Restoring Natural Language as a Computerised Mathematics Input Method

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joint work with
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A Bit of Mathematics

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Omega	`(inverse-exist R op (struct-unit R op)).`

Goal: Smoothing and strengthening transitions.

MathLang grammatical categories

term	Common mathematical objects.	"R", "0", "$a + b$"
set	Sets of mathematical objects.	"$\mathbb{R}$"
noun	Categories to classify **terms**	"ring"
adjective	Modifiers for **nouns**	"Abelian"
statement	Assertions of truth	"$a = b$"
declaration	Type signature designations	"Addition is denoted $a + b$"
definition	New symbol introductions	"A ring is. . ."
step	A group of mathematical assertions.	"We have. . . . . . and also. . ."
context	Assertions preliminary to a **step**	"Given a ring R, . . ."

Example

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Anatomy of a box

<interp> contents

Color	Grammatical category
Contents	Original mathematics
<interp>	Logical interpretation

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Examples:

<equal> a is equal to b

<equal> $a = b$

<ident> $a = b$

<A> A <reals> $\mathbb{R}$

<inter> <apples> A $\cap$ <oranges> O

Another Example

$$a = b = c$$

Another Example

$$\boxed{a} = \boxed{b} = \boxed{c}$$

Another Example

$$a = b = c$$

Another Example

$$a = b = c$$

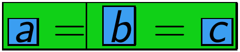
What next?

Another Example

$$a = b = c$$

?

Another Example



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What does this mean?

How do we cope?

$$a = b = c$$

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How do we cope?

$$a = b = c$$

- Compound statement
- Short for "$a = b$ and $b = c$"
- Must be translated before computerisation

Syntax sugaring:

- Common in many computer languages
- Used for pretty-printing
- Eases human use of languages
- Always: nice for computers $\longrightarrow$ nice for humans

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Syntax souring:

- A new transformation: Syntax souring
- Syntax souring solves the problem of $a = b = c$.
- Other direction: nice for humans $\longrightarrow$ nice for computers

The ULTRA Solution

Another look at the problem

$$a = b = c$$

- The relation "$=$" is binary: it takes *two* arguments
- The term "b":
  - Appears only once
  - Is actually provided as argument twice
  - Is "shared"
- **Goal:** tell $=$ to be nice and share

The ULTRA Solution

$$a = \qquad\qquad b \quad = c$$

The ULTRA Solution

$$\boxed{a} = \qquad \boxed{b} \quad = \boxed{c}$$

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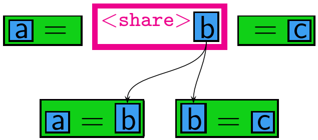

Kinds of Souring

share	Natural splitting of single argument	}	Duplication
chain	More flexible forwarding of entities		
fold	Recursion upon lists	}	List operations
map	Iteration over lists		
position	Reordering of arguments	}	Reordering

Kinds of Souring: Duplication

share • chain • fold • map • position

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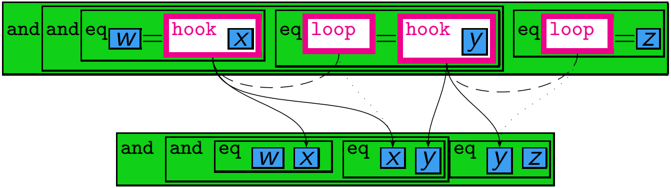
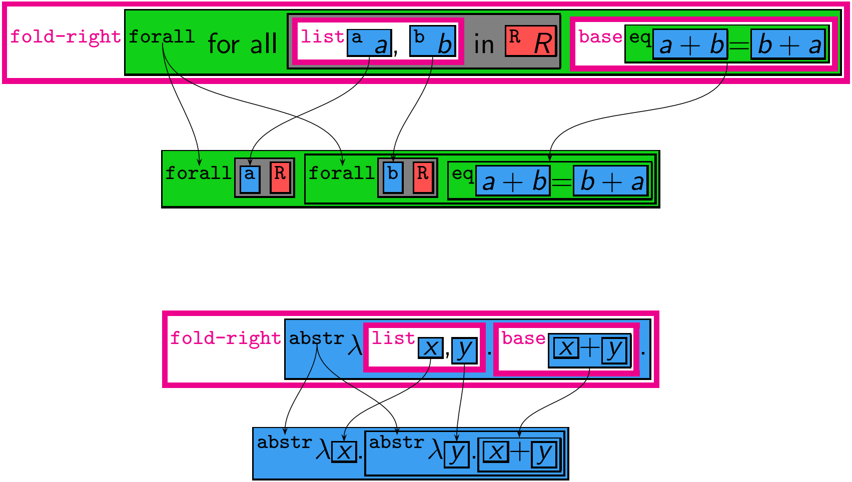


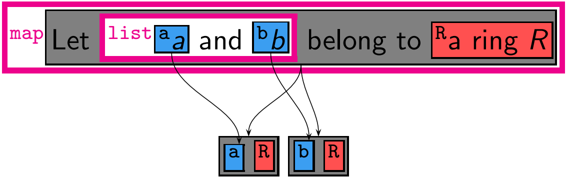
Kinds of Souring: Lists

share • chain • **fold** • map • position



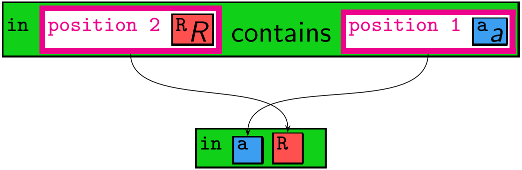
Kinds of Souring: Lists

share • chain • fold • map • position



Kinds of Souring: Reordering

share • chain • fold • map • position



Conclusion

- Five kinds of souring:

  `share` • `chain` • `fold` • `map` • `position`
- Common goal: elucidating the intent of language

- Future Work:
  - Look for other souring needs
  - Automate the annotation process
  - Identify appropriate granularity for annotation
  - Arrive at recommendations/conventions for annotation
  - Cope with ellipsis.

$$\overbrace{x + \ldots + x}^{n \text{ times}} \qquad 2^{2^{\cdot^{\cdot^{\cdot^{2}}}}} \qquad \frac{1}{1 + \frac{1}{1+\cdots}}$$

Text and Symbol

Box Annotation

Souring annotation

Souring Examples

Conclusion