Wave energy losses in intermediate depths

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Recent work on wave power devices has encouraged interest in the processes whereby waves lose energy and change direction in shoaling water, and especially in detailed calculations of their effects.[1,2] Here one of the most comprehensive sets of measurements available is examined, for four sites in depths of 15 to 100 m off South Uist in the Hebrides. The mean directional spectrum is recalculated for each site, and a proper allowance found for refraction which raises the estimates of net energy flux in intermediate depths by up to 10%. Indeed, the pattern of losses between 100 m and 23 m depths fits well with that expected from bottom friction. The estimated friction coefficient is quite high, as might be expected in view of the very rough sea bottom in the area.[3]

Key Words: wave power devices, lose energy, change direction, shoaling water, pattern of losses, friction coefficient.

BACKGROUND

Probably the most comprehensive sets of measurements for the evaluation of wave power are those made by the UK Institute of Oceanographic Sciences at sites in various depths of water off South Uist in the Hebrides. The principal data set is a long series (1976-present) of 3-hourly wave records from a waverider buoy sited in 42 m depth of water.[4–6] From this series a sample of 399 records has been selected, as representative of long term conditions at the site, by a stratified sampling procedure using long term wind records.[4,7] Simultaneous comparison measurements have been made over shorter periods with buoys in 100 m, 23 m and 15 m depths[6,8] (see Fig. 1), from which frequency-dependent correction factors have been calculated for these sites (see Fig. 2).[9]

These correction factors, however, apply only to the gross (measured) power levels. To estimate the wave power resource, that is the net energy flux across a line facing out to sea, knowledge of the directional distribution of power is crucial. Our only information on this directional distribution is a synthesis made by Crabb[4] for the representative sample of 399 records for the 42 m site, and this has been widely used in estimates of the resource and of device productivity. Crabb's synthesis is a fine achievement in making full use of the available meteorological data, but unfortunately takes no account of the finite water depth: and refraction, while negligible in 100 m depth for all but the lowest frequencies, is significant in 42 m depth for the wavelengths which carry most wave power.

METHODOLOGY

Wave climate data for South Uist thus include Crabb's long term mean spectrum for the 42 m site, but with a directional distribution more appropriate to the deep water (100 m) site: and frequency dependent corrections between the gross power levels at 42 m and the sites in other depths.

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The only simple way to unscramble these data appears to be the following procedure, which calculates a mean directional spectrum for each depth:

(1) Calculate a mean spectrum for the 100 m site by applying the simple frequency dependent correction to Crabb's spectrum.

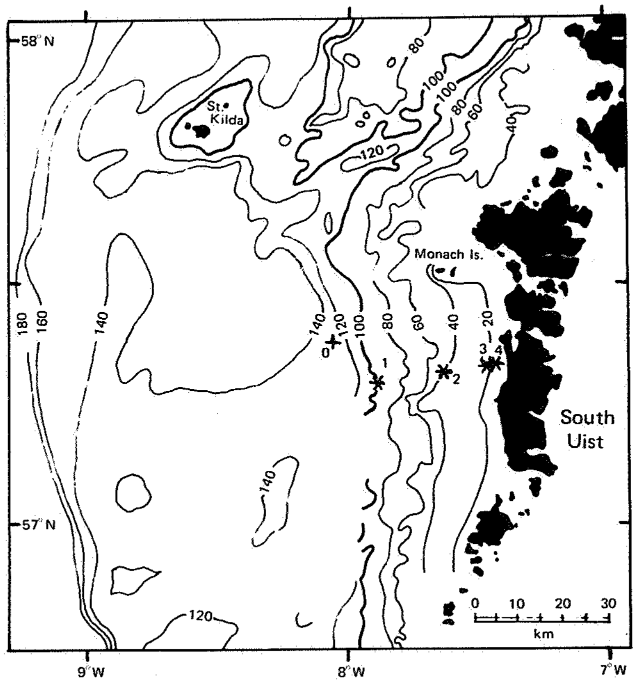

*Figure 1. The continental shelf west of South Uist, with depth contours in metres, showing data sites as follows: *1, 100 m IOS Waverider buoy; *2, 42 m IOS Waverider buoy; *3, 23 m IOS Waverider buoy; *4, 15 m IOS Waverider buoy; +0, 'S Uist' gridpoint of Meteorological Office hindcast model.*

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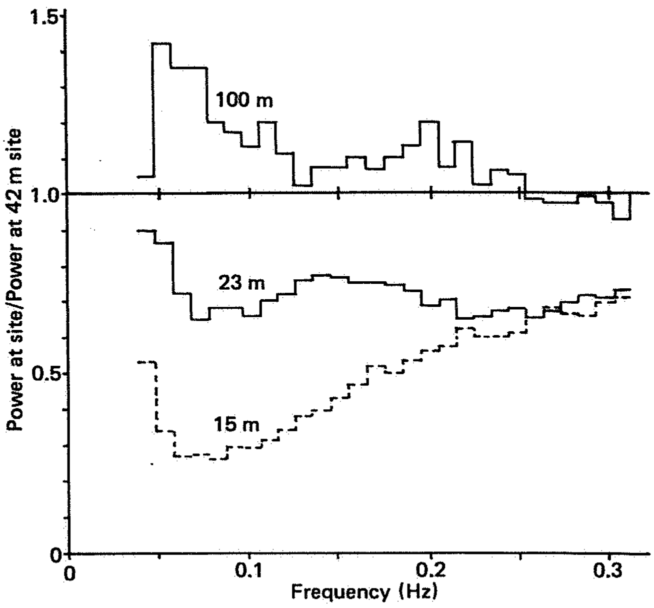

Figure 2. Ratios of gross power at other sites (100, 23 and 15 m) to gross power at the 42 m site, calculated in frequency bandwidths of 1/102.4 Hz.

(2) Follow each spectral component in to 42 m depth, taking account of refraction.
(3) For each frequency, scale down the spectral components so that the gross power level matches the measured figure for the 42 m site. The power loss is shared between the different directional components approximately in proportion to the distance travelled.
(4) Steps (2) and (3) are repeated to transfer the resulting spectrum first to 23 m and then to 15 m depth.

Several approximations are involved in this procedure, especially in step (2). Firstly, we assume parallel bathymetry (with contours running N-S), and ignore variations in the degree to which each site is sheltered, as for instance by the shallow water around the Monach Islands. Partly this is because of the sparsity of soundings and the consequent methodological difficulties: such studies as have been done have shown no clear pattern in their results.[1,10] Also, we are in any case more interested in the effect of shoaling water on the wave climate in general than on the peculiarities of the South Uist measurement sites *per se*.

Secondly, it is inappropriate to apply step (2) to the shorter wave periods, for which local generation is significant, so that some power will be travelling offshore. The refraction calculation (step (2)), which assumes that energy is only travelling in onshore directions, has therefore only been applied to components of period longer than a cut-off period (7.8 s for the 100 m to 42 m stage, 5.7 s for the inshore stages). For shorter periods, Crabb's synthesis is assumed to apply at all sites: this use of a simple cut-off is clearly rather a crude approximation, but it should be noted that less than 10% of the net energy flux at the 100 m site is associated with periods of under 7.8 s, and less than 2% with periods of under 5.7 s.

RESULTS

Gross and net power levels, and their quotient, the 'directionality factor', are shown in Table 1, together with net power levels previously calculated assuming Crabb's spectrum for the 42 m site. Note how in the previous calculation the net loss in the first stage, between 100 m and 42 m, appears anomalously high: roughly equal to that in the second stage: this led to the suggestion that the measurement of power in 100 m might be 10% too high.[11] With refraction in the first stage now taken into account, it seems that the explanation is rather that the calculated net power in 42 m was 10% too low.

Some confirmation of the estimates for the 100 m site is provided by data from the UK Meteorological Office's wind-wave hindcasting model[12] for a nearby deepwater site (see Fig. 1). Data for October 1978 to March 1981 yield an estimate of 67 kW/m annual average for the gross power, 45 kW/m net; the gross figure is likely to be too high because the Meteorological Office model allows unrealistic exposure to both north and south, but this should have only a marginal effect on the net figure.

The pattern of loss between 100 m and 23 m is consistent with a straightforward bottom friction mechanism, of loss through small-scale turbulent vortices, as first considered by Bagnold.[13] The friction loss rate is

$$(4\sqrt{2}/3\pi) f \rho \omega^3 H_b^3 \text{ watts/m}^2$$

taking Vitale's[14] definition of the dimensionless friction coefficient f rather than Bagnold's $(= (2/3)f)$, and letting H_b denote the rms wave movement at the sea bed. If we define $h_b = H_b/H_{rms}$, where H_{rms} is the rms surface movement for a wave of the same power in deep water, so that

Table 1. Gross and net wave power averages (and their ratio, the 'directionality coefficient') at each site, in kW/m, calculated (a) as described in the text, and (b) similarly, but neglecting refraction in water deeper than 42 m

Site	(a) Present calculation				(b) Previous calculation			
	Power			Directionality coefficient	Power			Directionality coefficient
	Gross	Net	Loss		Gross	Net	Loss	
100 m	58.1	46.0		0.81	58.1	46.0		0.81
			5.2 (11%)				8.6 (19%)	
42 m	47.8	40.8		0.85	47.8	37.4		0.78
			11.2 (27%)				8.4 (23%)	
23 m	32.8	29.6		0.90	32.8	29.0		0.885
			16.1 (54%)				15.7 (54%)	
15 m	14.9	13.5		0.91	14.9	13.3		0.893

the power flux takes the simple form $(1/2)\rho g^2 \omega^{-1} H_{\rm rms}$ watts/m; then the proportional power loss rate is

$$(8\sqrt{2}/3\pi)\,(fH_b)\,\omega^4 h_b/g^2 \text{ per metre}$$

Now both Bagnold and Vitale found that friction coefficients vary nearly inversely with wave amplitude; from Bagnold in particular we have $f = 0.018 H_b^{-0.75}$ for oscillations over small sand ripples. For the present calculation we therefore adopt the convenient approximation, fH_b = constant, so that the proportional power loss rate is independent of the amplitude of oscillations. It also means that our one further approximation, that losses can be calculated separately for separate spectral components, should not be too inaccurate.

With these approximations, we find that the losses between 100 m and 42 m, and between 42 m and 23 m, both lead to an estimated friction coefficient of $0.025/H_b$. Not surprisingly, in view of the very rough sea bottom off South Uist,[3] this is about twice as high as the values found by Bagnold for sand ripples (see above), but it is well within the range of observations and experiments quoted by Vitale.

The frequency dependence of losses is compared with what would be expected from bottom friction in Fig. 3. The agreement is especially good between 42 m and 23 m, where we might expect our method to be most reliable. Alternative mechanisms such as reflection, and backscattering of one sort or another,[1,15] should not give rise to energy losses varying so little with period. In any case, they are unlikely to be of the order of magnitude observed; it should be noted that Ewing[16] found no significant amounts of reflected energy in measurements of directional spectra off South Uist.

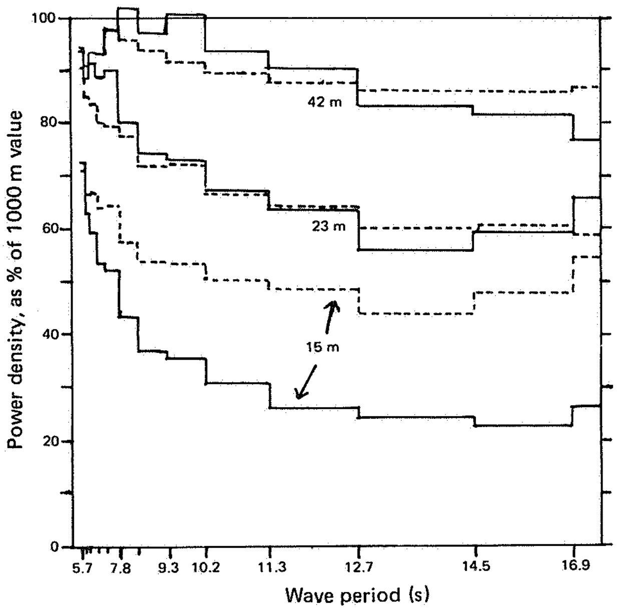

Figure 3. Power density off South Uist from the 270° (±15°) sector, for each of the shallower sites as a percentage of the value for the 100 m site: —— calculated; – – – fitted, $f = 0.025/H_b$. The horizontal axis shows period, stretched proportionately to the power density in 100 m. Thus the area under each curve represents the overall power density at that site (from the 270° sector = 20.8 kW/m in 100 m, 17.9 kW/m in 42 m (calculated and fitted), 13.2 kW/m in 23 m (calculated and fitted), 6.1 kW/m in 15 m calculated, 10.1 kW/m fitted)

Assuming a bottom friction mechanism with $f = 0.025/H_b$, we can estimate average energy losses for westerly waves between the edge of the continental shelf and the 100 m site: this comes to about 4%, almost all at periods of 12 s or more.

The losses between 23 m and 15 m depth are higher than expected from bottom friction, especially for longer period components where there is barely half as much power as might be expected. This may be tentatively explained as due to a combination of the following factors: (1) As already mentioned, measured friction coefficients decrease slightly slower with H_b than the assumed inverse relationship; and should therefore be larger between 23 m and 15 m than in deeper water (between 100 m and 23 m depth, H_b increases by a factor of five or more for waves of period less than 14 s). (2) Layers of kelp (*Laminaria Hyperborea*) 2 m long grow in this area,[17] which again may indicate a higher 'friction' coefficient than in deeper water. (3) Wave breaking is apparently not significant, but areas where waves break frequently occur both to north and south of the 15 m site;[17] this, together, with the concave bathymetry, suggests that local refractive defocusing is contributing to the low power levels at the 15 m site (it should be remembered that the present study simply assumes parallel bathymetry).

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