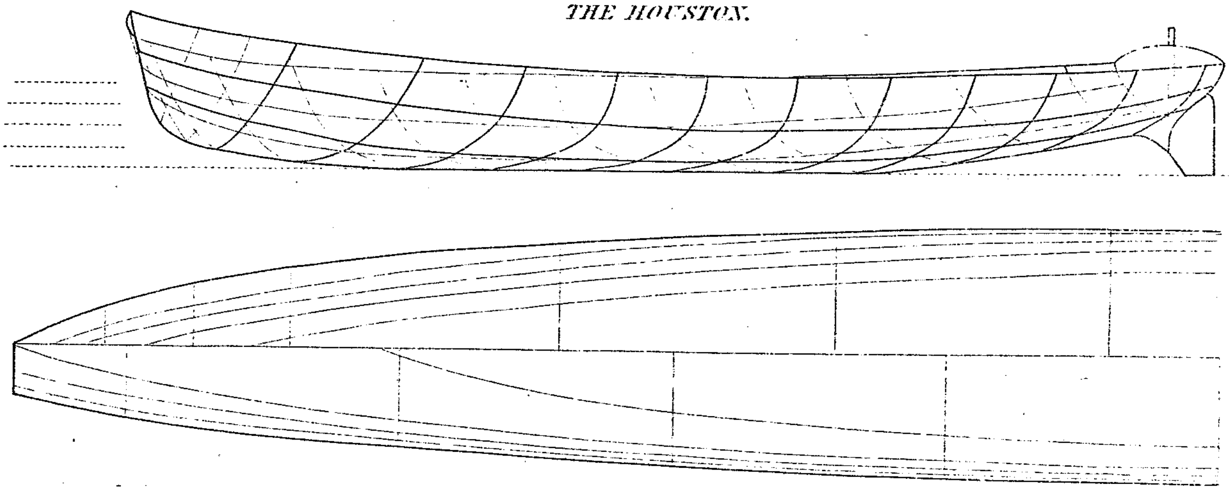


Fig. 3.

THE HOUSTON.



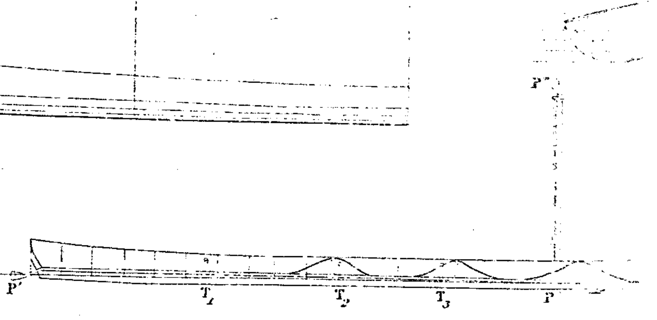


Fig. 5.

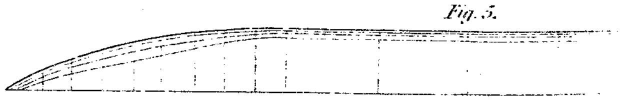
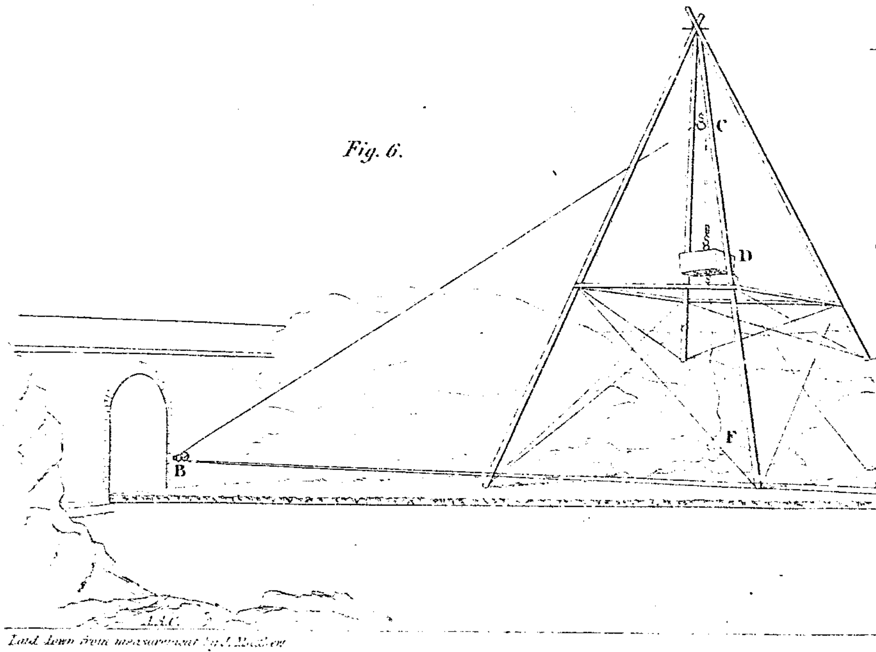


Fig. 6.



Laid down from measurement by J. [illegible]