Proof-Carrying-Code

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Motivation

Downloading software over the network is nowadays common-place.

But who says that the software does what it promises to do?

Who protects the consumer from malicious software or other undesirable side-effects?

$\Longrightarrow$ **Mechanisms for ensuring that a program is "well-behaved" are needed.**

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Authentication for Mobile Code

The main mechanisms used nowadays are based on authentication.
Java:

- Originally a sandbox model where all code is untrusted and executed in a secure environment (**sandbox**)
- In newer versions security policies can be defined to have more fine-grained control over the level of security defined. Managed through cryptographic signatures on the code.

Authentication for Mobile Code

Windows:

- Microsoft's Authenticode attaches cryptographic signatures to the code.
- User can distinguish code from different providers.
- Very widely used — more or less compulsory in Windows XP for device drivers.

But, all these mechanisms say nothing about the code, only about the supplier of the code!

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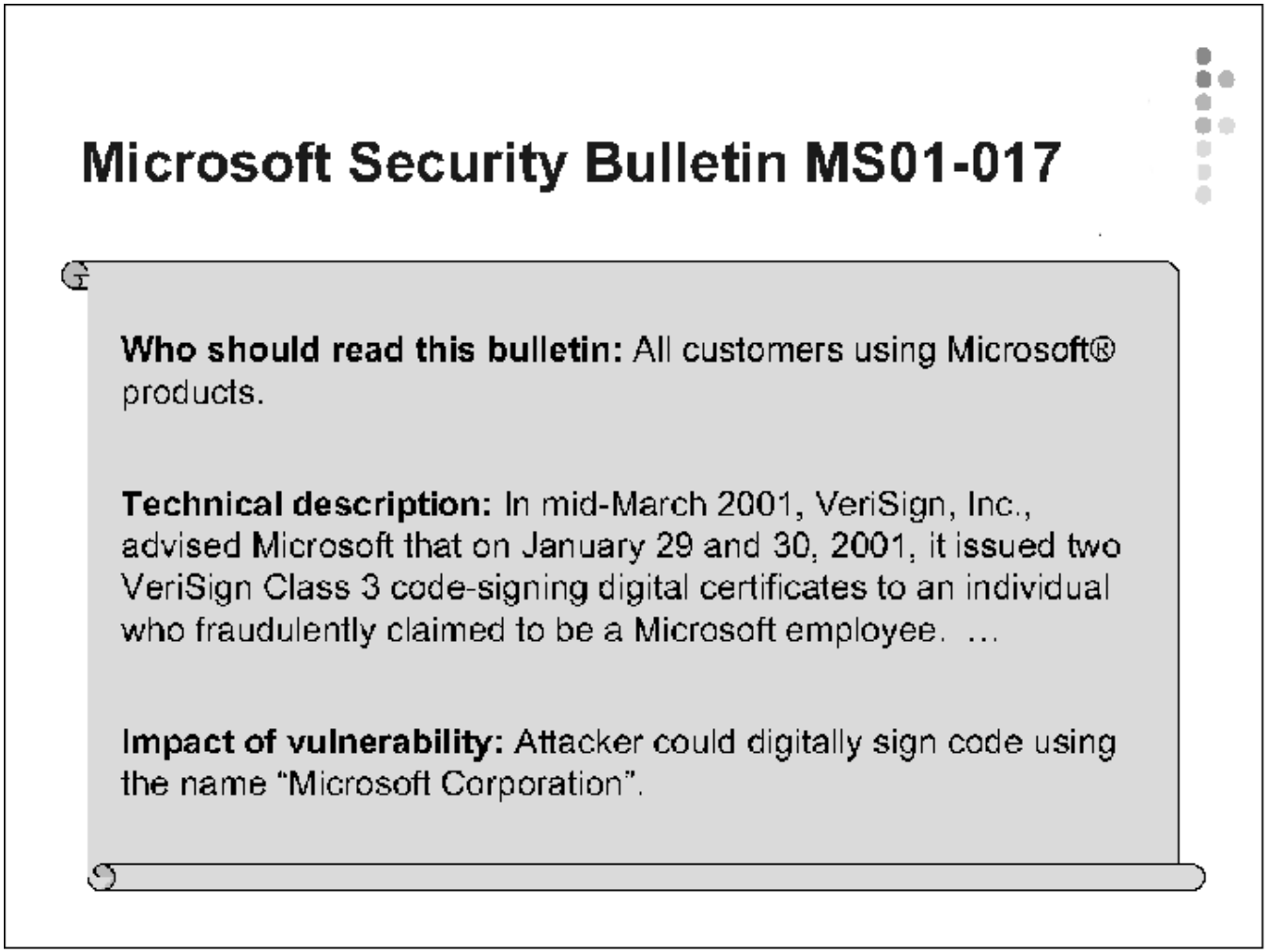
Whom do you trust completely?

Maybe that's not such a good idea!



Microsoft Security Bulletin MS01-017

Who should read this bulletin: All customers using Microsoft® products.

Technical description: In mid-March 2001, VeriSign, Inc., advised Microsoft that on January 29 and 30, 2001, it issued two VeriSign Class 3 code-signing digital certificates to an individual who fraudulently claimed to be a Microsoft employee. …

Impact of vulnerability: Attacker could digitally sign code using the name "Microsoft Corporation".

Proof-Carrying-Code (PCC): The idea

Goal: Safe execution of untrusted code.

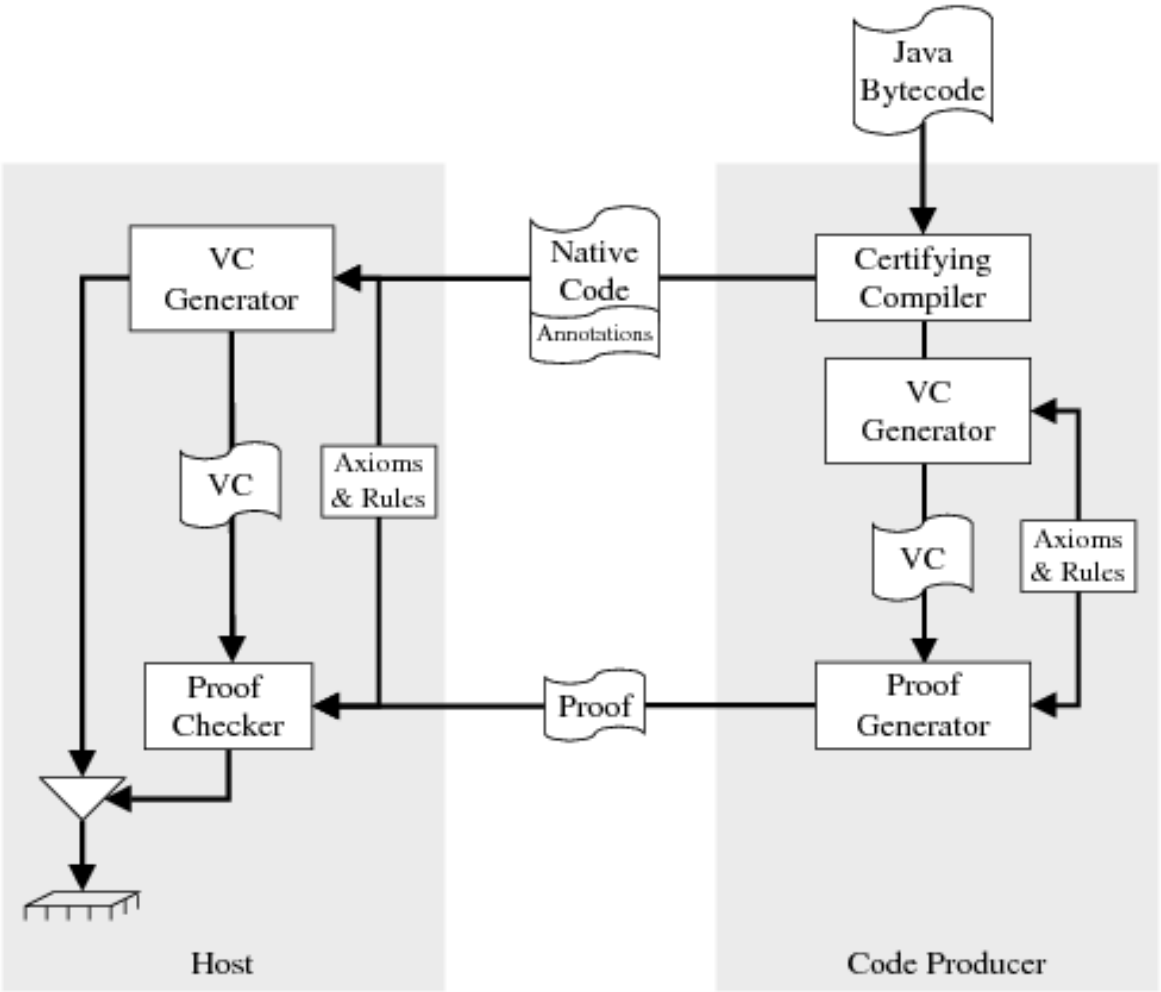
> *PCC is a software mechanism that allows a host system to determine with certainty that it is safe to execute a program supplied by an untrusted source.*

Method: Together with the code, a *certificate* describing its behaviour is sent.

This certificate is a condensed form of a formal proof of this behaviour.

Before execution, the consumer can check the behaviour, by running the proof against the program.

A PCC architecture



Program Verification Techniques

Many techniques for PCC come from the area of **program verification**. Main differences:

General program verification

- is trying to **verify good behaviour** (correctness).
- is usually interactive
- requires at least programmer annotations as invariants to the program

PCC

- is trying to **falsify bad behaviour**
- must be automatic
- may be based on inferred information from the high-level

Observation: Checking a proof is much simpler than creating one

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PCC: Selling Points

Advantages of PCC over present-day mechanisms:

- General mechanism for many different safety policies
- Behaviour can be checked before execution
- Certificates are tamper-proof
- Proofs may be hard to generate (producer) but are easy to check (consumer)

What does “well-behaved” mean?

PCC is a general framework and can be instantiated to many different **safety policies**.

A safety policy defines the meaning of “well-behaved”.

Examples:

- (functional) correctness
- type correctness ([1])
- array bounds and memory access (CCured)
- resource-consumption (MRG)

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Further Reading

- George Necula, *Proof-carrying code* in POPL'97 — Symposium on Principles of Programming Languages, Paris, France, 1997.
  `http://raw.cs.berkeley.edu/Papers/pcc_popl97.ps`
- George Necula, *Proof-Carrying Code: Design and Implementation* in Proof and System Reliability, Springer-Verlag, 2002.
  `http://raw.cs.berkeley.edu/Papers/marktoberdorf.pdf`
- *CCured Demo*,
  `http://manju.cs.berkeley.edu/ccured/web/index.html`

Main Challenges of PCC

PCC is a very powerful mechanism. Coming up with an efficient implementation of such a mechanism is a challenging task.

The main problems are

- Certificate size
- Size of the trusted code base (TCB)
- Performance of validation
- Certificate generation

Certificate Size

A certificate is a formal proof, and can be encoded as e.g. LF Term.

BUT: such proof terms include a lot of repetition
$\Longrightarrow$ huge certificates

Approaches to reduce certificate size:

- Compress the general proof term and do reconstruction on the consumer side
- Transmit only hints in the certificate (oracle strings)
- Embed the proving infrastructure into a theorem prover and use its tactic language

Size of the Trusted Code Base (TCB)

The PCC architecture relies on the correctness of components such as VC-generation and validation.

But these components are complex and implementation is error-prone.

Approaches for reducing size of TCB:

- Use proven/established software
- Build everything up from basics **foundational PCC** (Appel)

Performance

Even though validation is fast compared to proof generation, it is on the critical path of using remote code
$\Longrightarrow$ performance of the validation is crucial for the acceptance of PCC.

Approaches:

- Write your own specialised proof-checker (for a specific domain)
- Use hooks of a general proof-checker, but replace components with more efficient routines, e.g. arithmetic

LF Terms

The Logical Framework (LF) is a generic description of logics.

- Entities on three levels: objects, families of types, and kinds.
- Signatures: mappings of constants to types and kinds
- Contexts: mappings of variables to types
- Judgements:

$$\Gamma \vdash_\Sigma A : K$$

meaning A has kind K in context Γ and signature Σ.

$$\Gamma \vdash_\Sigma M : A$$

meaning M has type A in context Γ and signature Σ.

Styles of Program Logics

Two styles of program logics have been proposed.

- Hoare-style logics: $\{P\}\mathtt{e}\{Q\}$
  Assertions are parameterised over the "current" state.
  Example: Specification of an exponential function

$$\{0 \leq y \ \wedge \ x = X \wedge \ y = Y\} \ \mathtt{exp}(x, y) \ \{r = X^Y\}$$

  Note: X, Y are **auxiliary variables** and must not appear in e
- VDM-style logics: $\mathtt{e} : P$
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A Simple while-language

Language:

$$
\begin{array}{rcl}
e & ::= & \texttt{skip} \\
 & | & x := t \\
 & | & e_1 ; e_2 \\
 & | & \texttt{if } b \texttt{ then } e_1 \texttt{ else } e_2 \\
 & | & \texttt{while } b \texttt{ do } e \\
 & | & \texttt{call}
\end{array}
$$

A judgement has this form (for now!)

$$\vdash \{P\}\ e\ \{Q\}$$

A judgement is valid if the following holds

$$\forall z\ s\ t.\ s \overset{e}{\rightsquigarrow} t \Rightarrow P\ z\ s \Rightarrow Q\ z\ t$$

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A Simple Hoare-style Logic

$$\frac{}{\vdash \{P\}\ \texttt{skip}\ \{P\}}\ (\text{SKIP}) \qquad \frac{}{\vdash \{\lambda z\ s.\ P\ z\ s[t/x]\}\ x := t\ \{P\}}\ (\text{ASSIGN})$$

$$\frac{\vdash \{P\}\ e_1\ \{R\} \quad \{R\}\ e_2\ \{Q\}}{\vdash \{P\}\ e_1;e_2\ \{Q\}}\ (\text{COMP})$$

$$\frac{\vdash \{\lambda z\ s.\ P\ z\ s\ \wedge\ b\ s\}\ e_1\ \{Q\} \quad \vdash \{\lambda z\ s.\ P\ z\ s\ \wedge\ \neg(b\ s)\}\ e_2\ \{Q\}}{\vdash \{P\}\ \texttt{if}\ b\ \texttt{then}\ e_1\ \texttt{else}\ e_2\{Q\}}\ (\text{IF})$$

$$\frac{\vdash \{\lambda z\ s.\ P\ z\ s\ \wedge\ b\ s\}\ e\ \{P\}}{\vdash \{P\}\ \texttt{while}\ b\ \texttt{do}\ e\{\lambda z\ s.\ P\ z\ s\ \wedge\ \neg(b\ s)\}}\ (\text{WHILE})$$

$$\frac{\vdash \{P\}\ body\ \{Q\}}{\vdash \{P\}\ \texttt{CALL}\ \{Q\}}\ (\text{CALL})$$

A Simple Hoare-style Logic (structural rules)

The consequence rule allows us to weaken the pre-condition and to strengthen the post-condition:

$$\frac{\forall s\ t.\ (\forall z.\ P'\ z\ s \Rightarrow P\ z\ s) \quad \vdash \{P'\}\ e\ \{Q'\} \quad \forall s\ t.\ (\forall z.\ Q\ z\ s \Rightarrow Q'\ z\ s)}{\vdash \{P\}\ e\ \{Q\}}$$

(CONSEQ)

Recursive Functions

In order to deal with recursive functions, we need to collect the knowledge about the behaviour of the functions.

We extend the judgement with a context Γ, mapping expressions to Hoare-Triples:

$$\Gamma \vdash \{P\}\ e\ \{Q\}$$

where Γ has the form $\{\ldots, (P', e', Q'), \ldots\}$.

Recursive Functions

Now, the call rule for recursive, parameter-less functions looks like this:

$$\frac{\Gamma \cup \{(P, \texttt{CALL}, Q)\} \vdash \{P\}\ body\ \{Q\}}{\Gamma \vdash \{P\}\ \texttt{CALL}\ \{Q\}} \quad (\textsc{call})$$

We collect the knowledge about the (one) function in the context, and prove the body.

Note: This is a rule for partial correctness: for total correctness we need some form of measure.

Recursive Functions

To extract information out of the context we need and axiom rule

$$\frac{(P, e, Q) \in \Gamma}{\Gamma \vdash \{P\}\ e\ \{Q\}} \quad (\text{AX})$$

Note that we now use a **Gentzen-style** logic (one with contexts) rather than a Hilbert-style logic.

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But how can we prove $\{i = N - 1\}\texttt{CALL}\{i = N - 1\}$ from $\{i = N\}\texttt{CALL}\{i = N\}$?

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But how can we prove $\{i = N - 1\}\texttt{CALL}\{i = N - 1\}$ from $\{i = N\}\texttt{CALL}\{i = N\}$?
We need to **instantiate** N with $N - 1$!

Recursive functions

To be able to instantiate auxiliary variables we need a more powerful consequence rule:

$$\frac{\Gamma \vdash \{P'\}\ e\ \{Q'\} \quad \forall s\ t.\ (\forall z.\ P'\ z\ s \Rightarrow Q'\ z\ t) \ \Rightarrow\ (\forall z.\ P\ z\ s \Rightarrow Q\ z\ t)}{\Gamma \vdash \{P\}\ e\ \{Q\}}\ (\text{CONSEQ})$$

Now we are allowed to proof $P \Rightarrow Q$ under the knowledge that we can choose z freely as long as $P' \Rightarrow Q'$ is true.
This complex rule for **adaptation** is one of the main disadvantages of Hoare-style logics.

Extending the Logic with Termination

The Call and While rules need to use a well-founded ordering $<$ and a side condition saying that the body is smaller w.r.t. this ordering:

$$\frac{\begin{array}{c} wf\ < \\ \forall s'.\ \{(\lambda z\ s.P\ z\ s \wedge\ s < s', \texttt{CALL}, Q)\} \\ \vdash_T \{\lambda z\ s.P\ z\ s \wedge\ s = s'\} body\ \{Q\} \end{array}}{\vdash_T \{P\}\ \texttt{CALL}\{Q\}}$$

Note the explicit quantification over the state s'. Read it like this

> *The pre-state s must be smaller than a state s′, which is the post-state.*

Extending the Logic with Mutual Recursion

To cover mutual recursion a different derivation system $\vdash_M$ is defined. Judgements in $\vdash_M$ are extended to sets of Hoare triples, informally:

$$\Gamma \vdash_M \{(P_1, e_1, Q_1), \ldots, (P_n, e_n, Q_n)\}$$

The Call rule is generalised as follows

$$\frac{\bigcup p.\ \{(P\ p, \texttt{CALL p}, Q\ p)\} \vdash_M \bigcup p.\{(P\ p, body\ p, Q\ p)\}}{\emptyset \vdash_M \bigcup p.\ \{(P\ p, \texttt{CALL p}, Q\ p)\}}$$

Further Reading

- Thomas Kleymann, *Hoare Logic and VDM: Machine-Checked Soundness and Completeness Proofs*, Lab. for Foundations of Computer
  Science, Univ of Edinburgh, LFCS report ECS-LFCS-98-392, 1999.
  `http://www.lfcs.informatics.ed.ac.uk/reports/98/ECS-LFCS`
- Tobias Nipkow, *Hoare Logics for Recursive Procedures and Unbounded Nondeterminism*, in CSL 2002 — Computer Science Logic, LNCS 2471, pp. 103–119, Springer, 2002.

Challenge: Minimising the TCB

This aspect is the emphasis of the **Foundational PCC** approach.

An infrastructure developed by the group of Andrew Appel at Princeton [1].

Motivation: With complex logics and VCGs, there is a big danger of introducing bugs in software that needs to be trusted.

Validator

What exactly is proven?

The safety policy is typically encoded as a pre-post-condition pair (P/Q) for a program e, and a logic describing how to reason.

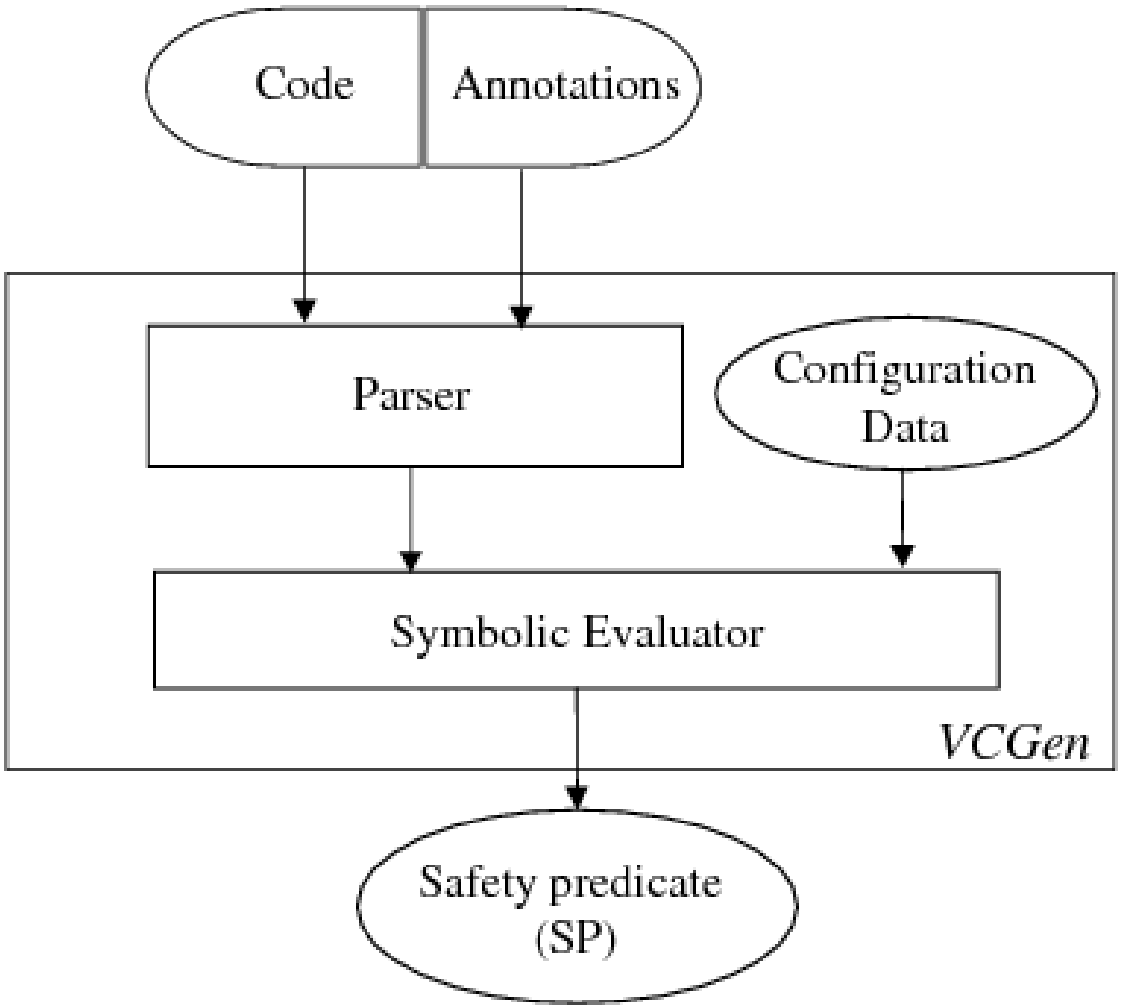
Running the verification condition generator VCG over e and Q, generates a set of conditions, that need to be fulfilled in order for the program to be safe.

The condition that needs to be proven is:

$$P \Longrightarrow VC(e, Q)$$

.

Structure of the VCG



The Philosophy of Foundational PCC

Define safety policy directly on the **operational semantics** of the code.

Certificates are proofs over the operational semantics.

It minimises the TCB because no trusted verification condition generator is needed.

Pros and cons:

- ☺ **more flexible**: not restricted to a particular type system as the language in which the proofs are phrased;
- ☺ **more secure**: no reliance on VCG.
- ☹ **larger proofs**

Conventional vs Foundational PCC

Re-examine the logic for memory safety, eg.

$$\frac{m \vdash e : \tau\ \mathit{list} \quad e \neq 0}{\begin{array}{l} m \vdash e : \mathit{addr} \ \wedge\ m \vdash e + 4 : \mathit{addr} \wedge \\ m \vdash \mathit{sel}(m, e) : \tau \ \wedge\ m \vdash \mathit{sel}(m, e + 4) : \tau\ \mathit{list} \end{array}} \text{(ListElim)}$$

The rule has **built-in knowledge about the type-system**, in this case representing the data layout of the compiler ("*Type specialised PCC*")
$\Longrightarrow$ dangerous if soundness of the logic is not checked mechanically!

Logic rules in Foundational PCC

In foundational PCC the rules work on the operational semantics:

$$\frac{m \models e : \tau\ \mathit{list} \quad e \neq 0}{\begin{array}{l} m \models e : \mathit{addr} \ \wedge\ m \models e + 4 : \mathit{addr} \ \wedge \\ m \models \mathit{sel}(m, e) : \tau \ \wedge\ m \models \mathit{sel}(m, e + 4) : \tau\ \mathit{list} \end{array}} \text{(ListElim)}$$

This looks similar to the previous rule but has a very different meaning: $\models$ is a predicate over the formal model of the computation, and the above rule can be proven as a lemma, $\vdash$ is an encoding of a type-system on top of the operational semantics and thus needs a **soundness proof**.

Components of a foundational PCC infrastructure

Operational semantics and safety properties are directly encoded in a **higher-order logic**.

As language for the certificates, the LF metalogic framework is used.

For development and for proof-checking the Twelf theorem proofer is used.

Specifying safety

To specify safety, the operational semantics is written in such a way, that it gets stuck whenever the safety condition is violated.

Example: operational semantics on assembler code.
Safety policy: "only readable addresses are loaded".
Define a predicate: $readable(x) \equiv 0 \leq x \leq 1000$
The semantics of a load operation `LD r_i, c(r_j)` is now written as follows:

$$\begin{aligned} load(i,j,c) \quad \equiv \quad & \lambda\, r\ m\ r'\ m'. \\ & r'(i) = m(r(j)+c) \ \wedge \ readable(r(j)+c) \ \wedge \\ & (\forall x \neq i.\ r'(x) = r(x)) \ \wedge \ m' = m \end{aligned}$$

Note: the clause for nothing else changes, quickly becomes awkward when doing these proofs
$\Longrightarrow$ Separation Logic (Reynolds'02) tackles this problem.

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Further Reading

Andrew Appel, *Foundational Proof-Carrying Code* in LICS'01 — Symposium on Logic in Computer Science, 2001.
`http://www.cs.princeton.edu/~appel/papers/fpcc.pdf`

PCC for Resources: Motivation

Resource-bounded computation is one specific instance of PCC.

Safety policy: resource consumption is lower than a given bound.

Resources can be (heap) space, time, or size of parameters to system calls.

Strong demand for such guarantees for example in embedded systems.

Mobile Resource Guarantees

Objective:
Development of an infrastructure to endow mobile code with independently verifiable certificates describing resource behaviour.

Approach:
Proof-carrying code for **resource-related properties**, where proofs are generated from typing derivations in a **resource-aware type system**.

Motivation

Restrict the execution of mobile code to those adhering to a certain resource policy.

Application Scenarios:

- A user of a **handheld device** might want to know that a downloaded application will definitely run within the limited amount of memory available.
- A provider of **computational power in a Grid infrastructure** may only be willing to offer this service upon receiving dependable guarantees about the required resource consumption.

Proof-Carrying-Code with High-Level-Logics

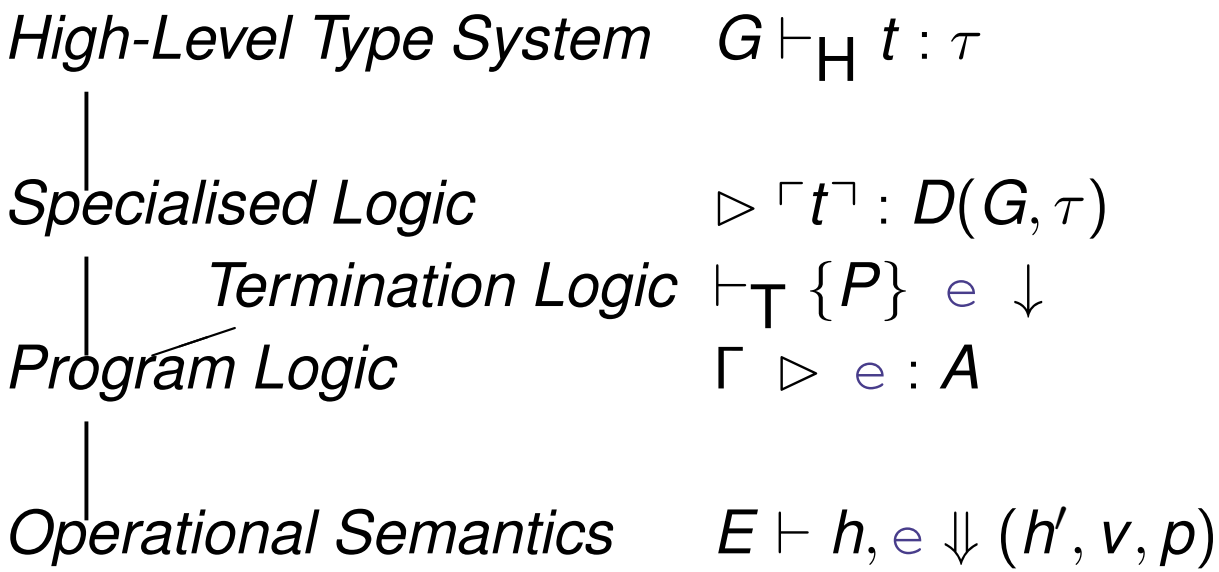
Our approach to PCC: Combine high-level type-systems with program logics and build a **hierarchy of logics** to construct a logic tailored to reason about resources.

Everything is **formalised in a theorem prover**.

Classic vs Foundational PCC: best of both worlds

- **Simple reasoning**, using specialised logics;
- **Strong foundations**, by encoding the logics in a theorem prover

Proof-Carrying-Code with High-Level-Logics



Motivating Example of this Hierarchical Approach

High-level language: ML-like.

Safety policy: well-formed datatypes.

Define a predicate $h \models_t a$, expressing that an address a in heap h is the start of a (high-level) data-type t.

Prove: $f :: \tau\ list \to \tau\ list$ adheres to this safety policy.

Directly on the program logic

$$\triangleright\, f(x)\ :\ \lambda E\ h\ h'\ v\ .\ h \models_{list} E\langle x\rangle\ \longrightarrow\ h' \models_{list} v$$

NOT: reasoning on this level generates huge side-conditions.

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Instead, define a higher-level logic $\vdash_H$ that abstracts over the details of datatype representation, and that has the property

$$G \vdash_H t : \tau \implies \triangleright \ulcorner t \urcorner : D(\Gamma, \tau)$$

We specialise the form of assertions like this

$$D(\{x : list, y : list\}, list) \equiv$$
$$\lambda E\ h\ h'\ v. \quad h \models_{list} E\langle x \rangle \wedge h \models_{list} E\langle y \rangle \longrightarrow$$
$$h' \models_{list} E\langle x \rangle \wedge h' \models_{list} E\langle y \rangle \wedge h' \models_{list} v$$

Now we can formulate rules, that match translations from the high-level language:

$$\frac{\triangleright \ulcorner t_1 \urcorner : D(\Gamma, \tau) \qquad \triangleright \ulcorner t_2 \urcorner : D(\Gamma, \tau\ list)}{\triangleright \ulcorner cons(t_1, t_2) \urcorner : D(\Gamma, \tau\ list)}$$

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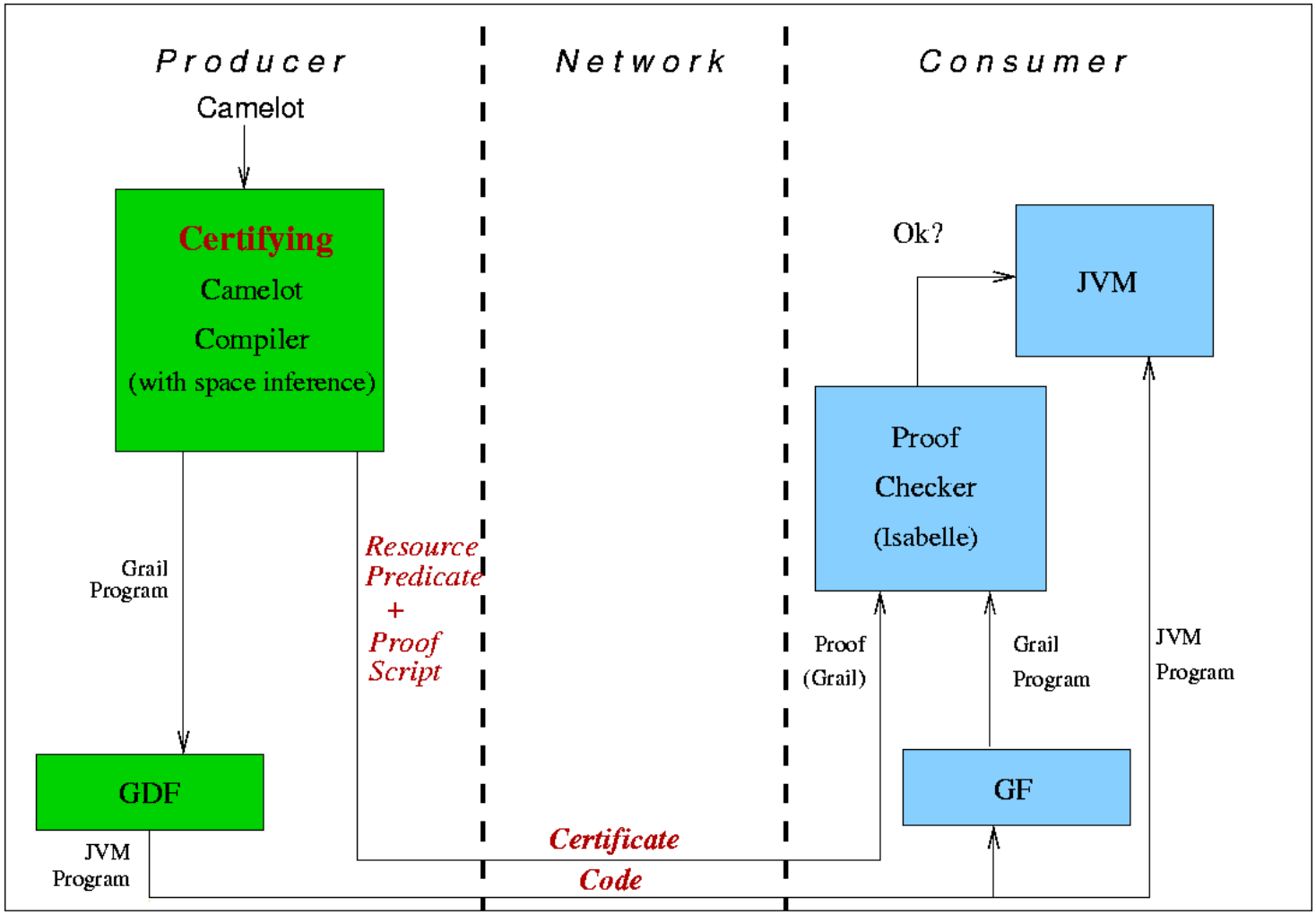
A Proof-Carrying-Code Infrastructure for MRG



Camelot

- Strict, first-order functional language with CAML-like syntax and object-oriented extensions
- Compiled to subset of JVM (Java Virtual Machine) bytecode (Grail)
- Memory model: 2 level heap
- Security: Static analyses to prevent deallocation of live cells in Level-1 Heap: linear typing (folklore + Hofmann), readonly typing (Aspinall, Hofmann, Konencny), layered sharing analysis (Konencny).
- Resource bounds: Static analysis to infer linear upper bounds on heap consumption (Hofmann, Jost).

Example: Insertion Sort

Camelot program:

```
let ins a l = match l with
                Nil -> Cons(a,Nil)
              | Cons(x,t)@_ -> if a < x then Cons(a,Cons(x
                                          else Cons(x, ins a

let sort l =  match l with  Nil -> Nil
                          | Cons(a,t)@_ -> ins a (sort t)
```

In-place Operations via a Diamond Type

Using operators, such as `Cons`, amounts to heap allocation.

Additionally, Camelot provides extensions to do **in-place operations** over arbitrary data structures via a so called **diamond type** $\diamond$ with **d** $\in \diamond$:

```
match l with Nil@d => e1
           | Cons (h,t)@d => ... Cons (x,t)@d ...
```

The memory occupied by the cons cell can be **re-used** via the diamond `d`.
Note:

- $\diamond$ is an abstract data-type
- structured use of diamonds in branches of pattern matches

How does this fit with referential transparency?

Using a diamond type, we can introduce side effects:

```
type ilist = Nil | Cons of int*ilist
let insert1 x l =
    match l with Nil -> Cons (x, l)
                | Cons(h,t)@d ->
                    if x <= h then Cons(x, Cons(h,t)@d)
                              else Cons(h, insert1 x t)@d

let sort l =  match l with Nil -> Nil
                         | Cons(h,t) -> insert1 h (sort t)
```

Now, what's the result of

```
let start args = let l = [4,5,6,7] in
                 let l1 = insert1 6 l in
                 print_list l
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Linearity saves the day

We can characterise the class of programs for which referential transparency is retained.

Theorem

*A **linearly typed** Camelot program computes the function specified by its purely functional semantics (Hofmann, 2000).*

Beyond Linearity

But: linearity is too restrictive in many cases; we also want to use diamonds in programs where **only the last access to the data structure is destructive**.

More expressive type systems to express such access patterns are **readonly types** (Aspinall, Hofmann, Konecny, 2001) and types with **layered sharing** (Konecny 2003).

As with pointers, diamonds can be a powerful gun to shoot yourself in the foot. We need a **powerful type system** to prevent this, and want a **static analysis** to predict resource consumption.

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Space Inference

Goal: Infer a linear upper bound on heap consumption.

Given Camelot program containing a function

```
start : string list -> unit
```

find **linear function** *s* such that `start`(*l*) will not call `new()` (only `make()`) when evaluated in a heap *h* where

- **the freelist has length not less than** $s(n)$
- *l* points in *h* to a linear list of some length *n*
- the freelist which forms a part of *h* is well-formed
- the freelist does not overlap with *l*

Composing `start` with *runtime environment* that binds input to, e.g., `stdin` yields a standalone program that runs within predictable heap space.

Extended (LFD) Types

Idea: Weights are attached to constructors in an extended type-system.

```
ins          : 1, int -> list(...<0>) -> list(...<0>),
```

says that the call `ins x xs` requires 1 heap-cell plus 0 heap cells for each `Cons` cell of the list `xs`.

```
sort         : 0, list(...<0>) -> list(...<0>), 0
```

`sort` does not consume any heap space.

```
start        : 0, list(...<1>) -> unit, 0;
```

gives rise to the desired linear bounding function $s(n) = n$.

High-level Type System: Function Call

A, B, C are types, $k, k', n, n' \in \mathbb{Q}^+$, $\mathtt{f}$ is a Camelot function and $\mathtt{x}_1, \ldots, \mathtt{x}_p$ are variables, Σ is a table mapping function names to types.

$$\frac{\begin{array}{c}\Sigma(\mathtt{f}) = (A_1, \ldots, A_p, k) \longrightarrow (C, k') \\ n \geq k \qquad n - k + k' \geq n'\end{array}}{\Gamma, \mathtt{x}_1 : A_1, \ldots, \mathtt{x}_p : A_p, n \vdash \mathtt{f}(\mathtt{x}_1, \ldots, \mathtt{x}_p) : C, n'} \quad (\textsc{Fun})$$

Grail

Grail is an abstraction over virtual machine languages such as the JVM.

$$
\begin{array}{lcl}
e \in \mathit{expr} & ::= & \texttt{null} \mid \texttt{int}\ i \mid \texttt{var}\ x \mid \texttt{prim}\ p\ x\ x \mid \texttt{new}\ c\ [t_1 := x_1, \ldots, t_n := x_n] \mid \\
 & & x.t \mid x.t := x \mid c \diamond t \mid c \diamond t := x \mid \texttt{let}\ x = e\ \texttt{in}\ e \mid e\ ;\ e \mid \\
 & & \texttt{if}\ x\ \texttt{then}\ e\ \texttt{else}\ e \mid \texttt{call}\ f \mid x \cdot m(\overline{a}) \mid c \diamond m(\overline{a}) \\
a \in \mathit{args} & ::= & \texttt{var}\ x \mid \text{null} \mid i
\end{array}
$$

Example: Insertion sort

Grail code:

```
method static public List ins (int a, List l) = ...Make(..
method static public List sort (List l) =
  let fun f(List l) =
    if l = null then null
                else let val h = l.HD
                         val t = l.TL
                         val () = D.free (l)
                         val l = List.sort (t)
                     in List.ins (h, l) end
  in f(l) end
```

This is a 1-to-1 translation of JVM code

Judgement of the Operational Semantics

Modelling resources: Resources are an extra component in operational and axiomatic semantics ("resource record").

$$\mathbf{p} \in \mathit{RRec} = (\!|\mathit{clock} : \mathit{nat}, \mathit{callcount} : \mathit{nat}, \mathit{invokedepth} : \mathit{nat}, \mathit{maxstack} : \mathit{nat}|\!)$$

A judgement in the functional operational semantics

$$E \vdash h, e \Downarrow_n (h', v, p)$$

is to be read as "starting with a heap h and a variable enviroment E, the Grail code e evaluates in n steps to the value v, yielding the heap h' as result and consuming p resources."

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Operational Semantics: Let- and Call-rules

$$\frac{E \vdash h, e_1 \Downarrow_n (h_1, w, p) \quad w \neq \bot \quad E\langle x := w\rangle \vdash h_1, e_2 \Downarrow_m (h_2, v, q)}{E \vdash h, \texttt{let}\ x = e_1\ \texttt{in}\ e_2 \Downarrow_{max(n,m)+1} (h_2, v, \mathbf{p_1} \smile \mathbf{p_2})} \quad \text{(LET)}$$

$$\frac{E \vdash h, body_f \Downarrow_n (h_1, v, p)}{E \vdash h, \texttt{call}\ f \Downarrow_{n+1} (h_1, v, \langle \mathbf{1\ 1\ 0\ 0}\rangle \oplus \mathbf{p_1})} \quad \text{(CALL)}$$

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A Program Logic for Grail

VDM-style logic with judgements of the form $\Gamma \rhd e : A$, meaning

"in context Γ *expression* e *fulfills the assertion* A*"*

Type of assertions (**shallow embedding**):

$$\mathcal{A} \equiv \mathcal{E} \to \mathcal{H} \to \mathcal{H} \to \mathcal{V} \to \mathcal{R} \to \mathcal{B}$$

No syntactic separation into pre- and postconditions.

Semantic validity $\models e : A$ means

"whenever $E \vdash h, e \Downarrow (h', v, p)$ *then* $A\ E\ h\ h'\ v\ p$ *holds"*

Note: Covers partial correctness; termination orthogonal.

A Program Logic for Grail

Simplified rule for parameterless function call:

$$\frac{\Gamma, (\texttt{Call f} : A) \rhd \texttt{e} : A^+}{\Gamma \rhd \texttt{Call f} : A} \qquad \text{(CallRec)}$$

where `e` is the body of the function `f` and

$$A^+ \equiv \lambda\, E\, h\, h'\, v\, p.\, A(E, h, h', v, p^+)$$

where p^+ is the updated cost component.
Note:

- Context Γ: collects hypothetical judgements for recursion
- Meta-logical guarantees: soundness, relative completeness

Program Logic Rules

$$\frac{\Gamma \rhd e_1 : P \quad \Gamma \rhd e_2 : Q}{\Gamma \rhd \texttt{let}\ x = e_1\ \texttt{in}\ e_2 : \lambda E\, h\, h'\, v\, p.\, \exists\, p_1\ p_2\ h_1\ w.\ \begin{array}{l} P\ E\ h\ h_1\ w\ p_1\ \wedge\ w \neq \bot \wedge \\ Q\ (E\langle x := w\rangle)\ h_1\ h'\ v\ p_2) \wedge \\ p = \mathbf{p_1} \smile \mathbf{p_2} \end{array}} \text{(VLET)}$$

$$\frac{\Gamma \cup \{(\texttt{call}\ f, P)\} \rhd body_f : \lambda E\, h\, h'\, v\, p.\, P\ E\ h\ h'\ v\ \langle \mathbf{1\ 1\ 0\ 0} \rangle \oplus \mathbf{p_1},}{\Gamma \rhd \texttt{call}\ f : A} \text{(VCALL)}$$

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Specific Features of the Program Logic

- Unusual rules for **mutually recursive methods** and for **parameter adaptation** in method invocations

$$\frac{(\Gamma, \mathsf{e} : A)\ \mathit{goodContext}}{\triangleright\, \mathsf{e} : A} \qquad (\textsc{Mutrec})$$

$$\frac{(\Gamma, c \diamond m(\overline{a}) : \mathit{MS}\ c\ m\ \overline{a})\ \mathit{goodContext}}{\triangleright c \diamond m(\overline{b}) : \mathit{MS}\ c\ m\ \overline{b}} \qquad (\textsc{Adapt})$$

- Proof via admissible Cut rule, **no extra derivation system**
- Global specification table *MS*, *goodContext* relates entries in *MS* to the method bodies

Example: Insertion sort

Specification:

$$
\begin{array}{lcl}
\mathit{insSpec} & \equiv & \mathit{MS}\ \mathrm{List\ ins}\ [a_1, a_2] = \\
 & & \quad \lambda\ E\ h\ h'\ v\ p\ .\forall\ i\ r\ n\ X\ . \\
 & & \quad (E\langle a_1\rangle = i \wedge E\langle a_2\rangle = \mathrm{Ref}\ r \wedge h, r \models_X n \\
 & & \quad \longrightarrow |\mathit{dom}\,(h)| + 1 = |\mathit{dom}\,(h')| \wedge \\
 & & \quad\ \ p \leq \ldots) \\
\mathit{sortSpec} & \equiv & \mathit{MS}\ \mathrm{List\ sort}\ [a] = \\
 & & \quad \lambda\ E\ h\ h'\ v\ p\ .\forall\ i\ r\ n\ X\ . \\
 & & \quad (\ E\langle a\rangle = \mathrm{Ref}\ r \wedge h, r \models_X n\ \longrightarrow |\mathit{dom}\,(h)| = |\mathit{dom}\,(h')|\ \wedge\ p \leq \ldots)
\end{array}
$$

Lemma: $\mathit{insSpec}\ \wedge\ \mathit{sortSpec} \longrightarrow\ \triangleright\ \mathrm{List} \diamond \mathit{sort}([xs]) : \mathit{MS}\ \mathrm{List}\ \mathit{sort}\ [xs]$

Discussion of the Program Logic

- Expressive logic for correctness and resource consumption
- Logic proven **sound and complete**
- Termination built on top of a logic for partial correctness
- Less suited for immediate program verification: not fully automatic (case-splits, $\exists$-instantiations,...), verification conditions large and complex
- Continue abstraction: loop unfolding in op. semantics $\rightarrow$ invariants in general program logics $\rightarrow$ specific logic for interesting (resource-)properties
- Aim: exploit structure of Camelot compilation (freelist) and program analysis

$$\begin{array}{rcl} \text{List.ins} & : & 1, \mathbf{I} \times \mathbf{L}(0) \rightarrow \mathbf{L}(0), 0 \\ \text{List.sort} & : & 0, \mathbf{L}(0) \rightarrow \mathbf{L}(0), 0 \end{array}$$

Heap Space Logic (LFD-assertions)

- Translation of Hofmann-Jost type system to Grail, types interpreted as relating initial to final freelist
- Fixed assertion format $[\![U, n, [\Delta] \blacktriangleright T, m]\!]$

$$\begin{array}{rcl} \text{List.ins} & : & [\![\{a, l\}, 1, [a \mapsto \mathbf{I}, l \mapsto \mathbf{L}(0)] \blacktriangleright \mathbf{L}(0), 0]\!] \\ \text{List.sort} & : & [\![\{l\}, 0, [l \mapsto \mathbf{L}(0)] \blacktriangleright \mathbf{L}(0), 0]\!] \end{array}$$

- LFD types express space requirements for datatype constructors, numbers n, m refer to the freelist length
- Semantic definition by expansion into core bytecode logic, derived proof rules using linear affine context management
- Dramatic reduction of VC complexity!

Semantic interpretation of $[\![U, n, [\Delta] \blacktriangleright T, m]\!]$

$$
\begin{array}{l}
[\![U, n, [\Delta] \blacktriangleright T, m]\!] \equiv \\
\lambda\ E\ h\ h'\ v\ p. \\
\quad \forall\ F\ N. \quad (\mathit{regionsExist}(U, \Delta, h, E) \wedge \mathit{regionsDistinct}(U, \Delta, h, E) \wedge \\
\qquad\qquad\ \mathit{freelist}(h, F, N) \wedge \mathit{distinctFrom}(U, \Delta, h, E, F)) \\
\qquad\qquad \longrightarrow \\
\qquad\qquad (\exists\ R\ S\ M\ G. \ \ v, h' \models_T R, S \ \wedge \mathit{freelist}(h', G, M) \wedge R \cap G = \emptyset \wedge \\
\qquad\qquad\qquad\qquad\ \mathit{Bounded}((R \cup G), F, U, \Delta, h, E) \wedge \mathit{modified}(F, U, \Delta, h, E, h') \wedge \\
\qquad\qquad\qquad\qquad\ \mathit{sizeRestricted}(n, N, m, S, M, U, \Delta, h, E) \wedge \mathit{dom}\, h = \mathit{dom}\, h')
\end{array}
$$

- Formulae defined by BC expansion:

$$
\begin{array}{l}
\mathit{regionsDistinct}(U, \Delta, h, E) \equiv \\
\quad \forall\ x\ y\ R_x\ R_y\ S_x\ S_y. \\
\qquad (\{x, y\} \subseteq U \cap \mathit{dom}\, \Delta \wedge x \neq y \wedge E\langle x \rangle, h \models_{\Delta(x)} R_x, S_x \wedge E\langle y \rangle, h \models_{\Delta(y)} R_y, S_y) \\
\qquad\ \longrightarrow R_x \cap R_y = \emptyset \\
\mathit{sizeRestricted}(n, N, m, S, M, U, \Delta, h, E) \equiv \\
\quad \forall\ q\ C.\ \mathit{Size}(E, h, U, \Delta, C) \wedge n + C + q \leq N \longrightarrow m + S + q \leq M
\end{array}
$$

- **You don't want to read this — and you don't need to!**

Proof System

Proof system with linear inequalities and linear affine type system (U, Δ) that guarantees benign sharing;

$$\frac{\Delta(x) = T \quad n \leq m}{\Gamma \rhd \texttt{var}\ x : [\![\{x\}, m, [\Delta] \blacktriangleright T, n]\!]} \quad \text{(VAR)}$$

$$\frac{\begin{array}{ll}\Gamma \rhd e_1 : [\![U_1, n, [\Delta] \blacktriangleright T_1, m]\!] & \Gamma \rhd e_2 : [\![U_2, m, [\Delta, x \mapsto T_1] \blacktriangleright T_2, k]\!] \\ U_1 \cap (U_2 \setminus \{x\}) = \emptyset & T_1 = \mathbf{L}(\_)\end{array}}{\Gamma \rhd \texttt{let}\ x = e_1\ \texttt{in}\ e_2 : [\![U_1 \cup (U_2 \setminus \{x\}), n, [\Delta] \blacktriangleright T_2, k]\!]} \quad \text{(LET)}$$

$$\frac{\Delta(x) = \mathbf{L}(k) \quad l = n + k \quad \Gamma \rhd e : [\![U, l, [\Delta, t \mapsto \mathbf{L}(k)] \blacktriangleright T, m]\!] \quad x \notin U \setminus \{t\}}{\Gamma \rhd \texttt{let}\ t = x.TL\ \texttt{in}\ e : [\![(U \setminus \{t\}) \cup \{x\}, n, [\Delta] \blacktriangleright T, m]\!]} \quad \text{(LETTL)}$$

Note: Linearity relaxed in rules for compiled `match`-expressions

Discussion of the Heap Space Logic

- ☺ Exploit program structure and compiler analysis: most effort done once (in soundness proofs), application straight-forward
- ☺ “Classic PCC”: independence of derived logic from Isabelle (no higher-order predicates, certifying constraint logic programming)
- ☺ “Foundational PCC”: can unfold back to core logic and operational semantics if desired
- ☹ Generalisation to all Camelot datatypes needed
- ☹ Soundness proofs non-trivial, and challenging to generalise to more liberal sharing disciplines

Certificate Generation

Goal: Automatically generate proofs from high-level types and inferred heap consumption.

Approach: Use inferred space bounds as invariants. Use powerful Isabelle tactics to automatically prove a statement on heap consumption in the heap logic.

Example certificate (for list append):

```
Γ ▷ snd (methtable Append append) : SPEC append
by (Wp append_pdefs)
```

```
▷Append.append([RNarg x0, RNarg x1]) : sMST Append append [RNarg x0, RNarg x1
by (fastsimp intro: Context_good GCInvs simp: ctxt_def)
```

Summary

MRG works towards **resource-safe global computing**:

- **check resource consumption** before executing downloaded code;
- **automatically generate certificate** out of a Camelot type.

Components of the picture

- Proof-Carrying-Code infrastructure
- Inference for space consumption in Camelot
- Specialised derived assertions on top of a general program logic for Grail
- Certificate = proof of a derived assertion
- Certificate generation from high-level types

Further Reading

- David Aspinall, Stephen Gilmore, Martin Hofmann, Donald Sannella and Ian Stark, *Mobile Resource Guarantees for Smart Devices* in CASSIS04 — Construction and Analysis of Safe, Secure, and Interoperable Smart Devices, LNCS 3362, 2005.
  `http://groups.inf.ed.ac.uk/mrg/publications/mrg/cassis20`
- David Aspinall and Lennart Beringer and Martin Hofmann and Hans-Wolfgang Loidl and Alberto Momigliano, *A Program Logic for Resource Verification*, in TPHOLs2004 — International Conference on Theorem Proving in Higher Order Logics, Utah, LNCS 3223, 2004.
- Martin Hofmann, Steffen Jost, *Static Prediction of Heap Space Usage for First-Order Functional Programs*, in POPL'03 — Symposium on Principles of Programming Languages, New Orleans, LA, USA, Jan 2003.

Further Reading

- K. Crary and S. Weirich, *Resource Bound Certification* in POPL'00 — Symposium on Principles of Programming Languages, Boston, USA, 2000.
  `http://www-2.cs.cmu.edu/ crary/papers/1999/res/res.ps.gz`
- Gilles Barthe, Mariela Pavlova, Gerardo Schneider, *Precise analysis of memory consumption using program logics* in International Conference on Software Engineering and Formal Methods (SEFM 2005), 7–9 September 2005, Koblenz, Germany.
  `http://www-sop.inria.fr/everest/soft/Jack/doc/papers/gmg`

Summary

PCC is a powerful, general mechanism for providing safety guarantees for mobile code.

It provides these guarantees without resorting to a trust relationship.

It uses techniques from the areas of type-systems, program verification and logics.

It is a very active research area at the moment.

Current Trends

Using formal methods to check specific program properties.

- Program logics as the basic language for doing these checks attract renewed interest in PCC.
- A lot of work on program logics for low-level languages.
- Immediate applications for smart cards and embedded systems.

Future Directions

Embedded Systems as a domain for formal methods.

- Some of these systems have strong security requirements.
- Formal methods are used to check these requirements.
- Model checking is a very active area for automatically checking properties.

Links to other areas

Checking program properties is closely related to inferring quantitative information.

- **Static analyses** deal with extracting quantitative information (e.g. resource consumption)
- A lot of research has gone into making these techniques efficient.
- **Model checking** can deal with a larger class of problems (e.g. specifying safety conditions in a system)
- Just recently these have become efficient enough to be used for main stream programming.

Reading List:

`http://www.tcs.ifi.lmu.de/~hwloidl/PCC/reading.html`