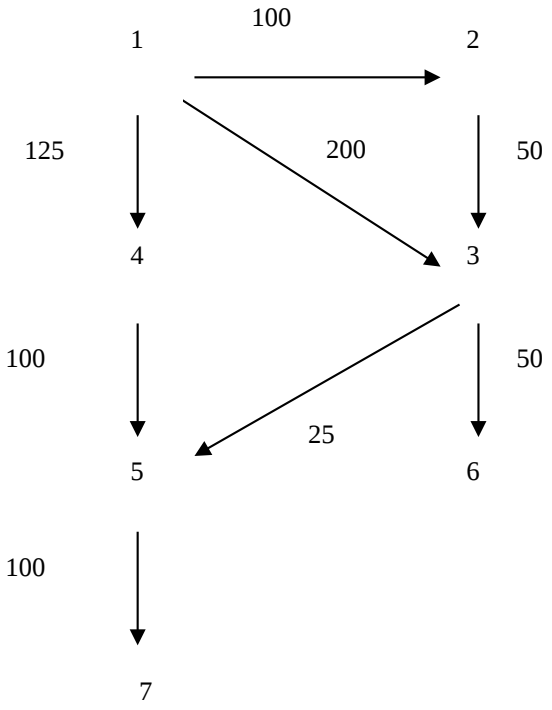
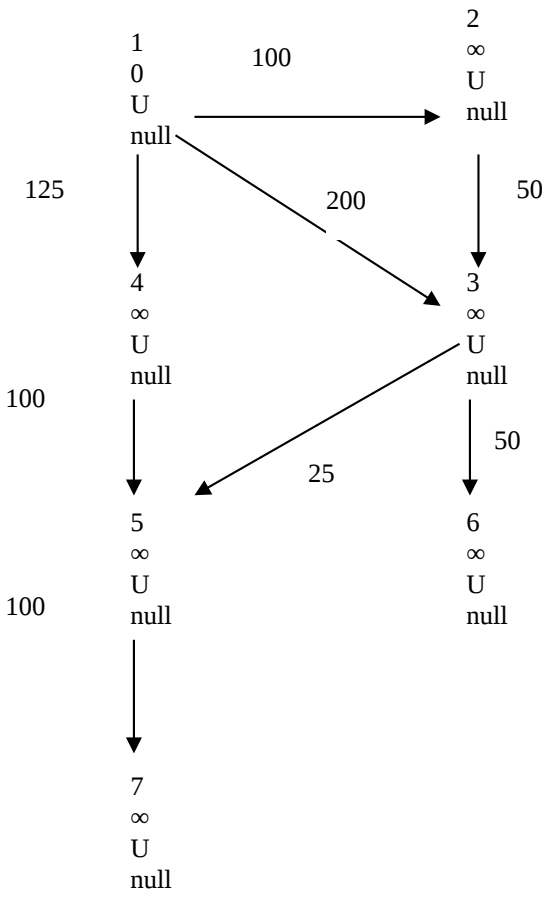
Graph Search Algorithms

1. The following weighted directed graph describes the power-up sequence of devices on a space vehicle, i.e. device 1 (the Backbone Computer) must be on before devices 2,3 and 4 (Navigation, Yaw&Pitch Control and Atmosphere Control respectively) can be initiated. The weights represent the wattage required to initiate each device.



The following figure shows the graph and `NewReachables` set after the initialization phase of Dijkstra's shortest path algorithm, where each vertex is labeled with `distanceFromSource`, `known` (K for known and U for unknown), and `path` i.e. *preceding vertex* in the path (null if none).



`NewReachables` = {1}

a) For the next (first) step of Dijkstra's algorithm,
   i) Explain which node will next be selected, and why.
   ii) Show the updated graph and `NewReachables` set after processing the next node

b) For the following (second) step of Dijkstra's algorithm,
   i) Explain which node will next be selected, and why.
   ii) Show the updated graph and `NewReachables` set after processing the next node

c) Show the graph after Dijkstra's shortest path algorithm completes

d) The batteries in the vehicle have limited power and for a course correction maneuver mission control asks:

i) What sequence should the devices be started to initiate device 3 (Yaw&Pitch Control) with minimal power usage?

ii) What sequence should the devices be started to initiate device 7 (the Side Thrusters) with minimal power usage? For each vertex v_i on the path, show $v_i.path$.

e) After the maneuver all devices except device 1 (the Backbone Computer) are turned off for 2 days to conserve power, by which time the maximum power output from the batteries is 200W. Mission control now asks the following questions

i) Can device 5 (Roll Control) be initiated? If so using what sequence, and what is the total power consumed?

ii) Can device 7 (Side Thrusters) be initiated? If so using what sequence, and what is the total power consumed?

iii) Can devices 5 and 6 (Roll Control and Cabin Heater) be initiated simultaneously? If so using what sequence, and what is the total power consumed?

iv) What are the *largest* sequences of devices that can be initiated with the power available?
N.B. Report only the largest sequences, e.g. {1,2,3} implies that {1,2} and {1} are both possible.

e) Did the vehicle land safely ;-) ???

2.

i) Adapt the pseudocode for Dijkstra's shortest path algorithm to find the longest path from a source node to every destination node in an acyclic graph.

ii) Walkthrough of your algorithm on an example graph (e.g. those above) to check it works correctly.

3. With reference to the graph in figure 1 below:

(i) Illustrate the effect of a depth first search of the graph: list the sequence of nodes visited and show how the stack at each point in the search

(ii) Illustrate the effect of a breadth first search of the graph (as above, but show the queue data structure being used).

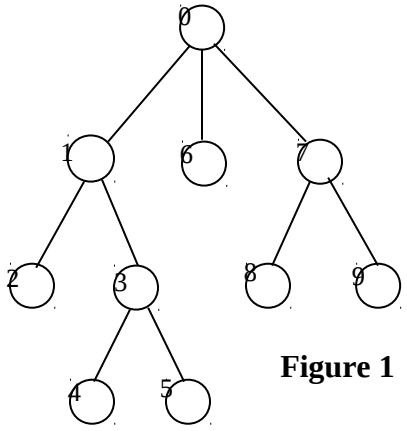


Figure 1

4. What does it mean to determine the shortest path in a graph?
   - For a weighted graph
   - For a non-weighted (or uniform weighted) graph

5. Think about an interesting application (i.e. something that interests you!) that could benefit from a graph structure. If you are stuck for an example, think about the following:
   - Some sort of game
   - A route finding tool for a user

   Think about some of the things that you might need to consider given the particular application. What type of graph structure may help you most – list the characteristics of the graph structure and how that may help the application.