



Faculty of Science



Eden: Parallel Processes, Patterns and Skeletons

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Contents

- ① The Language Eden (in a nutshell)
- ② Skeleton-Based Programming
- ③ Small-Scale Skeletons: Map and Reduce
- ④ Process Topologies as Skeletons
- ⑤ Algorithm-Oriented Skeletons: Two Classics
- ⑥ Summary



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- 2 Skeleton-Based Programming
- 3 Small-Scale Skeletons: Map and Reduce
- 4 Process Topologies as Skeletons
- 5 Algorithm-Oriented Skeletons: Two Classics
- 6 Summary

Learning Goals:

- Writing programs in the parallel Haskell dialect Eden
- Reasoning about the behaviour of Eden programs.
- Applying and implementing parallel skeletons in Eden



Eden Constructs in a Nutshell

- Developed since 1996 in Marburg and Madrid
- Haskell, extended by communicating **processes** for **coordination**



Eden Constructs in a Nutshell

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Eden constructs for Process abstraction and instantiation

```
process :: (Trans a, Trans b) => (a -> b) -> Process a b
( # ) :: (Trans a, Trans b) => (Process a b) -> a -> b
spawn :: (Trans a, Trans b) => [ Process a b ] -> [a] -> [b]
```

- Distributed Memory (Processes do not share data)
- Data sent through (hidden) **1:1 channels**
- Type class **Trans**:
 - **stream communication** for lists
 - **concurrent evaluation** of tuple components
- **Full evaluation** of process output (if any result demanded)
- Non-functional features: **explicit communication**, $n : 1$ channels



Quick Sidestep: WHNF, NFData and Evaluation

- Weak Head Normal Form (WHNF):
Evaluation up to the top level constructor
-



Quick Sidestep: WHNF, NFData and Evaluation

- **Weak Head Normal Form (WHNF):**
Evaluation up to the top level constructor

- **Normal Form (NF):**
Full evaluation (recursively in sub-structures)

From *Control.DeepSeq*

```
class NFData a where
  rnf :: a -> ()      -- This was a _Strategy_ in 1998
  rnf a = a 'seq' () -- returning unit ()

instance NFData Int
instance NFData Double
...
instance (NFData a) => NFData [a] where
  rnf [] = ()
  rnf (x:xs) = rnf x 'seq' rnf xs
...
instance (NFData a, NFData b) => NFData (a,b) where
  rnf (a,b) = rnf a 'seq' rnf b
```



Essential Eden: Process Abstraction/Instantiation

Process Abstraction: `process ::... (a -> b) -> Process a b`

`multproc = process (\x -> [x*k | k <- [1,2..]])`



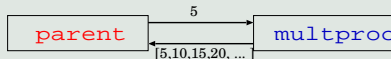
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`multiple5 = multproc # 5`



- Full evaluation of argument (concurrent) and result (parallel)
- Stream communication for lists



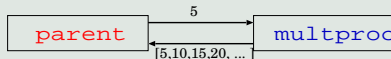
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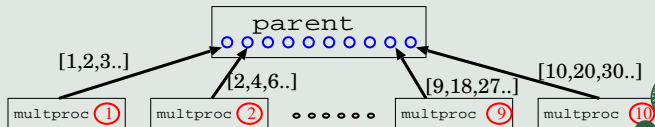
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- Full evaluation of argument (concurrent) and result (parallel)
- Stream communication for lists

Spawning multiple processes: `spawn ::... [Process a b] -> [a] -> [b]`

`multiples = spawn (replicate 10 multproc) [1..10]`



A Small Eden Example¹

- Subexpressions evaluated in parallel
- ... in different **processes** with separate heaps

simpleeden.hs

```
main = do args <- getArgs
        let first_stuff = (process f_expensive) # (args!!0)
            other_stuff = g_expensive $# (args!!1) -- syntax variant
        putStrLn (show first_stuff ++ '\n':show other_stuff)
```

¹(compiled with option `-parcp` or `-parmpi`)



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... which will not produce any speedup!

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  putStrLn (show first_stuff ++ '\n':show other_stuff)
```

... which will not produce any speedup!

simpleeden2.hs

```
main = do args <- getArgs
  let [first_stuff, other_stuff]
      = spawnF [f_expensive, g_expensive] args
  putStrLn (show first_stuff ++ '\n':show other_stuff)
```

- Processes are created when there is demand for the result!
- Spawn both processes at the same time using special function.

¹(compiled with option `-parcp` or `-parmpi`)



Basic Eden Exercise: Hamming Numbers

The Hamming Numbers are defined as the **ascending sequence** of numbers:

$$\{2^i \cdot 3^j \cdot 5^k \mid i, j, k \in \mathbb{N}\}$$



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Dijkstra:

The first Hamming number is 1. Each following Hamming number H can be written as $H = 2K$, $H = 3K$, or $H = 5K$; with a suitable smaller Hamming number K .



Basic Eden Exercise: Hamming Numbers

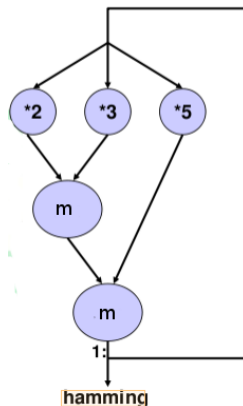
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Dijkstra:

The first Hamming number is 1. Each following Hamming number H can be written as $H = 2K$, $H = 3K$, or $H = 5K$; with a suitable smaller Hamming number K .

- Write an Eden program that produces Hamming numbers using parallel processes. The program should take one argument n and produce the numbers up to position n .
- Observe the parallel behaviour of your program using EdentV.



Non-Functional Eden Constructs for Optimisation

Location-Awareness: `noPe, selfPe :: Int`

`spawnAt :: (Trans a, Trans b) => [Int] -> [Process a b] -> [a] -> [b]`

`instantiateAt :: (Trans a, Trans b) =>`
`Int -> Process a b -> a -> IO b`



Non-Functional Eden Constructs for Optimisation

Location-Awareness: `noPe, selfPe :: Int`

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Int -> Process a b -> a -> IO b`

Explicit communication using primitive operations (monadic)

`data ChanName = Comm (Channel a -> a -> IO ())`

`createC :: IO (Channel a , a)`

```
class NFData a => Trans a where
  write :: a -> IO ()
  write x = rdeepseq x 'pseq' sendData Data x
  createComm :: IO (ChanName a, a)
  createComm = do (cx,x) <- createC
                 return (Comm (sendVia cx) , x)
```

Nondeterminism! `merge :: [[a]] -> [a]`

Hidden inside a Haskell module, only for the library implementation.



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The Idea of Skeleton-Based Parallelism

You have already seen one example:

- Divide and Conquer, as a higher-order function

```
divConqB :: (a -> b) -> a      -- base case fct., input
          -> (a -> Bool)      -- parallel threshold
          -> (b -> b -> b)   -- combine
          -> (a -> Maybe (a,a)) -- divide
          -> b
```

```
divConqB baseF input doSeq combine divide = ...
```

(type will be modified later)

- Parallel structure (binary tree) exploited for parallelism
- Abstracted from concrete problem



The Idea of Skeleton-Based Parallelism

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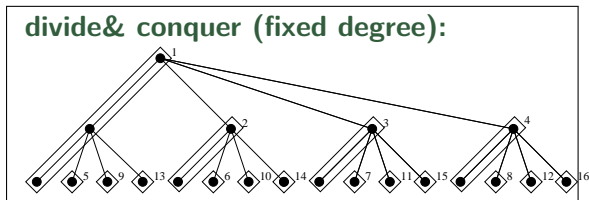
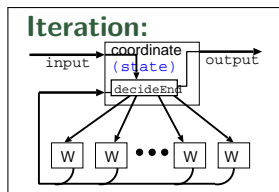
- Parallel structure (binary tree) exploited for parallelism
- Abstracted from concrete problem

And another one, much simpler, much more common:

```
parMap :: (a->b) -> [a] -> [b]
```



Algorithmic Skeletons for Parallel Programming

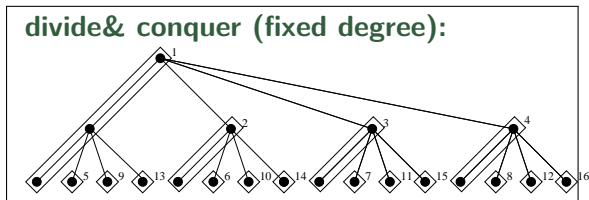
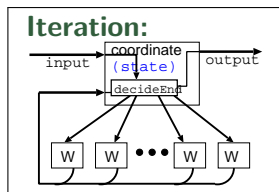


Algorithmic Skeletons [Cole 1989]: Boxes and lines – **executable!**

- Abstraction of algorithmic structure as a higher-order function



Algorithmic Skeletons for Parallel Programming

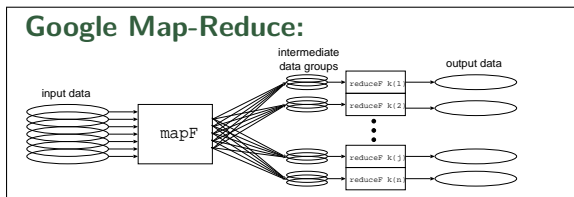
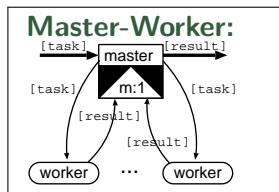


Algorithmic Skeletons [Cole 1989]: Boxes and lines – **executable!**

- Abstraction of algorithmic structure as a higher-order function
- Embedded “worker” functions (by application programmer)
- Hidden parallel library implementation (by system programmer)



Algorithmic Skeletons for Parallel Programming

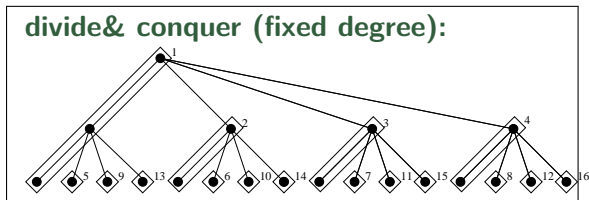
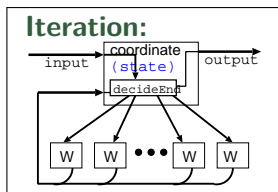


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- Abstraction of algorithmic structure as a higher-order function
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- Different kinds of skeletons: topological, small-scale, algorithmic



Algorithmic Skeletons for Parallel Programming



Algorithmic Skeletons [Cole 1989]: Boxes and lines – **executable!**

- Abstraction of algorithmic structure as a higher-order function
- Embedded “worker” functions (by application programmer)
- Hidden parallel library implementation (by system programmer)
- Different kinds of skeletons: topological, small-scale, algorithmic

Explicit parallelism control and functional paradigm are a good setting to implement and use skeletons for parallel programming.



Types of Skeletons

Common Small-scale Skeletons

- encapsulate common parallelisable operations or patterns
- parallel behaviour (concrete parallelisation) hidden

Structure-oriented: Topology Skeletons

- describe interaction between execution units
- explicitly model parallelism

Proper Algorithmic Skeletons

- capture a more complex algorithm-specific structure
- sometimes domain-specific



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Basic Skeletons: Higher-Order Functions

- **Parallel transformation:** Map

`map :: (a -> b) -> [a] -> [b]`

independent elementwise transformation

... probably the most common example of parallel functional programming (called "embarrassingly parallel")



Basic Skeletons: Higher-Order Functions

- **Parallel transformation: Map**

`map :: (a -> b) -> [a] -> [b]`

independent elementwise transformation

... probably the most common example of parallel functional programming (called "embarassingly parallel")

- **Parallel Reduction: Fold**

`fold :: (a -> a -> a) -> [a] -> a`

with commutative and associative operation.

- **Parallel Scan:**

`parScanL :: (a -> a -> a) -> [a] -> [a]`

reduction keeping the intermediate results.

- **Parallel Map-Reduce:**

combining transformation and groupwise reduction.



Embarassingly Parallel: map

map: apply **transformation** to all elements of a list

- Straight-forward element-wise parallelisation

```
parmap :: (Trans a, Trans b) => (a -> b) -> [a] -> [b]
parmap = spawn . repeat . process
      -- parmap f xs = spawn (repeat (process f)) xs
```



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Much too fine-grained!



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parmap = spawn . repeat . process
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```

Much too fine-grained!

- Group-wise processing: **Farm** of processes

```
farm :: (Trans a, Trans b) => (a -> b) -> [a] -> [b]
farm f xs = join results
  where results = spawn (repeat (process (map f))) parts
        parts   = distribute noPe xs -- noPe, so use all nodes
        join    = ...
        distribute n = ... -- join . distribute n == id
```

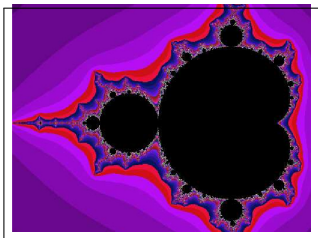


Example

Mandelbrot set visualisation $z_{n+1} = z_n^2 + c$ for $c \in \mathbb{C}$

Mandelbrot (Pseudocode)

```
pic :: .picture-parameters.. -> PPMAscii
pic threshold ul lr dimx np s = ppmheader ++ concat (parMap computeRow rows)
  where rows = ...dimx..ul..lr..
        parMap = ...np..s..
```

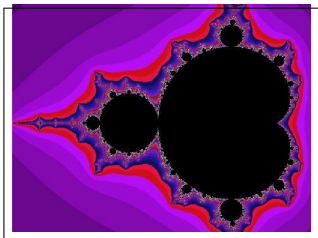


Example / Exercise

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        parMap = ...np..s.. -- you define it
```



Exercise:

- Implement `parMap` in 2 different ways
- Run the Mandelbrot program with both versions, compare the behaviour.

Framework programs can be found on the course pages...

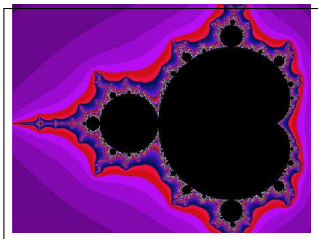


Example / Exercise: Chunked Tasks

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pic :: ..picture-parameters.. -> PPMAscii
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  where rows = ...dimx..ul..lr..
        parMap = ..using chunks..
```

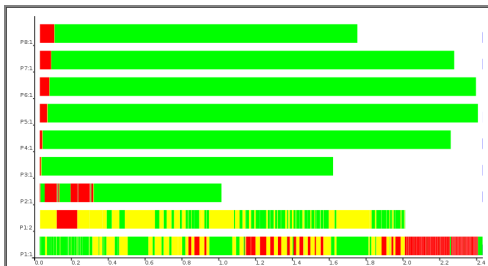
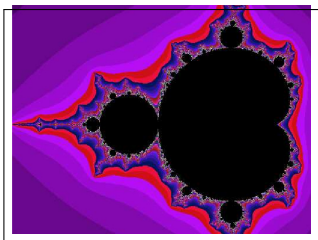


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Simple chunking leads to **load imbalance** (task complexities differ)

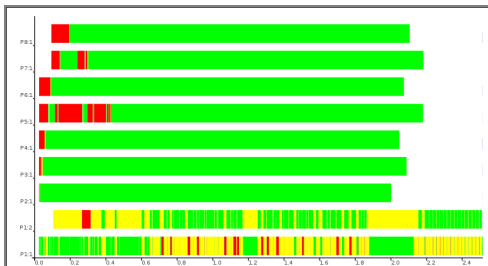
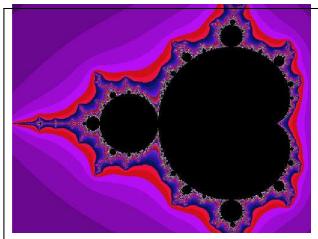


Example / Exercise: Round-robin Tasks

Mandelbrot set visualisation $z_{n+1} = z_n^2 + c$ for $c \in \mathbb{C}$

Mandelbrot (Pseudocode)

```
pic :: .picture-parameters.. -> PPMAscii
pic threshold ul lr dimx np s = ppmheader ++ concat (parMap computeRow rows)
  where rows = ...dimx..ul..lr..
        parMap = ..distributing round-robin..
```



Better: round-robin distribution, but still not well-balanced.



Master-Worker Skeleton

Worker nodes transform elementwise:

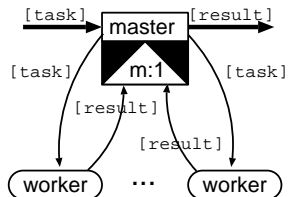
```
worker :: task -> result
```

Master node manages task pool

```
mw :: Int -> Int ->
      ( a -> b ) -> [a] -> [b]
mw np prefetch f tasks = ...
```

Parameters: no. of workers, prefetch

- Master sends a new task each time a result is returned (needs many-to-one communication)
- Initial workload of `prefetch` tasks for each worker:
Higher prefetch \Rightarrow more and more static task distribution
Lower prefetch \Rightarrow dynamic load balance



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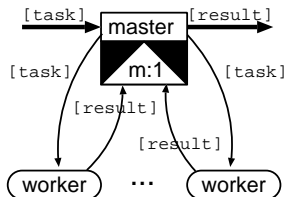
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- Master sends a new task each time a result is returned (needs many-to-one communication)
- Initial workload of `prefetch` tasks for each worker:
Higher prefetch \Rightarrow more and more static task distribution
Lower prefetch \Rightarrow dynamic load balance
- **Result order needs to be reestablished!**



Master-Worker: An Implementation

Master-Worker Skeleton Code

```
mw np prefetch f tasks = results
  where
    fromWorkers      = spawn workerProcs toWorkers
    workerProcs      = [process (zip [n,n..] . map f) | n<-[1..np]]
    toWorkers        = distribute tasks requests
```

- Workers tag results with their ID (between 1 and `np`).



Master-Worker: An Implementation

Master-Worker Skeleton Code

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mw np prefetch f tasks = results
  where
    fromWorkers          = spawn workerProcs toWorkers
    workerProcs          = [process (zip [n,n..] . map f) | n<-[1..np]]
    toWorkers            = distribute tasks requests

    (newReqs, results) = (unzip . merge) fromWorkers
    requests            = initialReqs ++ newReqs
    initialReqs         = concat (replicate prefetch [1..np])
```

- Workers tag results with their ID (between 1 and np).
- Result streams are non-deterministically merged into one stream.



Master-Worker: An Implementation

Master-Worker Skeleton Code

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    (newReqs, results) = (unzip . merge) fromWorkers
    requests           = initialReqs ++ newReqs
    initialReqs        = concat (replicate prefetch [1..np])

    distribute :: [t] -> [Int] -> [[t]]
    distribute tasks reqs = [taskList reqs tasks n | n<-[1..np]]
      where taskList (r:rs) (t:ts) pe | pe == r    = t:(taskList rs ts pe)
          | otherwise = taskList rs ts pe
          taskList _ _ _ = []
  
```

- Workers tag results with their ID (between 1 and np).
- Result streams are non-deterministically merged into one stream.
- The `distribute` function supplies new tasks according to requests.



Parallel Reduction, Map-Reduce

Reduction (`fold`) usually has a `direction`

- `foldl :: (b -> a -> b) -> b -> [a] -> b`
`foldr :: (a -> b -> b) -> b -> [a] -> b`

Starting from the `left` or `right`, implying different reduction function.

- To parallelise: break into sublists and pre-reduce in parallel.
- Better options if order does not matter.



Parallel Reduction, Map-Reduce

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- Better options if order does not matter.

Example: $\sum_{k=1}^n \varphi(k) = \sum_{k=1}^n |\{j < k \mid \gcd(k, j) = 1\}|$ (Euler Phi)

sumEuler

```
result = foldl (+) 0 (map phi [1..n])
phi k = length (filter (\ n -> gcd n k == 1) [1..(k-1)])
```



Parallel Map-Reduce: Restrictions

- `parmapReduceStream :: Int ->`
 `(a -> b) -> (b -> b -> b) -> b ->`
 `[a] -> b`
`parmapReduceStream np mapF redF neutral list = foldl redF neutral subRs`
 where `sublists = distribute np list`
 `subFold = process (foldl' redF neutral . (map mapF))`
 `subRs = spawn (replicate np subFold) sublists`



Parallel Map-Reduce: Restrictions

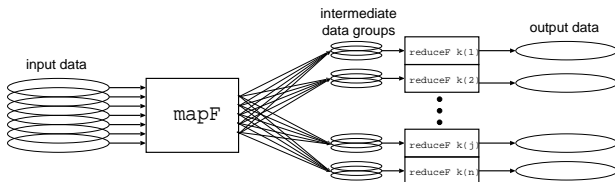
- `parmapReduceStream :: Int ->`
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`parmapReduceStream np mapF redF neutral list = foldl redF neutral subRs`
 where `sublists = distribute np list`
 `subFold = process (foldl' redF neutral . (map mapF))`
 `subRs = spawn (replicate np subFold) sublists`
- Associativity and **neutral element** (essential).
- **commutativity** (desired, more liberal distribution)
- **need to narrow type** of the reduce parameter function!
- ... Alternative `fold` type: `redF' :: [b] -> b`
 `redF' [] = neutral`
 `redF' (x:xs) = foldl' redF x xs`



Google Map-Reduce: Grouping Before Reduction

```

gMapRed :: (k1 -> v1 -> [(k2,v2)])      -- mapF
          -> (k2 -> [v2] -> Maybe v3)  -- reduceF
          -> Map k1 v1 -> Map k2 v3    -- input / output
  
```



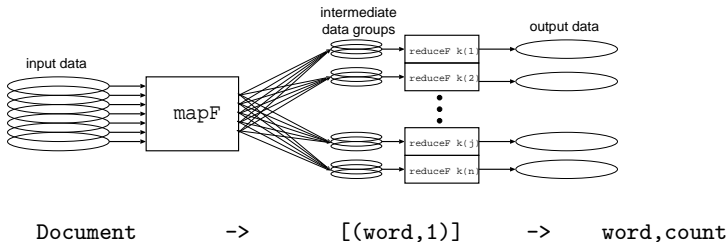
- ① Input: key-value pairs (k_1, v_1) , **many** or **no** outputs (k_2, v_2)
- ② Intermediate grouping by key k_2
- ③ Reduction per (intermediate) key k_2 (maybe without result)
- ④ Input and output: Finite mappings



Google Map-Reduce: Grouping Before Reduction

```

gMapRed :: (k1 -> v1 -> [(k2,v2)])    -- mapF
         -> (k2 -> [v2] -> Maybe v3) -- reduceF
         -> Map k1 v1 -> Map k2 v3    -- input / output
  
```



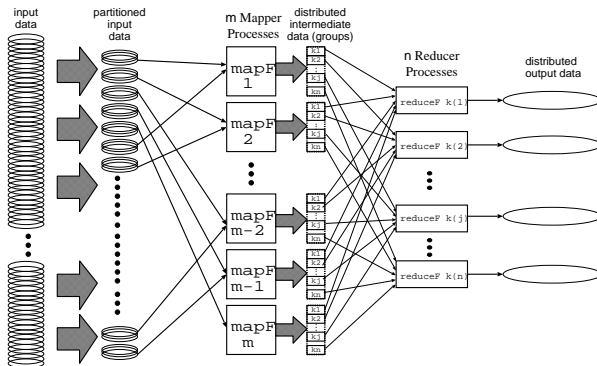
Word Occurrence

```

mapF :: URL -> String -> [(String,Int)]
mapF _ content = [(word,1) | word <- words content ]
reduceF :: String -> [Int] -> Maybe Int
reduceF word counts = Just (sum counts)
  
```



Google Map-Reduce (parallel)



R.Lämmel,
Google's
Map-Reduce
Program-
ming Model
Revisited.
In: SCP 2008

```
gMapRed :: Int -> (k2->Int) -> Int -> (v1->Int) -- parameters
          (k1 -> v1 -> [(k2,v2)])                -- mapper
          -> (k2 -> [v2] -> Maybe v3)             -- pre-reducer
          -> (k2 -> [v3] -> Maybe v4)             -- final reducer
          -> Map k1 v1 -> Map k2 v4               -- input / output
```



Outline

- ① The Language Eden (in a nutshell)
- ② Skeleton-Based Programming
- ③ Small-Scale Skeletons: Map and Reduce
- ④ Process Topologies as Skeletons**
- ⑤ Algorithm-Oriented Skeletons: Two Classics
- ⑥ Summary

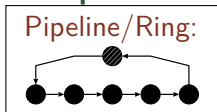


Process Topologies as Skeletons: Explicit Parallelism

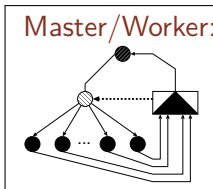
- describe typical patterns of parallel interaction structure
- (where node behaviour is the function argument)
- to structure parallel computations

Examples:

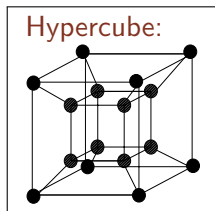
Pipeline/Ring:



Master/Worker:



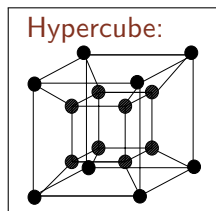
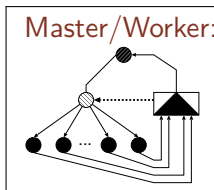
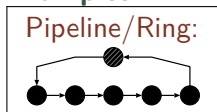
Hypercube:



Process Topologies as Skeletons: Explicit Parallelism

- describe typical patterns of parallel interaction structure
- (where node behaviour is the function argument)
- to structure parallel computations

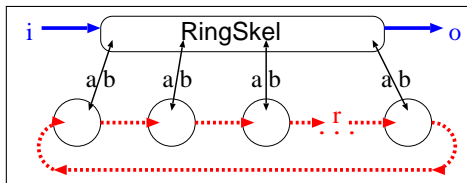
Examples:



⇒ well-suited for functional languages (with explicit parallelism).
Skeletons can be **implemented** and **applied** in Eden.



Process Topologies as Skeletons: Ring



```
type RingSkel i o a b r = Int -> (Int -> i -> [a]) -> ([b] -> o) ->
    ((a, [r]) -> (b, [r])) -> i -> o
```

```
ring size makeInput processOutput ringWorker input = ...
```

- Good for exchanging (updated) global data between nodes
- All ring processes connect to parent to receive input/send output
- Parameters: functions for
 - decomposing input, combining output, ring worker



Outline

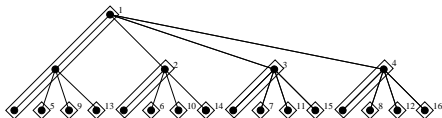
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Two Algorithm-oriented Skeletons

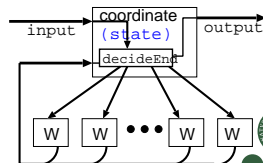
- Divide and conquer

```
divCon :: (a -> Bool) -> (a -> b)      -- trivial? / then solve
      -> (a -> [a]) -> ([b] -> b)    -- split / combine
      -> a -> b                       -- input / result
```



- Iteration

```
iterateUntil :: (inp -> ([ws],[t],ms)) ->      -- split/init function
              (t -> State ws r) ->            -- worker function
              ([r] -> State ms (Either out [t])) -- manager function
              -> inp -> out
```



Divide and Conquer Skeletons

- General version: no assumptions on problem characteristics

```
divCon :: (a -> Bool) -> (a -> b)      -- trivial? / then solve
        -> (a -> [a]) -> ([b] -> b) -- split / combine
        -> a -> b                    -- input / result
divCon trivial solve split combine = ...
```

- Implementation will make (parallel?) recursive calls to itself (with same parameters as the initial call).



Divide and Conquer Skeletons

- General version: no assumptions on problem characteristics

```
divCon :: (a -> Bool) -> (a -> b)      -- trivial? / then solve
        -> (a -> [a]) -> ([b] -> b)  -- split / combine
        -> a -> b                    -- input / result
divCon trivial solve split combine = ... -- you write one
```

- Implementation will make (parallel?) recursive calls to itself (with same parameters as the initial call).

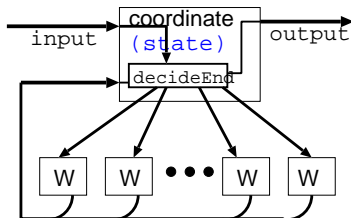
Exercise:

- Implement this general divide-and-conquer version.
Write a sequential version first, then make recursive calls parallel.
Add **one Int parameter** to limit the parallel depth.



Iteration Skeleton

- Fixed set of workers
- Lock-step execution, solving a set of tasks
- Manager decides end



```
iterateUntil :: (inp -> ([ws],[t],ms)) ->           -- split/init function
               (t -> State ws r) ->              -- worker function
               ([r] -> State ms (Either blue [t])) -- manager function
               -> inp -> out
```

Worker: computes result r from task t
using and updating a local state ws

Manager: decides whether to continue,
based on master state ms and all worker results.
produce tasks for all workers



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Summary

- Eden: Explicit parallel processes, mostly functional face
- Two levels of Eden: Skeleton implementation and skeleton use
 - Skeletons: High-level specification exposes parallel structure
 - and enables programmers to think in parallel patterns.
- Different skeleton categories (increasing abstraction)
 - Small-scale skeletons (map, fold, map-reduce, ...)
 - Process topology skeletons (ring, ...)
 - Algorithmic skeletons (divide & conquer, iteration)



Summary

- Eden: Explicit parallel processes, mostly functional face
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- Different skeleton categories (increasing abstraction)
 - Small-scale skeletons (map, fold, map-reduce, ...)
 - Process topology skeletons (ring, ...)
 - Algorithmic skeletons (divide & conquer, iteration)
- More information on Eden:

<http://www.mathematik.uni-marburg.de/~eden>

(<http://hackage.haskell.org/package/edenskel/>)

(<http://hackage.haskell.org/package/edenmodules/>)



Exercises for the Lab

- 1 Complete the **Hamming number** program
File: `hamming-.hs`
Execute the program and look at an execution trace using EdenTV
- 2 Implement **two versions of parMap** which increase granularity
Files: `ParMap.hs`, `mandel.hs`
Test your versions using the Mandelbrot program.
- 3 Implement the **Divide-And-Conquer skeleton**
Files: `DC.hs`, `mergesort.hs`
Test your skeleton implementation using the provided mergesort program.
- 4 (**Bonus**) Implement a simple quicksort program using the skeleton



Usage example:

Compile example, (with tracing `-eventlog`):

```
berthold@bwlf01$ COMPILER -parcp -eventlog -O2 -rtsopts --make mandel.hs
[1 of 2] Compiling ParMap          ( ParMap.hs, ParMap.o )
[2 of 2] Compiling Main              ( mandel.hs, mandel.o )
Linking mandel ...
```

Run, second run with tracing:

```
berthold@bwlf01$ ./mandel 0 200 1 -out +RTS -qp4 > out.ppm
==== Starting parallel execution on 4 processors ...
berthold@bwlf01$ ./mandel 0 50 1 +RTS -qp4 -l
==== Starting parallel execution on 4 processors ...
Done (no output)
Trace post-processing...
  adding: berthold=mandel#1.eventlog (deflated 65%)
  adding: berthold=mandel#2.eventlog (deflated 59%)
  adding: berthold=mandel#3.eventlog (deflated 58%)
  adding: berthold=mandel#4.eventlog (deflated 58%)
berthold@bwlf01$ edentv berthold\=mandel_0_50_1_+RTS_-qp4_-l.parevents
```

