GAUGE INDEPENDENCE OF MASS COUNTERTERMS IN THE ABELIAN HIGGS MODEL AND GRAVITY

D. JOHNSTON

Laboratoire de Physique Théorique et Hautes Energies*, Université de Paris XI, Bâtiment 211, 91405 Orsay, France

We re-examine the work of Antoniadis et al. [1] on the apparent gauge-parameter dependence of the mass counterterm for a scalar field coupled to gravity and show that the same effect appears in a spontaneously broken abelian Higgs model. In both cases the Nielsen identities assure the gauge-parameter independence of the pole masses.

In ref. [1] Antoniadis, Iliopoulos and Tomaras observed that

$$\frac{\partial \Sigma}{\partial \xi}\bigg|_{\text{on-mass-shell}} = 0 \tag{1.1}$$

in a Yang-Mills/scalar system without spontaneous symmetry breaking, where Σ is the scalar self-energy and ξ is the gauge-parameter, whereas this was not true for a scalar minimally coupled to gravity. They suggested that this constituted an important difference between the two examples, although gauge invariance and unitarity were still guaranteed in the gravitational case by virtue of an identity describing the scale variance of the generating functionals. We shall show in this paper that (1.1) is the result of carrying out the calculation for the unbroken symmetry case, where the vacuum expectation value of the scalar field, $\langle \phi \rangle$, is zero. If we have $\langle \phi \rangle \neq 0$, eq. (1.1) is no longer true, but the appropriate Nielsen identity guarantees the gauge-parameter independence of the mass at the pole of the propagator. The gravitational case is similar to the latter, because here we have the full gravitational field acquiring a "vacuum expectation value". We have

$$\sqrt{-g}\, g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}, \tag{1.2}$$

where $h^{\mu\nu}$ is the graviton field and $\eta^{\mu\nu}$ is the Minkowski metric, so

$$\left\langle \sqrt{-g}\, g^{\mu\nu} \right\rangle = \eta^{\mu\nu} \tag{1.3}$$

but in this case the gauge symmetry is not broken. Amusingly, for the "symmetric" case in the gravitational theory

$$\left\langle \sqrt{-g}\, g^{\mu\nu} \right\rangle = 0 \tag{1.4}$$

we find that (1.1) would still hold true.

In sect. 2 of this paper we give a brief review of the Antoniadis et al. result at one-loop level and in sect. 3 we present a similar calculation for a spontaneously broken abelian (for simplicity) Higgs model, showing that (1.1) does not hold in this case. In sects. 4 and 5 we discuss these results in the light of the Nielsen identities for the theories.

2. One-loop results of Antoniadis et al.

We shall consider an abelian gauge theory for simplicity, non-abelian gauge theories merely introduce more indices and no further features. The lagrangian in ref. [1] was given by

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{1}{2} \left(\partial_\mu \phi_i \right) \left(\partial^\mu \phi_i \right)$$

$$- e \varepsilon_{ij} \left(\partial_\mu \phi_i \right) \phi_j A^\mu + \tfrac{1}{2} e^2 A^2 \phi^2 - \tfrac{1}{2} m^2 \phi^2 - \frac{1}{2\xi} G(A)^2, \tag{2.1}$$

where $\varepsilon_{10} = -\varepsilon_{01} = 1$, $i, j = 1, 2$ and $G(A)$ is the gauge-fixing function. A non-standard form was taken for the gauge-fixing

$$G(A) = \frac{\Box + \mu^2}{\mu^2} \left(\partial_\mu A^\mu \right), \tag{2.2}$$

which provided an infrared regulator via μ^2 (we solve this problem ourselves by adopting an R_ξ gauge in sect. 3). In the gauge of (2.2) the gauge-boson propagator is given by

$$iD_{\mu\nu} = \frac{1}{k^2} \left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) + \xi \frac{k_\mu k_\nu}{k^4} \frac{\mu^4}{\left(k^2 - \mu^2 \right)^2}, \tag{2.3}$$

which allows one to calculate the gauge dependent part of Σ, which we shall call Σ_ξ,
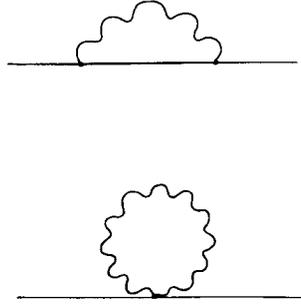
Fig. 1. One-loop self-energy graphs for the scalar field which contribute to Σ_ξ in the gauge used by Antoniadis et al.

from the two diagrams of fig. 1

$$\Sigma_\xi \sim \int d^4k \frac{(2p-k)^\mu (2p-k)^\nu k_\mu k_\nu}{k^4 (k^2-\mu^2)^2 [(p-k)^2 - m^2]} \bigg|_{p^2=m^2} - \int d^4k \frac{k^2}{k^4(k^2-\mu^2)} = 0. \quad (2.4)$$

From (2.4) we can see that (1.1) holds.

In the gravitational case the appropriate part of the lagrangian was given by

$$\mathscr{L} = \sqrt{-g} \left[-\frac{2}{k^2} R + \tfrac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - \tfrac{1}{2} m^2 \phi^2 \right] - \frac{1}{2\xi}(G^\nu)^2, \quad (2.5)$$

where R is the gravitational curvature, ϕ is the uncharged scalar field and

$$G^\nu = \frac{\Box + \mu^2}{\mu^2} \partial_\mu h^{\mu\nu}. \quad (2.6)$$

The gravitational equivalent of the diagrams in fig. 1 then gives us

$$\Sigma_\xi \sim -\xi m^2 \mu^2 \int d^4k \frac{1}{k^2 (k^2 - \mu^2)^2} \neq 0. \quad (2.7)$$

The dependence on μ^2 cancels after doing the integration

$$\Sigma_\xi \sim -\xi m^2, \quad (2.8)$$

so this is not an artifact of the chosen gauge. In the next section we shall see how to reproduce this non-zero result for the gauge-dependent part of Σ in a spontaneously broken abelian Higgs model.

3. One-loop results for a spontaneously broken abelian Higgs model

We shall now calculate Σ for spontaneously broken abelian Higgs model in an R_ξ gauge [2, 3]. We choose to work in this gauge rather than (2.2) because we have already done the relevant calculations ourselves in this gauge in ref. [3]! We take the lagrangian to be

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{1}{2} \left(\partial_\mu \phi_i \right)\left(\partial^\mu \phi_i \right)$$

$$- e\varepsilon_{IJ}\left(\partial_\mu \phi_i \right)\phi_j A^\mu + \tfrac{1}{2} e^2 A^2 \phi^2$$

$$+ \tfrac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!}\phi^4 - \frac{1}{2\xi}\left(\partial_\mu A^\mu + e\xi\varepsilon_{ij}\langle\phi_j\rangle\phi_i \right)^2$$

$$+ \partial_\mu C^* \, \partial^\mu C - e^2 \xi C^* C \varepsilon_{ij}\varepsilon_{il}\langle\phi_l\rangle\phi_j , \tag{3.1}$$

which differs from (2.1) in the sign of the mass term, the introduction of a quartic ϕ coupling and a different gauge choice. In this case the ghosts do not decouple and we have retained them (C^* and C). The theory given by (3.1) is spontaneously broken with, at the classical level

$$\langle\phi\rangle = \left(\frac{6m^2}{\lambda} \right)^{1/2} . \tag{3.2}$$

To calculate Σ at the one-loop level we must consider the diagrams of figs. 2a and 2b. To make our task more tractable, following ref. [2], we choose $\lambda \sim O(e^4)$ and
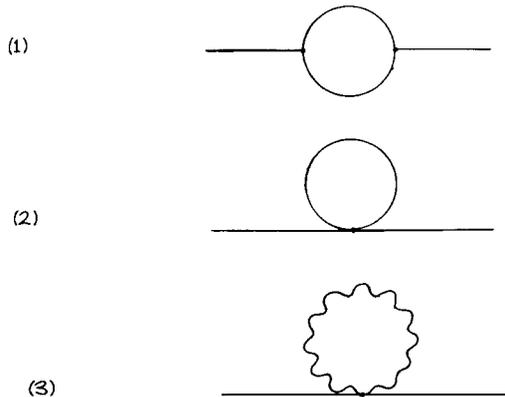

Fig. 2a. One-loop self-energy graphs which do not contribute to $\partial\Sigma/\partial p^2$ in our expansion scheme. Diagram 2a.1 is of $O(\lambda^2)$ and the other two are p^2 independent.
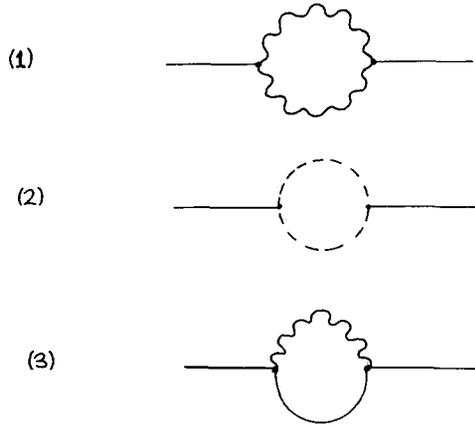
Fig. 2b. One-loop self-energy graphs which do give a contribution to $\partial \Sigma / \partial p^2$.

expand to the lowest order at which gauge dependence appears, which we know (with hindsight) to be $O(e^2\lambda)$. As we are already working to one-loop order the argument of Σ is the tree-level pole mass

$$m_1^2 = \tfrac{1}{3}\lambda\langle\phi\rangle^2 \tag{3.3}$$

so, expanding Σ to order $e^2\lambda$, we find

$$\Sigma\left(m_1^2\right) = \Sigma(0) + m_1^2 \frac{\partial\Sigma\left(p^2\right)}{\partial p^2}\bigg|_{p^2=0} + \cdots . \tag{3.4}$$

In the calculation of the second term in (3.4) the graphs in fig. 2a do not contribute because of the reasons stated in the figure captions. If we calculate the remaining graphs of fig. 2b in an $\overline{\text{MS}}$ renormalization scheme, with M as the arbitrary renormalization mass, we find

(2b.1) $\qquad \dfrac{\partial\Sigma}{\partial p^2}\bigg|_{p^2=0} = \dfrac{e^2\xi}{16\pi^2}\left[\dfrac{3}{\xi-1}\ln\xi - \dfrac{11}{6}\right],$

(2b.2) $\qquad \dfrac{\partial\Sigma}{\partial p^2}\bigg|_{p^2=0} = \dfrac{e^2\xi}{16\pi^2}\left[\dfrac{1}{6}\right],$

(2b.3) $\qquad \dfrac{\partial\Sigma}{\partial p^2}\bigg|_{p^2=0} = \dfrac{e^2\xi}{16\pi^2}\left[\dfrac{2}{3} + \ln\dfrac{e^2\xi\langle\phi\rangle^2}{M^2} - \dfrac{3}{\xi-1}\ln\xi\right], \tag{3.5}$

which sum to give

$$\frac{\partial \Sigma_{total}}{\partial p^2}\bigg|_{p^2=0} = \frac{e^2\xi}{16\pi^2}\left[\ln\frac{e^2\xi\langle\phi\rangle^2}{M^2} - 1\right]. \tag{3.6}$$

To complete our calculation of (3.4) we note that

$$\frac{\partial^2 V^{(1)}}{\partial^2\bar{\phi}}\bigg|_{\bar{\phi}=\langle\phi\rangle} = m_1^2 + \Sigma(0), \tag{3.7}$$

where $V^{(1)}$ is the one-loop effective potential. This gives us [2, 3]

$$\Sigma(0) = \frac{e^2\xi\lambda\langle\phi\rangle^2}{32\pi^2}\left[\frac{1}{3}\ln\frac{e^2\xi\langle\phi\rangle^2}{M^2} - \frac{1}{3}\right] - \frac{1}{3}\lambda\langle\phi\rangle^2 \tag{3.8}$$

and for (3.4)

$$\Sigma(m_1^2) = \frac{e^2\xi\lambda\langle\phi\rangle^2}{32\pi^2}\left[\ln\frac{e^2\xi\langle\phi\rangle^2}{M^2} - 1\right] - \frac{1}{3}\lambda\langle\phi\rangle^2, \tag{3.9}$$

which is manifestly gauge-parameter dependent when $\langle\phi\rangle \neq 0$

$$\frac{\partial\Sigma(m_1^2)}{\partial\xi} = \frac{e^2\lambda\langle\phi\rangle^2}{32\pi^2}\ln\frac{e^2\xi\langle\phi\rangle^2}{M^2}. \tag{3.10}$$

We have thus reproduced the apparent gauge-parameter dependence of the scalar self-energy at the mass pole in the simplest of gauge-boson/scalar systems, the abelian Higgs model. In the next section we explain why we have written "apparent" gauge parameter dependence and why this is linked to $\langle\phi\rangle \neq 0$.

4. The Nielsen identities: generalities

We shall follow closely the introductory discussion of ref. [2], which the reader is urged to consult. The Nielsen identities describe the gauge-parameter dependence of the generating functionals and objects derived from them, such as the effective potential V

$$-\Omega V = \Gamma|_{constant\ field}, \tag{4.1}$$

where Ω is the spacetime volume and Γ is the 1PI generating functional, or the pole mass for the scalar field given by the zero of

$$\frac{\delta^2\Gamma}{\delta\bar{\phi}(x)\delta\bar{\phi}(y)} \tag{4.2}$$

the inverse propagator. If we consider first the effective potential, we note that, being an off-shell object, it will depend on the gauge-parameter ξ (see, for example, the one-loop calculations in ref. [4]) as well as the semiclassical field which we will denote generically in this section by $\bar{\phi}$. The ground state will be given by the solution to

$$\frac{\partial V}{\partial \bar{\phi}}(\bar{\phi}, \xi) = 0 \tag{4.3}$$

and we will have spontaneous symmetry breaking if we can find a non-zero $\bar{\phi}$, say $\langle \phi \rangle$, such that (4.3) is satisfied. This solution will be gauge-invariant if

$$V(\langle \phi \rangle + \delta \langle \phi \rangle, \xi + \delta \xi) = V(\langle \phi \rangle, \xi) = V_{\min} \tag{4.4}$$

(remember that $\langle \phi \rangle$ is calculated using a gauge-fixed lagrangian, so a change in $\xi, \delta \xi$, will produce a corresponding change $\delta \langle \phi \rangle = \delta \langle \phi \rangle \delta \xi / \delta \xi$). Rewriting (4.4) in differential terms we find

$$\frac{\partial V}{\partial \bar{\phi}}\bigg|_{\xi} \frac{d \bar{\phi}}{d \xi} + \frac{\partial V}{\partial \xi}\bigg|_{\langle \phi \rangle} = 0, \tag{4.5}$$

which can also be written as

$$\frac{dV}{d\xi} = 0, \tag{4.6}$$

where $d/d\xi$ represents the total derivative with respect to ξ, which takes account of both the explicit ξ dependence and the implicit dependence via $\langle \phi \rangle$.

We have an equation similar to (4.5) for the gauge-parameter independence of other quantities, such as the pole mass for the scalar

$$\frac{\partial m^2}{\partial \bar{\phi}}\bigg|_{\xi} \frac{d \bar{\phi}}{d \xi} + \frac{\partial m^2}{\partial \xi}\bigg|_{\langle \phi \rangle} = 0. \tag{4.7}$$

Nielsen showed that it was, indeed, possible to derive equations of the form (4.5) and (4.7) by making use of the BRS [5] invariances of the theory concerned, thus guaranteeing the gauge-parameter independence of physical quantities. We emphasize that we do not find

$$\frac{\partial m^2}{\partial \xi}\bigg|_{\langle \phi \rangle} = 0 \tag{4.8}$$

as the condition for this gauge-parameter independence but rather (4.7) which takes

account of the implicit dependence in $\langle\phi\rangle$. Eq. (4.7) of course implies a similar condition for the gauge-parameter independence of the self-energy

$$\frac{\mathrm{d}\Sigma(m^2)}{\mathrm{d}\xi} = 0, \tag{4.9}$$

where the total derivative is again used. For the symmetric theory with $\langle\phi\rangle = 0$ eq. (4.8) will, however, be true which explains the observation that $\Sigma_\xi = 0$ to one-loop order in an unbroken abelian Higgs model and assures us that it will remain true to all orders.

5. The Nielsen identities: particulars

We shall derive the Nielsen identity for the inverse scalar propagator (4.2) in this section for both the gauge-boson/Higgs model and the gravity/scalar model, following the methods of refs. [3] and [6]. To facilitate our derivation of the identity we write the gauge-fixing for (3.1) in auxiliary field form

$$\tfrac{1}{2}\xi B^2 + B\big(\partial_\mu A^\mu + e\xi\varepsilon_{ij}\langle\phi_j\rangle\phi_i\big) \tag{5.1}$$

and introduce an additional BRS transformation on the gauge-fixing parameter

$$\delta\xi = \varepsilon\chi \tag{5.2}$$

where ε is the global, anticommuting BRS parameter and χ is also a global, anticommuting object. We must introduce compensating terms into the lagrangian

$$\mathcal{L}_\chi = \tfrac{1}{2}\chi C^* B + e\chi C^*\xi\varepsilon_{ij}\langle\phi_j\rangle\phi_i \tag{5.3}$$

to maintain its BRS invariance. Carrying out a BRS transformation on the augmented generating functional Z,

$$Z = \int[\mathrm{D}\Phi]\exp\left[\frac{i}{\hbar}\int\left(\mathcal{L}+\mathcal{L}_\chi+J_\mu A^\mu+\eta^*C+C^*\eta+J_B B+J_i\phi_i\right.\right.$$

$$\left.\left.+\rho_\mu\frac{\delta A^\mu}{\delta\varepsilon}+K_i\frac{\delta\phi_i}{\delta\varepsilon}\right)\right], \tag{5.4}$$

where we have introduced sources to couple to the BRS variations of ϕ_i and A_μ, and Legendre-transforming gives an equation of the following form for Γ

$$\int\left(\frac{\delta\Gamma}{\delta\bar{A}^\mu}\frac{\delta\Gamma}{\delta\rho_\mu}+\frac{\delta\Gamma}{\delta\bar{\phi}_i}\frac{\delta\Gamma}{\delta K_i}+\bar{B}\frac{\delta\Gamma}{\delta\bar{C}^*}\right)+\chi\frac{\partial\Gamma}{\partial\xi}=0. \tag{5.5}$$

If we differentiate (5.5) with respect to χ, using

$$\frac{\partial \Gamma}{\partial \chi}\bigg|_{\chi=0} = \Gamma\left(-\frac{1}{2\xi}\left(\partial_\mu A^\mu - e\xi\varepsilon_{ij}\langle\phi_j\rangle\phi_i\right)\right), \tag{5.6}$$

where P denotes an operator insertion, we find

$$\int\left(\frac{\delta\Gamma}{\delta\bar{A}^\mu}\frac{\delta\Gamma(P)}{\delta\rho_\mu} + \frac{\delta\Gamma}{\delta\bar{\phi}_i}\frac{\delta\Gamma(P)}{\delta K_i} + \bar{B}\frac{\delta\Gamma(P)}{\delta\bar{C}^*} + \frac{\partial\bar{B}}{\partial\chi}\frac{\delta\Gamma}{\delta\bar{C}^*}\right) - \frac{\partial\Gamma}{\partial\xi} = 0. \tag{5.7}$$

If we take the 1 direction in Higgs space as the physical scalar differentiating (5.7) twice with respect to ϕ_1 gives the Nielsen identity for the inverse propagator (4.2)

$$\int\left(\frac{\delta^3\Gamma}{\delta\bar{A}^\mu\delta^2\bar{\phi}_1}\frac{\delta\Gamma(P)}{\delta\rho_\mu} + 2\frac{\delta^2\Gamma}{\delta^2\bar{\phi}_1}\frac{\delta^2\Gamma(P)}{\delta K_1\delta\bar{\phi}_1} + \bar{B}\frac{\delta^3\Gamma(P)}{\delta\bar{C}^*\delta^2\bar{\phi}_1} + \frac{\delta^3\Gamma}{\delta^3\bar{\phi}_1}\frac{\delta\Gamma(P)}{\delta K_1}\right)$$

$$-\frac{\partial}{\partial\xi}\frac{\delta^2\Gamma}{\delta^2\bar{\phi}_1} = 0, \tag{5.8}$$

where we have been careless with the spacetime arguments for brevity. The first term vanishes because $\delta\Gamma(P)/\delta\rho_\mu$ is given by a sum of vacuum diagrams and, with only scalar legs on shell, there is no way for a vacuum diagram to carry just one index. The second term will vanish at the pole (if we were careful with spacetime arguments we would have to disentangle a convolution to demonstrate this), the third will vanish because $\bar{B} = 0$. We thus have at the pole (or, equivalently, zero of the inverse propagator)

$$\left(\frac{\partial}{\partial\xi} - \int\frac{\delta\Gamma(P)}{\delta K_1}\frac{\delta}{\delta\bar{\phi}_1}\right)\frac{\delta^2\Gamma}{\delta^2\bar{\phi}_1} = 0, \tag{5.9}$$

which implies

$$\left(\frac{\partial}{\partial\xi} - \int\frac{\delta\Gamma(P)}{\delta K_1}\frac{\delta}{\delta\phi_1}\right)m^2 = 0, \tag{5.10}$$

where m^2 is the pole mass. This is of precisely the form (4.7) (*not* (4.8)) which we

lowed was necessary for gauge-parameter independence. At the one-loop level
[, 3]

$$\int \frac{\delta \Gamma(P)}{\delta K_1} = \frac{e^2 \xi \langle \phi \rangle}{32\pi^2} \ln \frac{e^2 \xi \langle \phi \rangle^2}{M^2},$$ (5.11)

) if we had $\langle \phi \rangle = 0$ we would find

$$\frac{\partial m^2}{\partial \xi} = 0$$ (4.8)

; did Antoniadis et al. in their gauge. For the gravitational theory we consider

$$\mathscr{L} = -\frac{2}{K^2} \sqrt{-g} \left(R + \Lambda \right) + \tfrac{1}{2} \xi B_\mu B_\nu \eta^{\mu\nu} + B_\mu G^\mu + C_\mu^* M_\nu^\mu C^\nu$$

$$+ \sqrt{-g} \left(\tfrac{1}{2} (\nabla_\mu \phi)(\nabla_\nu \phi) g^{\mu\nu} - \tfrac{1}{2} m^2 \phi^2 \right),$$ (5.12)

here Λ is the cosmological constant, $C^{*\mu}$ the antighost, C^ν the ghost, B_μ the
ιxiliary gauge-fixing field, M the Fadeev-Popov determinant, and ∇_μ the gravita-
ɔnal covariant derivative. The G^μ is again given by (2.6)

$$G^\mu = \frac{\Box + \mu^2}{\mu^2} \partial_\rho h^{\mu\rho}$$ (2.6)

ɪ going through the same procedure that led to (5.10) we find, again on-scalar
ass-shell

$$\left(\frac{\partial}{\partial \xi} - \int \frac{\delta \Gamma(P)}{\delta K} \frac{\delta}{\delta \bar{\phi}} - \int \frac{\delta \Gamma(P)}{\delta \rho_{\mu\nu}} \frac{\delta}{\delta \bar{h}^{\mu\nu}} \right) m^2 = 0,$$ (5.13)

here K couples to the BRS variation of ϕ

$$\delta \phi = -\varepsilon C^\alpha \partial_\alpha \phi$$ (5.14)

ιd $\rho_{\mu\nu}$ couples to the BRS variation of $h_{\mu\nu}$

$$\delta h^{\mu\nu} = \varepsilon \big[\partial^\mu C^\nu + \partial^\nu C^\mu - \eta^{\mu\nu} (\partial_\alpha C^\alpha)$$

$$- \partial_\alpha (C^\alpha h^{\mu\nu}) + h^{\alpha\nu} \partial_\alpha C^\mu + h^{\alpha\mu} \partial_\alpha C^\nu \big].$$ (5.15)

ow $\langle \phi \rangle = 0$, so the second term in (5.13) will vanish but the last term need not

because we can have

$$\int \frac{\delta\Gamma(P)}{\delta\rho_{\mu\nu}} = -\tau(\bar{\phi}, \bar{h}, \xi)\eta^{\mu\nu} \qquad (5.16)$$

which we could not have if $\langle \sqrt{-g}\, g^{\mu\nu} \rangle = 0$, so

$$\left(\frac{\partial}{\partial\xi} + \tau\eta^{\mu\nu} \frac{\delta}{\delta\bar{h}_{\mu\nu}} \right) m^2 = 0. \qquad (5.17)$$

This means, following the discussion of sect. 4, that under the transformation $\xi \to \xi + \delta\xi$ we preserve gauge-parameter independence with a compensating transformation $\langle h^{\mu\nu} \rangle \to \langle h^{\mu\nu} \rangle + \tau\eta^{\mu\nu}\delta\xi$. However, we have defined

$$\langle \sqrt{-g}\, g^{\mu\nu} \rangle = \eta^{\mu\nu}, \qquad (1.3)$$

which means that

$$\langle h^{\mu\nu} \rangle = 0 \qquad (5.18)$$

so we must reinterpret (5.17) as implying a rescaling of $\eta^{\mu\nu}$

$$\eta^{\mu\nu} \to (1 + \tau\delta\xi)\eta^{\mu\nu}. \qquad (5.19)$$

This, in turn, could be interpreted as a global scale transformation

$$x^\mu \to (1 + \tau\delta\xi)^{1/2} x^\mu. \qquad (5.20)$$

The action, apart from the gauge fixing, is invariant under (5.20) if we assign the following transformation properties to the fields

$$\delta' h^{\mu\nu} = -(n-2)\eta^{\mu\nu} - \left[n - 2 + x^\mu \partial_\mu \right] h^{\mu\nu},$$

$$\delta' F = -\left[d_F + x^\mu \partial_\mu \right] F, \qquad (5.21)$$

where F represents all the other fields apart from h and $d_c = d_{c^*} = d_\phi = 0$, $d_B = 1$. We find the following equation for the variation of Γ under (5.20) [1]

$$2(n+2)\xi \frac{\partial\Gamma}{\partial\xi} = \int \frac{\delta\Gamma}{\delta\bar{h}^{\mu\nu}} \left[n - 2 + x^\mu \partial_\mu \right] \bar{h}^{\mu\nu}$$

$$\times \int \frac{\delta\Gamma}{\delta\bar{\phi}} (d_\phi + x^\mu \partial_\mu)\bar{\phi} + (n-2) \int \frac{\delta\Gamma}{\delta\bar{h}^\mu_\mu}. \qquad (5.22)$$

When differentiated twice with respect to $\bar{\phi}$ and Fourier-transformed this gives

$$2(n+2)\xi\frac{\partial}{\partial\xi}\frac{\delta^2\Gamma}{\delta^2\bar{\phi}}\bigg|_{p^2=m^2} = (n-2)\frac{\delta^3\Gamma}{\delta\bar{h}^\mu_\mu\delta^2\bar{\phi}} - p_\mu\frac{\partial}{\partial p_\mu}\frac{\delta^2\Gamma}{\delta^2\bar{\phi}}\bigg|_{p^2=m^2}, \quad (5.23)$$

which gives a relation between functions of the same order when calculated in a loop expansion. It is thus an alternative expression of the result of (5.17), which, because the expansion of τ starts at one-loop order, mixes orders in the loop expansion. Both (5.17) and (5.23) state that the apparent gauge parameter dependence of the mass pole is balanced out by a global scale transformation which is analogous to the gauge parameter dependence in the abelian Higgs model being balanced by a change in $\langle\phi\rangle$.

References

[1] I. Antoniadis, J. Iliopoulos and T.N. Tomaras, Nucl. Phys. B267 (1986) 497
[2] I.J.R. Aitchison and C.M. Fraser, Ann. of Phys. (NY) 156 (1984) 1
[3] D. Johnston, Nucl. Phys. B253 (1985) 687
[4] R. Jackiw, Phys. Rev. D9 (1974) 1686
[5] C. Becchi, A. Rouet and R. Stora, Phys. Lett. B52 (1974) 344
[6] O. Piguet and K. Sibold, Nucl. Phys. B253 (1985) 517