Diagrams for Meaning Preservation

2003-06-13

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Useful
Logics,
Types,
Rewriting, and their
Automation

Overview

- **Motivation.**
- The AES framework.
- Methods for proving meaning preservation.
- Discussion.

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- The notation $\mathsf{has\text{-}nf}(R, t_1)$ means $t_1 \xrightarrow{R,\mathsf{nf}}\!\!\!\!\twoheadrightarrow t_2$ for some t_2.
- Trm, Conf, LConf, and Std are short names for *termination*, *confluence local confluence*, and *standardization*.

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Operational meaning:
$$\mathsf{result}(t) = \begin{cases} \mathsf{diverges} & \text{if } \neg\text{has-nf}(\mathbb{E}, t) \\ \mathsf{halt} & \text{if } t \xrightarrow{\mathbb{E},\mathsf{nf}} \lambda x.\, t' \\ \mathsf{stuck} & \text{if } t \xrightarrow{\mathbb{E},\mathsf{nf}} t' \neq \lambda x.\, t'' \end{cases}$$

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Is it an *observational equivalence*, i.e., does $t_1 \longrightarrow t_2$ imply $\text{result}(C[t_1]) = \text{result}(C[t_2])$ for any context C?

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Let the *non-evaluation steps* be $\mathbb{N} = \mathbb{S} \setminus \mathbb{E}$.

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$$t_1 \xrightarrow{\mathcal{S}_1, \mathcal{S}_2} t_2 \Leftrightarrow \; t_1 \xrightarrow{\mathcal{S}_1 \cap \mathcal{S}_2} t_2$$

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- Evaluation steps must preserve results, i.e., $t_1 \xrightarrow{\mathbb{E}} t_2$ must imply that $\mathsf{result}(t_1) = \mathsf{result}(t_2)$.
- The intention is to model execution where the only way to observe a result is to do evaluation steps as long as possible and then inspect the halted term, which is unique even when evaluation is non-deterministic (a deliberate AES framework limitation).

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  - Low-level proof methods with elementary diagrams.
  - Marks (e.g., finite developments).
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- This talk will focus on rewriting-based methods: Plotkin [1975], Machkasova and Turbak [2000], Odersky [1993], Ariola and Blom [2002].

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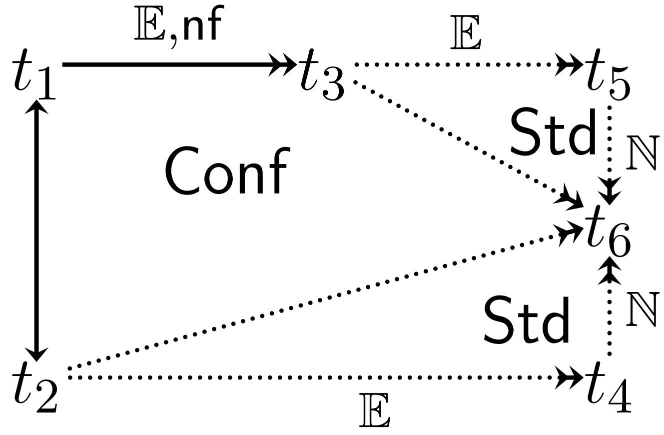

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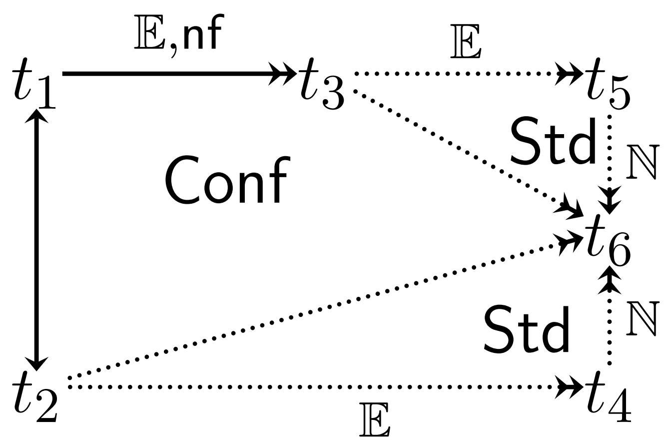

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Because t_5 has halted and $\mathbb{N}$-conversion preserves both this fact and results (important!), $\mathsf{result}(t_5) = \mathsf{result}(t_4)$.

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- Requiring standardization tends to force the evaluation contexts and rewrite rules to look arbitrarily deep into the term and inspect an arbitrary number of tree nodes.

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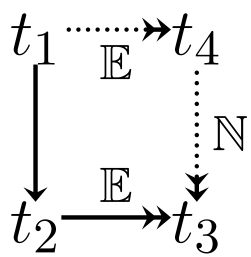

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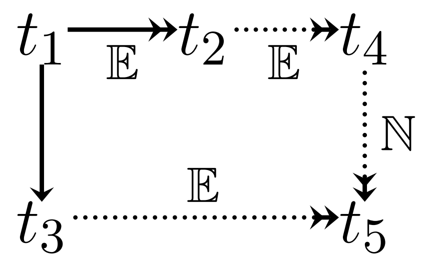

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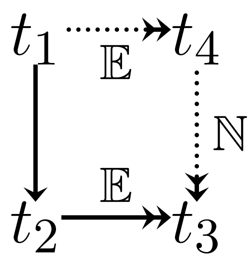

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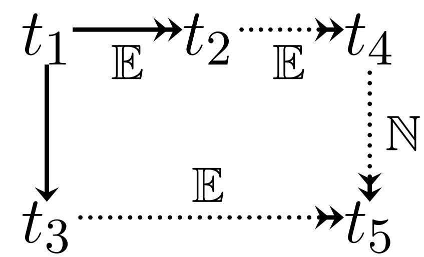

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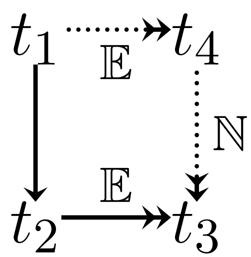

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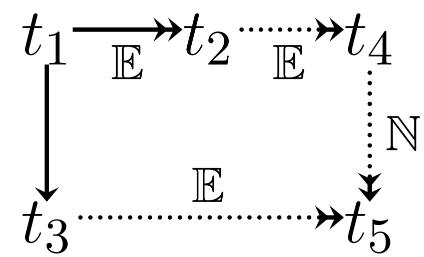

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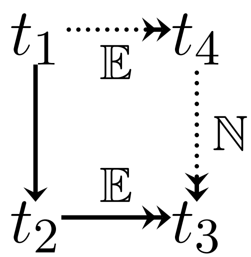

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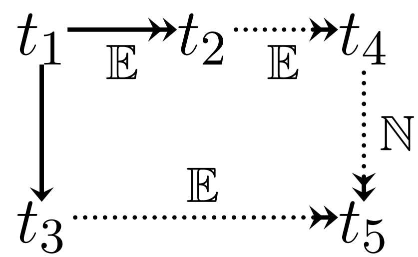

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In particular, Lift and Project can be used to prove correctness of Ariola/Blom/Klop-style rewrite rules for letrec.

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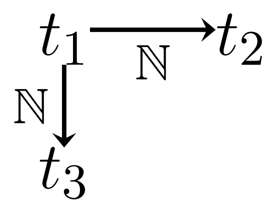

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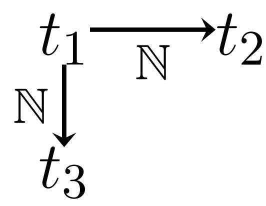
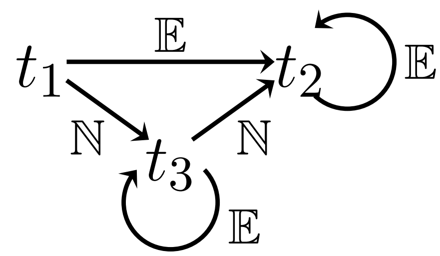


- (*New result:*) Confluence & standardization can handle cases for which Lift & Project fail, e.g.:



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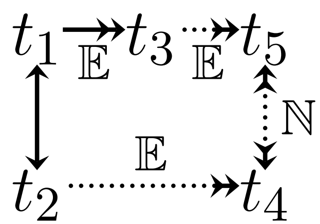
Weakening the Proof Burden (1)

- By carefully inspecting how confluence & standardization and Lift & Project prove meaning preservation, we obtained the following weaker *Lift/Project* (LP) diagram which implies meaning preservation:

LP :



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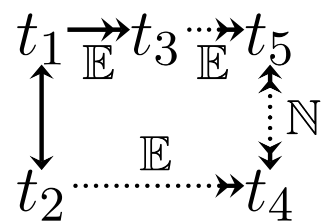
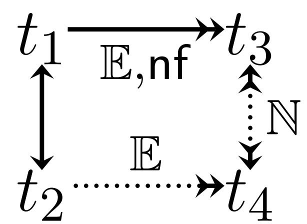


- We further weakened LP to the following *Lift/Project when Terminating* (LPT) diagram:

LPT :



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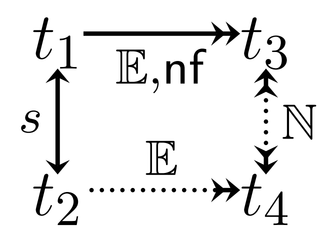

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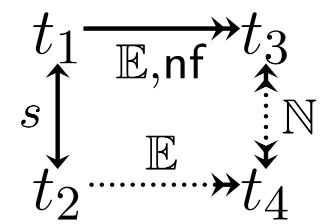

  - This is important because sometimes different methods are needed for different parts of a rewriting system.

Overview

- Motivation.
- The AES framework.
- **Methods for proving meaning preservation.**
  - Previous high-level proof methods.
  - New high-level proof methods.
  - **Low-level proof methods with elementary diagrams.**
  - Marks (e.g., finite developments).
- Discussion.

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- We present 2 new meaning preservation proof methods which are *low-level* because their conditions are only *elementary diagrams* and simple (to understand, not necessarily to prove) kinds of termination.
- It is expected that a rewriting system will be partitioned into step sets that are closed under "residuals w.r.t. evaluation" and the right method will be used for each partition. Often, each partition will contain all of the steps for some subset of the rewrite rules.

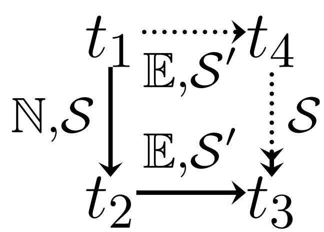
Low-level Method 1

Well Behaved with Standardization:

$$\mathsf{WB{+}Std}(\mathcal{S}) \iff \mathsf{Trm}(\mathbb{E} \cap \mathcal{S}) \wedge \mathsf{WL1}(\mathcal{S}, \mathcal{S}) \wedge \mathsf{WL1}(\mathcal{S}, \mathbb{S}) \wedge \mathsf{WP1}($$

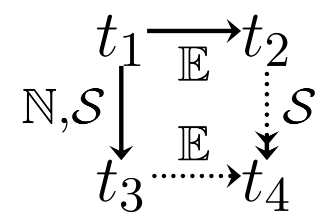
Weak Lift 1-Step:

$$\mathsf{WL1}(\mathcal{S}, \mathcal{S}') \iff$$



Weak Project 1-Step:

$$\mathsf{WP1}(\mathcal{S}) \iff$$



Useful for difficult rewrite step sets which do not have finite developments, e.g., Ariola/Blom/Klop-style letrec rewrite rules.

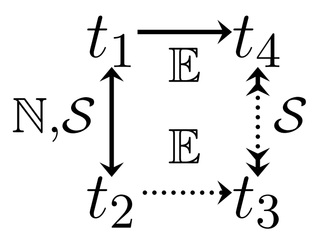
Low-level Method 2

Well Behaved without Standardization:

$$\mathsf{WB“Std}(\mathcal{S}) \iff \mathsf{Trm}(\mathcal{S}) \wedge \mathsf{LConf}(\mathcal{S}) \wedge \mathsf{WLP1}(\mathcal{S}) \wedge \mathsf{NE}(\mathcal{S})$$

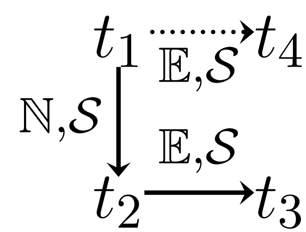
Weak Lift/Project 1-Step:

$$\mathsf{WLP1}(\mathcal{S}) \iff$$



$\mathbb{N}$-Steps Do Not Create $\mathbb{E}$-Step

$$\mathsf{NE}(\mathcal{S}) \iff$$



Useful for difficult rewrite step sets which do not have standardization but do have termination.

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- The marks typically force termination by forbidding contracting unmarked redexes and ensuring "created" redexes are unmarked.
- The rewriting system is embedded in a larger marked system with additional marked terms and rewrite steps. Proving the larger system correct is enough.
- We give conditions on marking such that proving LPT for $\mathcal{S} \cap \mathbb{M}$ (i.e., the marked fragment of the larger marked system) is sufficient to prove LPT for $\mathcal{S}$.

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Related Work

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Related Work

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- The work of Ariola and Blom [2002] has important similarities at a deep level. Their framework does not make it easy to prove operational properties, e.g., the user must prove a connection between *infinite normal forms* and operational behavior. Also, there are no low-level abstract proof methods.
- Odersky [1993] gives conditions that a transformation is an observational equivalence. Despite similarities, the formal presentation is quite different and tied to a particular syntactic formalism.

Conclusions

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- Our proof methods are designed to be easy for someone who is not a rewriting specialist to read, understand, and apply to their programming language calculi.
- We expect that our methods will help in making the expertise of the rewriting community accessible and useful to the outside world.

References

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