Pseudo-Adsorption Isotherm Matching for Oilfield Scale Inhibitor Squeeze Treatments

Oscar Vazquez de la Varga
Dissertation
MSc in Information Technology
School of Mathematical and Computer Sciences (MACS)
Heriot-Watt University
August 2011
Declaration

I, Oscar Vazquez de la Varga, confirm that this work submitted for assessment is my own and is expressed in my words. Any uses made within it of the words of other authors in any form e.g., ideas, equations, figures, text, tables, programs, etc are properly acknowledged. A list of the references employed is included.

Signed

Date Edinburgh, 18 August 2011
Abstract

Oilfield scale formation represents a very significant flow assurance challenge to the oil and gas industry, with increasing water production worldwide and higher oil prices. Scale Inhibitor (SI) Squeeze treatment is the most effective method to combat downhole scaling. In order to predict SI squeeze treatments accurately for further optimisation, it is necessary to simulate the SI retention in the formation, which may be described by a pseudo-adsorption isotherm. While these are often derived from core flood experiments, issues of representativity mean they are inappropriate for large field scale simulations. In practice the parameters of an analytic form of the isotherm equation are modified by trial and error until a match is obtained between the prediction and the return profile of the first treatment.

The main purpose of this study is to propose a Simple Stochastic Hill Climbing Algorithm for the automatic pseudo-adsorption isotherm matching. The algorithm performance was assessed against two test problems and validated using two field cases. The results demonstrated that for this particular problem, overfitting is not significant. In addition, the existence of the multiple local minima was demonstrated. The equivalence of these local minima was studied. And finally, the proposed algorithm was validated against two field cases, which were treated several times, with very good results.
Acknowledgement

I wish to thank David Corne, my supervisor, for his advice and time during the preparation of this thesis.

I would like to thank Myles Jordan for providing the field cases.

Also, I would like to thank Patrick Corbett, Head of the Institute of Petroleum Engineering and Keith Cornwell, Head of School of Mathematical and Computer Science back in 2006 for agreeing waiving the fees for this Master.

Last but not the least; I would like to thank my wife, for her encouragement and support.
# Table of contents

1 Introduction ........................................................................................................... 1  
   1.1 Oilfield Scale Inhibitor Squeeze Treatments ........................................... 2  
   1.2 Problem description .............................................................................. 5  
2 Literature review .............................................................................................. 7  
   2.1 Previous work in automatic pseudo isotherm matching ..................... 7  
   2.2 Optimization techniques ..................................................................... 9  
      2.2.1 Approximate search .................................................................. 10  
   2.3 Multiobjective optimization .............................................................. 11  
3 Proposed method ........................................................................................... 13  
   3.1 Parameter setting ............................................................................... 15  
   3.2 Test Problems .................................................................................... 16  
   3.3 Overfitting ........................................................................................ 18  
   3.4 Comparison between Train/Eval and 100% Training ......................... 21  
      3.4.1 Equilibrium Test Problem, analytical solution \( k = 100, n = 0.8 \) .... 21  
      3.4.2 Kinetic Test Problem, analytical solution \( k = 100, n = 0.3, r =20 \) .... 23  
   3.5 Analysis of the results of the Kinetic Test Problem ......................... 24  
   3.6 Are the suggested and the analytical solution equivalent? ............. 31  
   3.7 Conclusions ...................................................................................... 33  
4 Field Cases analysis ...................................................................................... 35  
5 Final Conclusions .......................................................................................... 41  
6 Future recommendations ............................................................................. 43  
7 References .................................................................................................... 44  
Appendix A Extra Comparison between Train/Eval and 100% Training ....... 47  
   A.1 Equilibrium Test Problem, analytical solution \( k = 100, n = 0.8 \) .... 47  
   A.2 Kinetic Test Problem, analytical solution \( k=100,n=0.3,r=20 \) .......... 48  
Appendix B Scatter Density plots for 100% Training and Train/Eval .......... 51
Table of Figures

Figure 1.1 Schematic showing scale inhibitor squeeze treatment process (Zavala et al, 2008)..........................................................................................................................4
Figure 1.2 Illustration of the SI Field Return Concentration Profile. ....................5
Figure 3.1 Equilibrium Synthetic case with added noise, 2-Dimensional problem. ........................................................................................................................................17
Figure 3.2 Kinetic Synthetic cases with added noise, 3-Dimensional problem. 17
Figure 3.3 Absolute difference between the Last and the Minimum evaluation error, the First, the Minimum and the Last evaluation error for Equilibrium, 2D problem........................................................................................................................................18
Figure 3.4 Absolute difference between the Last and the Minimum evaluation error, the First, the Minimum and the Last evaluation error for Kinetic, 3D problem........................................................................................................................................19
Figure 3.5 Training misfit error is shown in blue, validation misfit error in red, both as a function of the number of training cycles for equilibrium...........................20
Figure 3.6 Training misfit error is shown in blue, validation misfit error in red, both as a function of the number of training cycles for kinetic.........................................20
Figure 3.7 Distribution of best candidates for Equilibrium (k,n) with No Noise..22
Figure 3.8 Distribution of best candidates for Equilibrium (k,n) with Extreme Noise............................................................................................................................22
Figure 3.9 Distribution of best candidates for Kinetic (k,n,r) with No Noise......23
Figure 3.10 Distribution of best candidates for Kinetic (k,n,r) with Extreme Noise............................................................................................................................24
Figure 3.11 Scatter density plots of best candidates for Kinetic (k,n,r) with No Noise, 100% Train. ...............................................................................................................25
Figure 3.12 Scatter density plots of best candidates for Kinetic (k,n,r) with Extreme Noise, 100% Train. ...............................................................................................................26
Figure 3.13  Scatter plots of parameter $k$ versus $n$ and $r$ of the best candidates for Kinetic $(k,n,r)$ with No added and Extreme Noise, 100% Train. .......................... 27
Figure 3.14  Scatter density plots of best candidates for Kinetic $(k,n,r)$ with No Noise, Train/Eval. ........................................................................................................ 28
Figure 3.15  Scatter density plots of best candidates for Kinetic $(k,n,r)$ with Extreme Noise, Train/Eval. ..................................................................................... 28
Figure 3.16  Scatter plots of parameter $k$ versus $n$ and $r$ of the best candidates for Kinetic $(k,n,r)$ with No added and Extreme Noise, 100% Train. ..................... 29
Figure 3.17  Contour maps for the 3D cases with the logarithmic misfit function, misfit values in the white regions are higher than 20 and out of scale. ............. 30
Figure 3.18  Comparison of SI return concentration profiles obtained using the Target and the HC Kinetic pseudo-isotherm candidate for different treatment strategies 100% Train. .............................................................................. 32
Figure 3.19  Comparison of SI return concentration profiles obtained using the Target and the HC Kinetic pseudo-isotherm candidate for different treatment strategies Train/Eval. ................................................................. 33
Figure 4.1  SI return concentration Well A, Treatment 1. Best solution trio $(k,n,r)$ = (500.5, 0.18, 0.78) assuming single phase flow. .............................................. 36
Figure 4.2  SI return concentration Well A, Treatment 2. Best solution trio $(k,n,r)$ = (500.5, 0.18, 0.78) assuming single phase flow. .............................................. 37
Figure 4.3  SI return concentration Well A, Treatment 3. Best solution trio $(k,n,r)$ = (500.5, 0.18, 0.78) assuming single phase flow. .............................................. 37
Figure 4.4  SI return concentration Well A, Treatment 4. Best solution trio $(k,n,r)$ = (500.5, 0.18, 0.78) assuming single phase flow. .............................................. 38
Figure 4.5  SI return concentration Well B, Treatment 1. Best solution trio $(k,n,r)$ = (729,0.63,1.93) assuming single phase flow. ................................................... 39
Figure 4.6  SI return concentration Well B, Treatment 2. Best solution trio $(k,n,r)$ = (729,0.63,1.93) assuming single phase flow. ................................................... 39
Figure 4.7  SI return concentration Well B, Treatment 3. Best solution trio $(k,n,r)$ = (729,0.63,1.93) assuming single phase flow. ................................................... 40
1 Introduction

Mineral scale precipitation is one of the most significant concerns in the oil and gas industry. The main types of scale are organic such as wax, asphaltene and naphthenate, and inorganic such as carbonate and sulphate scales. Solids deposition can occur in the surface facilities, pipelines, wellbore or even in the formation around the well. Oilfield scales typically consist of one or more types of inorganic deposit along with other debris such as organic precipitates, sand, corrosion products, etc.

Some of the main problems due to the scale deposition are formation damage in the area close to the wellbore, blockages in perforations or gravel pack, restriction or blockage of pipelines, safety valve and choke failure, pump wear, metal corrosion underneath scale deposits and finally, some inorganic scales are radioactive.

The main cause of the scale deposition is a disturbance of the reservoir equilibrium reached after thousands of years, which occurs at the start of production. The reservoir temperature decreases due to the contact of the fluid with the well infrastructure, and so the reservoir pressure. Sea water or other brines are injected to maintain the reservoir pressure, and so the fluid production. Generally, the brines injected are incompatible with the reservoir formation brine, which perturbs the chemical equilibrium and as a result oilfield scale is deposited.

The most common oilfield scales are Calcium Carbonate, caused by pressure drop and Barium Sulphate, Strontium Sulphate and Calcium Sulphate, caused by the mixing of incompatible brines.

If scale deposition occurs, there are a number of alternatives for damage removal and bring back the productivity back to its original value. The scale deposits may be dissolved by an appropriate dissolver. In certain scenarios dissolving the scale is not an option, for example for certain sulphate deposits. An alternative is to use mechanical means, such as milling, fracturing or re-
perforating. Mechanical techniques to remove scale should be the latest option as they can cause equipment failure at a high cost.

An alternative to these techniques, possibly the most preferable, is to apply a “squeeze” treatment, which is a preventing technique. It involves shutting the well at risk before scale deposition or just after having evidence of scale deposition, followed by the injection of a scale inhibitor solution. When the chemical is in contact with the rock formation, it adheres to the rock surface. Finally, the well is back in production and the scale inhibitor gradually desorbs/dissolves into the flowing brine stream. The produced scale inhibitor in the flowing brine prevents or delays scale deposition by interfering the nucleation and the subsequent crystal growth, (Salamone, 1998). Mackay and Sorbie reported that once a crystal has nucleated, active growth sites are created, where the scaling ions present in the solution will attach, leading to a bigger crystal, and the consequent new growth sites.

1.1 Oilfield Scale Inhibitor Squeeze Treatments

Preventing scale deposition in a producing oil well is of major importance, as scale removal treatments might be very costly and risky. There are different techniques to prevent scale deposition in the producing well, such as controlling the reservoir temperature and pressure, injected fluid modification (seawater sulphate removal, injecting produced water or using aquifer water), flow modification (shut-in the water producing layers), inducing precipitation deep in the reservoir. In certain situations, these methods could have a negative impact on productivity and generally are less effective than injecting a scale inhibitor.

Squeeze treatments is the most common method for the prevention of oilfield mineral scale in producing oil wells. A “squeeze” treatment consists of the injection of scale inhibiting chemical in an oil producing well under risk. The chemical is retained and then gradually released back in the flowing aqueous phase when the well is back in production. The well will remain protected as long as the returning chemical concentration is above a certain level, generally known as MIC (Minimum Inhibitor Concentration). Generally, every oilfield scale
squeeze treatment consists of the following stages, as described previously (Mackay and Sorbie, 1999), and depicted in Figure 1.1:

- **Preflush** – This stage serves as a buffer for the main treatment, it cleans the rock formation, as well as cooling the area around the wellbore to preventing adsorption and/or precipitation close to the wellbore, which allows the chemical to be transported deeper in the reservoir formation. In certain situations additives are included to condition the rock.

- **Main slug** – This stage is where the main volume of scale inhibitor is injected.

- **Overflush** – Normally water is injected to expose to cleaner and hotter formation zones to enhance retention.

- **Shut in** – A period of no flow for period no longer than 24 hours to allow the soaked zones to be heated up further.

- **Production** – The well is brought back in production.

It is generally accepted that precipitation and adsorption on the rock surface are the principal mechanism of scale inhibitor (SI) retention, which ultimately determine the shape of the SI return concentration profile. According to Vazquez (Vazquez et al., 2009), there are two schools of thought: Rice university workers described the SI retention as deposition and subsequent dissolution of Calcium-SI complex (Kan, 2005), whereas Heriot-Watt school of thought is based on an adsorption isotherm or pseudo adsorption isotherm when adsorption is not the only retention mechanism (Sorbie and Gdanski, 2005). In this study the later will be adopted for being more general, where an adsorption isotherm is referred as a “retention isotherm”, since it accounts for both retention mechanisms. The main advantage of this approach is that it can be applied straightforward with a well established software, (Zhang and Sorbie, 1997; Vazquez et al, 2006).
To assess the performance of a squeeze treatment, the SI return concentration in the produced brine versus time is plotted, known as the field return concentration profile. Generally, before any field application a series of coreflood tests are carried out, from which a preliminary pseudo isotherm is derived. This preliminary isotherm is used to design the first squeeze treatment, however as it might be expected, the first field return profile is not perfectly matched for a particular scale inhibitor for a specific formation (Sorbie et al, 1992; Yuan et al., 1994). Several reasons why this may occur are listed below (Mackay and Jordan, 2003):

- Variations in lithology in the formation
- Assumptions about inhibitor placement
- Variations in actual production rates and flow profiles
- Changes in brine chemistry of the produced fluid
- Differences between coreflood and near-wellbore flow rates and temperatures
- Errors in analysis due to high TDS (Total Dissolved Salts) brines.
Therefore, generally it is necessary to re-derive a pseudo isotherm from the first field return to provide accurate predictions of the subsequent treatments. The derivation of the new pseudo isotherm is commonly carried on by trial and error varying numerical parameters that describe the shape of the pseudo adsorption isotherm until a reasonable match is obtained.

1.2 Problem description

As mentioned above plotting the SI return concentration versus time provides SI return concentration profile, as depicted in Figure 1.2, which might be described by a pseudo adsorption isotherm.

Figure 1.2 Illustration of the SI Field Return Concentration Profile.

Adsorption isotherms determine the relation between the SI concentration in the aqueous phase and the mass of SI adsorbed or retained in the rock surface, which are commonly described by Langmuir or Freundlich functions. In this study only Freundlich functions are considered, as they can be derived from Langmuir isotherm and they have been used more often (Wang and Hung, 2006).
Freundlich isotherms have the form shown by equation (3.1), where k and n are constants.

\[ \Gamma = k \cdot C^n \]  

(1.1)

If the adsorption (retention) process is at equilibrium, i.e. instantaneous process, the description of the isotherm by the Freundlich equation is enough. The parameter space becomes two dimensional, with the following range \(1 \leq k \leq 1000\) and \(0.01 \leq n \leq 1\), which is believed to be reasonable, as it captures every possible candidate, (Vazquez et al., 2010). The adsorption (retention) process might not be at equilibrium, i.e. it is time dependent. Thus it is necessary to describe the rate to equilibrium, which can be described by equation (3.2):

\[ \left( \frac{\partial \Gamma}{\partial t} \right)_c = r \cdot (\Gamma_{eq} - \Gamma) \]  

(1.2)

where \(r\) is the kinetic desorption rate, \(\Gamma_{eq}\) is the equilibrium adsorption level and \(\Gamma\) is the actual adsorption at a given mobile concentration, \(C\) (Sorbie et al., 1989; Sorbie, 1991; Zhang and Sorbie, 1997; Vazquez et al., 2006). To reasonably characterise the non-equilibrium adsorption rate, Vazquez (Vazquez et al, 2010) suggested the following range \(0.001 \leq r \leq 100\).

Every field return profile might be described by a pseudo adsorption isotherm, which is assumed that can be described by a Freundlich form, defined by two numerical constants \(k\) and \(n\). The adsorption (retention) process might be at equilibrium conditions, which lead to a two dimensional optimization problem, where the parameter space is defined by the pair \((k,n)\). On the other hand, if the adsorption process is not at equilibrium the parameter space becomes three dimensional, defined by the trio \((k,n,r)\), where \(r\) defines the rate to adsorption equilibrium.
2 Literature review

This section is divided in two different sections; the first one will cover what approaches have already been proposed. Whereas the second section will focus on which techniques can be applied to the problem in hand.

2.1 Previous work in automatic pseudo isotherm matching

There have been some attempts to make the pseudo-isotherm matching process automatic. The first attempt was to apply the Bisection method to find an isotherm match, proposed by Vazquez (Vazquez et al., 2009). This is a deterministic approach, where the misfit function, $F$, was defined to weight more the tail of the field profile than the early peak.

This approach is based on the following assumptions: First, the objective function is one dimensional, i.e. treating the pair $(k,n)$ and the trio $(k,n,r)$ for equilibrium and non equilibrium adsorption process as a single value. Second, that the objective function is continuous, although they did not show a formal proof, they claimed that intuitively it seems clear that for any simulated profile, it is possible to find a isotherm match (pair or trio), which is as close as required.

They proposed the following algorithm: given a field return profile, the algorithm starts with an isotherm guess, then the numeric value of the pair or trio is reduced until an interval $[a,b]$, a and b being a pair or trio $(k,n)$ or $(k,n,r)$, is found such as $F(a) \cdot F(b) < 0$. This implies that each limit results in a profile above and below the given field profile. Finally, the bisection method is applied to the interval $[a,b]$ five times. This procedure is based on the fact that by choosing a sufficient High or Low isotherm, the simulated return profile rises or lowers at least for the tail of the profile. As shown in Figure 1.2, the SI return profile can be divided in an early peak and a long tail, which is the most valuable part of the profile, as it provides when the profile lies crosses the MIC. This value is used to determine the squeeze treatment lifetime.
The algorithm was tested against a synthetic and two field cases, the results were promising, but not satisfactory. The match for the synthetic case was significantly good; however the matches for the field cases were not satisfactory.

Based on the results of the approach described above, the behaviour mismatch between the field and the simulated return profile using two different misfit functions, “standard” and “tail”, was presented (Vazquez et al., 2010). Standard misfit is an L2-norm misfit, the most common for history matching (Oliver et al., 2008). Typically, it is applied to the whole profile as described by equation (3.3):

\[
\text{Standard Misfit} = \sum_{i=1}^{n} \left( \frac{O_i - S_i}{O_i} \right)^2
\]  

where \( n \) is the number of data points of the field return concentration profile, \( O_i \) is the \( i^{th} \) observed return concentration obtained from the field profile and \( S_i \) is the \( i^{th} \) simulated return concentration.

Tail misfit is a variation of the well known L1-norm, previously described (Vazquez et al., 2009), see equation (2.2). Although there might be some cancellations using this variation, it calculates the misfit of the tail of the profile, since the early part (30%) of the return profile is ignored. The rationale behind is that is preferable to match the tail of the profile rather than the characteristic high concentration early peak, since matching the tail may be used to make predictions on when the return profile will be below the MIC, or in other words, the squeeze treatment lifetime.

\[
\text{Tail Misfit} = \left| \sum_{i=0.3n}^{n} (O_i - S_i) \right|
\]  

To study the behaviour of the misfit functions, they used density maps and 3D scattered plots for synthetic cases and field cases. Which were created as
follows: (k, n) and (k,n,r) are sampled on a regular grid and for each simulation corresponding values of the tail and standard misfit functions were obtained.

The results from the synthetic studies showed that each misfit behaves differently. However, comparing the scattered plots there was a clear overlapping region of low misfit values, where excellent matches were obtained, under equilibrium and kinetic conditions. The scatter plots for the field cases using Tail and Standard misfits did not seem to have any significant overlapping region of low misfit values, possibly due to the complexity of the problem under study.

### 2.2 Optimization techniques

In the literature there are plenty of alternatives to search for the optimum answer, such as data mining techniques (machine learning) or optimization techniques. A machine learning approach will normally use an algorithm to learn and then produce a predictive model. Among the optimization techniques, there is a clear distinction between deterministic and approximate search. Deterministic search assumes that the objective (misfit) function can be evaluated, where this information is used to establish the search direction in a deterministic way at every stage of the algorithm. Examples of deterministic search are the bisection method, the steepest descent or gradient method and Newton Raphson, (Spall, 2003).

Onwubolu and Babu stated (Onwubolu and Babu, 2004) that rather than applying deterministic search techniques, which ultimately will provide the global optimum solution. They suggested that good approximate search techniques that could present a reasonable good solution, should be used for complex real cases problems. According to them, the main reasons why an approximate approach is preferable to deterministic search in complex real life problems are as follows. Although deterministic searches will provide ultimately the global optimum solution, it might be at a high computational price and they are not always applicable when the functional relationship between the input variables and the misfit function is not clear, i.e. if the surface or fitness landscape is
jagged, or even in certain problems it may be discontinuous. In addition, deterministic searches might not be easy to implement for certain problems, whereas approximate search are much easier to implement, and much more flexible.

### 2.2.1 Approximate search

According to Onwubolu and Babu approximate search techniques can be divided in two categories, namely “tailored or customized” and “general purpose techniques”. Stochastic optimization falls in the general purpose category and applied to optimize systems where the input variables and the misfit objective function is not known a priori, they are very robust, being relatively insensitive to noisy data and it has been proved to find good solutions in many real-life and practical applications. Successful stochastic optimization methods are classified in two general categories, local search and population based.

#### 2.2.1.1 Local search

This type of techniques explores the neighbours of the current candidate locally until a new candidate becomes the current candidate, and then its neighbours are explored, until a certain stopping condition is reached. The stochastic hill climber is one of the simplest and most popular search methods, which is described in the steps below:

1. Start with a solution candidate, S, with certain misfit value, S
2. Explore local neighbours of S by a random change, S’
3. If S’ has a lower misfit value than S, replace S with S’
4. Go to step ii) until stopping criterion is reached, such as convergence or maximum number of iterations.

One of the major drawbacks of this simple algorithm is that it can get trapped easily in local minima.

#### 2.2.1.2 Population-based search

The main difference with the local search methods is that the current candidate or solution is replaced by a collection or population of potential solutions, where
the current population is used to generate a new population. Which in fact generate almost infinite strategies about how to create new members of the population via mutation and recombination operations, and which members to maintain and which ones to replace in the current population. A population based search method may be described by the following steps:

i) Start with a population, $S$ and evaluate.
ii) Generate a new population $S'$ from $S$ via mutation and recombination operations.
iii) Evaluate $S'$ and replace some members of $S$ with $S'$ following some preconceived strategy.
iv) Go to step ii) until stopping criterion is reached, such as convergence or maximum number of iterations.

Evolutionary algorithms fall in this category. Examples of this type of algorithms are Genetic Algorithm, GA, based on natural genetics operations such as inheritance, mutation, selection, and crossover. (Goldberg, 1989). Particle Swarm Optimization, PSO, this technique was inspired by the social behaviour of a flock of birds; it basically move candidate solutions called particles in the search space based on particle fitness and a simple formula (Kennedy and Eberhart, 1995). Differential Evolution, DE, this algorithm creates new candidate solutions combining a target vector (calculated using a simple formulae of vector-crossover and -mutation) with existing ones, and then keeping the candidate solution with the lowest misfit value (Storn and Price, 1995; Wikipedia, 2010). These techniques have been used in petroleum reservoir history matching, which aims to find petrophysical and geological properties that can reproduce the reservoir history, GA (Erbas and Christie, 2007), PSO (Mohamed et al., 2010) and DE (Hajizadeh et al., 2009).

2.3 Multiobjective optimization

Many real problems may have multiple and possibly conflicting objectives, which deviates for the single optimization problems. In the problem that was described above in section 1.2, although it could be well treated as a single objective
problem, Vazquez (Vazquez et al., 2009; 2010) showed that there might several local minima. They also suggested that in some situations, only the early peak and the tail could be matched, where the objective was to match the tail as it provides more information about the future predictions. It is not clear, which of those apparent local minima produce the best combined match of peak and tail. Therefore, there might be to two conflicting objectives, the early peak and the tail.

Fonseca (Fonseca and Fleming, 1995) classified different alternatives to tackle multiobjective optimization, by aggregation such as the simple weighted to target vector optimization, and non-aggregating, which can further classified between non- and Pareto approaches. The Vega Evaluated Genetic Algorithm (VEGA) proposed by Schaffer in 1985 is an example of a non-Pareto approach that treats objectives separately. Zitler (Zitler et al., 2003) stated that in recent years most of research has been focused on the approximation of the Pareto set, which is based on the concept of Pareto dominance, commonly used to compare solutions of a multidimensional misfit function. A solution, \( S_1 \), is said to dominate another solution, \( S_2 \), if no component of misfit function \( S_1 \) is higher (worse) than the corresponding in solution \( S_2 \) and at least one component is lower (better). Corne (Corne et al., 2000) proposed a multiobjective evolutionary algorithm called PESA (the Pareto Envelope-based Selection Algorithm), and Knowles (Knowles et al., 2009) studied Pareto multiobjective optimization in noisy problems using a limited number of evaluations.
3 Proposed method

A description of the problem has been shown in section 1.2. The problem involves finding of a pseudo-adsorption isotherm matching a field SI return concentration profile. Some methods have been proposed showing interesting results, but nevertheless unsatisfactory.

In the literature, a significant number of different optimization techniques have been proposed. Among them, approximate search techniques are the best suited for the problem in hand, due to their flexibility to deal with possible “rugged” misfit landscape and noisy data.

It has been shown before that the problem under study is complex, due to nature of the functional relationship between the input variables and the misfit function. Although, it is difficult to describe the misfit landscape, it seems reasonable to believe that most likely; it will be at least “rugged”, due to the fact that the misfit function depends on the outcome of a computer simulation. The only knowledge of the behaviour of the landscape misfit is by contour maps for specific cases. Studies have been done previously, (Vazquez et al, 2010), where complexity has been reported.

In addition, it is almost inevitably that the field SI return concentration profiles have undesired noise. This is due to error analysis, challenging and/or changing conditions. The field SI return concentration profiles are generating by taking regular samples from the water phase of the effluent of a producing well. These samples will be consequently analysed, which are sealed in plastic bottles. The plastic bottles are then sent to the laboratory for analysis using specialised equipment. It is not difficult to list a significant number of possible sources of error, from samples being compromised to errors in the lab analysis. Lab analysis involves a certain manipulation of the samples, which can lead to further inaccuracies. In addition, intrinsic error of specialised equipment and techniques utilised to detect the SI in the samples will add more to undesired noise. And finally, sample analysis is considerably sensitivity to the presence of
hydrocarbons in the water samples, high salinity and contamination by drilling fluids, etc, (Thompson et al., 2008).

Among all the techniques and methods that have been mentioned in the literature. Possibly the best suited is the Hill Climber algorithm, it is a very robust algorithm, capable of coping with complex misfit landscapes and noisy data. Looking carefully at the profile, it seems that there might be multiple objectives, namely the early peak and the tail. However, in this study the whole profile will be considered as a unique objective. Despite the fact that the early peak and tail might be conflicting objectives, the tail of the profile is the one that must prevail. Although, some of the Pareto approaches could be applied to the problem, compromising the tail match for an early peak match might lead to lower the quality of the solutions in terms of the tail mismatch.

The main purpose of finding a match is to produce accurate predictions of the squeeze lifetime for subsequent treatments; this is given when the return profile is below a certain concentration threshold, known as MIC, therefore it seems plausible to consider a unique objective.

The misfit is evaluated using the L1 norm Base 10, see equation (3.1). The formulation of the misfit function might weight more the tail of the profile than the early peak, since usually, approximately the first 30% of the profile describes the early peak and the rest the tail. This is desirable feature, as the most important part of the profile is the tail. There has been many examples in the literature where reasonable predictions were obtained by matching the tail and not early the peak, (Mackay and Jordan, 2003; Jordan, 2009, Selle et a., 2003). In these studies, the match was obtained by trial and error.

\[ Misfit = \sum_{i=0}^{N} \left| \log \left( \frac{S_i}{O_i} \right) \right| \] 

(3.1)

The algorithm considered for this study is a simple stochastic hill climbing algorithm, where the new potential candidate results from the addition of a
stochastic perturbation to the current solution, a high level description can be found in the next section.

3.1 Parameter setting

To fully parameterize the hill climbing algorithm, the stochastic perturbation needs to be defined. To generate a new solution a Gaussian distribution was selected, $N(\mu, \sigma^2)$, due to its flexibility. To fully define the Gaussian distribution, the mean and variance has to be selected. The selection of this parameter is more delicate since the magnitude of each parameter needs to be considered. The ranges for the parameters are as follows: $1 \leq k \leq 1000$ and $0.01 \leq n \leq 1$ and $0.01 \leq r \leq 200$, higher than the one suggested in the literature. Since this parameter is the most difficult to identify, the limits are expanded for the sake of completeness and to study its behaviour in the test problems, described in the following section.

To generate a good distribution of the solutions, aiming to create solutions in any direction from the original solution, the following means and variances were selected for each parameter, see Table 3.2. A description of the Hill Climbing algorithm can be found in Table 3.2

<table>
<thead>
<tr>
<th></th>
<th>$\mu$</th>
<th>$\sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>1.0</td>
<td>100.0</td>
</tr>
<tr>
<td>$n$</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>$r$</td>
<td>0</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 3.1 Gaussian distribution mean and variance, $N(\mu, \sigma^2)$, for each parameter.
Table 3.2 The simple stochastic hill climbing algorithm.

3.2 Test Problems

To assess the algorithm performance, synthetic cases were used assuming equilibrium and kinetic conditions. Noise was added to the synthetic cases to study how the algorithm performs with the addition of noise. It is almost inevitable that field SI return concentration profiles would have some added noise, due to experimental error, uncertainty about the sampling, etc. Different level of noise was added to the synthetic case, from conventional uniform noise to unconventional extreme noise, equation (3.2). An illustration of the synthetic cases with the added noise can be found in Figure 3.1 and 3.2, assuming equilibrium and kinetic conditions, respectively.

The reason for adding unconventional noise is that in certain field cases the profiles can show a very significant scatter, which can be mimicked by the extreme noise.

\[
S' = S \cdot \frac{U(-10,10) + 1}{U(-10,10)}
\]  

(3.2)
Where $S'$ is the solution plus noise and $U(-10,10)$ is uniform distribution.

Figure 3.1  Equilibrium Synthetic case with added noise, 2-Dimensional problem.

Figure 3.2  Kinetic Synthetic cases with added noise, 3-Dimensional problem.
3.3 Overfitting

In certain occasions statistical models may describe noise or random error rather than the relationship overlying, this phenomenon is commonly known as overfitting. To study if the proposed algorithm shows any signs of overfitting, the profile is divided in two parts, the first 2/3 of the profile is used for training and the remaining 1/3 for evaluation.

The algorithm was executed 100 times with a maximum of 300 iterations. For practical reasons, it was not possible to show the training and evaluation misfit profiles for every run. However, the difference between the lowest evaluation misfit error and the last is shown in Figure 3.3 for equilibrium (2D) and Figure 3.4 for kinetic (3D). They provide a good representation of any overfitting if present. The results seem to suggest that there are not significant signs of it, as the evaluation error in most of the runs did not increase.

Figure 3.3 Absolute difference between the Last and the Minimum evaluation error, the First, the Minimum and the Last evaluation error for Equilibrium, 2D problem.
Although some runs show signs of some overfitting, the evaluation misfit error was not particularly high, as it can be concluded studying the first, minimum and last evaluation error.

The majority of the training and evaluation misfit error appear to be like Figure 3.5 and 3.6, assuming equilibrium and kinetic adsorption respectively. Only some of the profiles from the 100 runs show some evaluation error increase. Although, it might suggest the presence of some overfitting, the increase is not very important.
Figure 3.5  Training misfit error is shown in blue, validation misfit error in red, both as a function of the number of training cycles for equilibrium.

Figure 3.6  Training misfit error is shown in blue, validation misfit error in red, both as a function of the number of training cycles for kinetic.
3.4 Comparison between Train/Eval and 100% Training

Another approach to study if overfitting could be an issue, which should be considered, is to compare the distribution of the best candidates assuming the 100% Training and Train/Eval approach.

For the 100% Train approach, the best candidate is the one with lowest misfit error assuming the whole profile. On the other hand, for the Train/Evaluation approach, the best candidate is the one with lowest evaluation misfit error.

Below, the results from 100 runs allowing a maximum of 300 iterations, assuming equilibrium and kinetic conditions, 2D and 3D test problems, respectively, will be shown.

3.4.1 Equilibrium Test Problem, analytical solution k = 100, n = 0.8

Under equilibrium conditions the problem is 2-Dimensional, the analytical solution is described by the pair (k,n) = (100,0.8), Figure 3.7 and 3.8 show the distribution of the best candidates for both approaches with no added noise and extreme noise, (the results for 5%, 10% and 20% uniform added noise can be found in Appendix A). Although the response is similar for both approaches, it seems that the 100% Train approach gets the right answer slightly more times than Train/Eval, which in the case of extreme noise added is even slightly more significant, as shown in Figure 3.8.
Figure 3.7  Distribution of best candidates for Equilibrium (k,n) with No Noise.

Figure 3.8  Distribution of best candidates for Equilibrium (k,n) with Extreme Noise.
3.4.2 Kinetic Test Problem, analytical solution $k = 100$, $n = 0.3$, $r = 20$

The analytical solution for this test problem is given by the trio $(k,n,r) = (100,0.3,20)$. Figure 3.9 and 3.10 show the distribution of the best candidates for no added noise and extreme noise for both approaches, (the results for 5%, 10% and 20% uniform added noise can be found in Appendix A). For kinetic conditions, the distribution of the best solutions is much more spread than for equilibrium conditions, in particular parameters $k$ and $r$. It is difficult to decide which approach shows a better performance, as both distributions are very similar. Assuming kinetic conditions none of the approaches seem to find the right solution any significant number of times, apart from parameter $n$. A more detail study about this is presented in the next section.

Figure 3.9  Distribution of best candidates for Kinetic $(k,n,r)$ with No Noise.
Figure 3.10  Distribution of best candidates for Kinetic (k,n,r) with Extreme Noise.

3.5 Analysis of the results of the Kinetic Test Problem

Following the results in the section before, the Kinetic test problem is studied in more detail. As mentioned before, under kinetic conditions the problem becomes three dimensional, where a pseudo isotherm is described by the trio (k,n,r). Figure 3.11 and 3.12 show the scatter density plots as function of the misfit error with no added noise and extreme added noise, respectively, for the 100% Train approach. Similar results were obtained assuming 5%, 10% and 20% uniform added noise, see Appendix B.

To calculate the average distance to the analytical solution, a normalised L1 norm is used, see equation (3.3), where $P^{\text{Analytical}}$ is the parameter of the
analytical solution, \( P_{\text{Solution}} \) is the value of the parameter of the suggested solution, and finally \( n \) is the number of solutions for a given set of solutions.

\[
d = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{P_i^{\text{Solution}} - P_i^{\text{Analytical}}}{P_i^{\text{Analytical}}} \right|
\]  

(3.3)

Analysing the results, it is not surprising that the lowest mismatch correspond to the values close to the analytical solution, \((k,n,r) = (100,0.3,20)\). However, there is a significant cluster of solutions with low misfit error, depicted in red, with significant scatter. There does not seem to be any clear correlation between the parameters, where the lowest average distance to the analytical solution corresponds to the parameter “n” and “r”, as shown in Figure 3.11 and 3.12. The average distance of the parameter “k” is significantly higher than the other two.

Figure 3.11  Scatter density plots of best candidates for Kinetic \((k,n,r)\) with No Noise, 100% Train.
Figure 3.12 Scatter density plots of best candidates for Kinetic (k,n,r) with Extreme Noise, 100% Train.

Figure 3.13 shows the correlations of the first two sets of best solutions for the cases of no added and extreme noise. For the lowest misfit value set no matter the noise added, there is a quite clear linear correlation between the parameter “k” and “r”, where the effect of parameter “k” is counter balance by parameter “r”. However, for the next set of low misfit values, there does not seem to such a clear correlation.
Figure 3.13  Scatter plots of parameter “k” vs “n” and “r” for the first two sets of best solutions with No added and Extreme Noise, 100% Train.

Similar results were obtained for the Train/Eval approach with no added noise and extreme noise, Figure 3.14 and 3.15. Again the parameter “k” showed the highest distance to the analytical solution. The results for 5%, 10% and 20% added uniform noise can be found in Appendix B. The results obtained after analysing any correlation of the best two sets of solutions, are comparable to the previous results, see Figure 3.16. However, for this approach the values of the parameter “n” seem to be more scattered than for the 100% Train approach.
Figure 3.14  Scatter density plots of best candidates for Kinetic (k,n,r) with No Noise, Train/Eval.

Figure 3.15  Scatter density plots of best candidates for Kinetic (k,n,r) with Extreme Noise, Train/Eval.
The best solutions of the 100% Train approach seems to have slightly more scatter than the Train/Eval approach, as shown comparing average distance to the analytical solution of the lowest misfit error values. However, the closest solution to the analytical solution was found after 100 runs by the Train/Eval approach. Despite this fact, the 100% Train approach found more times solutions very close to the analytical solution.

The reason for the scatter in low misfit values can be explained looking at Figure 3.17, which shows a contour map of the Kinetic Test Problem of low misfit error values in the plane (k,n) as function of the third parameter “r”. It demonstrates the existence of local minima, where Hill Climber algorithm might get trapped.
Figure 3.17  Contour maps for the 3D cases with the logarithmic misfit function, misfit values in the white regions are higher than 20 and out of scale.
3.6 Are the suggested and the analytical solution equivalent?

In this section, the question posed above will be addressed. Some of the solutions under kinetic conditions suggested by the algorithm are significantly different from the analytical solution, see Table 3.3. To determine if the solution is equivalent to the analytical solution, a number of calculations were performed, where a number of treatment constraints were modified. The objective is to compare the return profiles, and so assess if the predictions are equivalent. The treatment parameters, which were varied, were scale inhibitor concentration, the flow rate and volume of the main slug and overflush.

Figure 3.18 shows the results after varying the treatment parameters listed before, using the solution suggested by the 100% training approach. The results show that for the original treatment design there is no significant difference, as it is expected since the misfit error value was very low. Therefore, it is not surprising that both profiles are very close. The results showed that if one treatment parameter is changed, the predictions, although different, were relatively close. The more parameters were modified the further away the predictions were. As shown in the last calculation, where the flow rate, SI concentration and overflush volume was varied. Similar results were obtained where the Train/Eval approach was adopted, see Figure 3.19.

The results suggest that if the treatment characteristics do not change dramatically, the predictions, although different, they provide very similar responses. Therefore, in those circumstances, the solutions are equivalent. On the other hand, if the treatment is significantly changed; the solutions might not be equivalent. However, in a real application, these predictions should not be discarded, as they would possibly provide the best predictions given the information available.
### Table 3.3 Parameters describing analytical solution and best solutions for 100% Train and Train/Eval approach.

<table>
<thead>
<tr>
<th></th>
<th>Analytical Solution</th>
<th>Best Solution 100% Train</th>
<th>Best Solution Train/Eval</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>100.0</td>
<td>269.75</td>
<td>215.754</td>
</tr>
<tr>
<td>n</td>
<td>0.3</td>
<td>0.225</td>
<td>0.257</td>
</tr>
<tr>
<td>r</td>
<td>20.0</td>
<td>161.18</td>
<td>198.33</td>
</tr>
</tbody>
</table>

Figure 3.18 Comparison of SI return concentration profiles obtained using the Target and the HC Kinetic pseudo-isotherm candidate for different treatment strategies 100% Train.
3.7 Conclusions

After analysing all the results, the main conclusion that may be drawn is that the algorithm does not show significant signs of overfitting, and if so, it is very mild. Therefore, overfitting is not a major concern in this problem, and it will not be considered.

Overall, comparing both approaches with the test problems with a budget of 300 iterations, the 100% Train seemed to perform better than the Train/Eval approach. Although the distribution of the best candidates was very similar, the

Figure 3.19  Comparison of SI return concentration profiles obtained using the Target and the HC Kinetic pseudo-isotherm candidate for different treatment strategies Train/Eval.
candidates with lowest misfit error show slightly more scatter for the Train/Eval
than for 100% Train approach. And although, after 100 runs the Train/Eval
obtained the closest solution to the analytical, the 100% Train found more
solutions closer to the analytical solution.

After analysing the parameter distribution of the best solutions as function of
the misfit error values, it appears that the parameters “n” and “r” have the lowest
scatter with values very close to the analytical solution. It appears that these two
parameters might be correlated, and possibly the cause of the existence of local
minima. This is not completely surprising, since the parameter “k” controls the
level of maximum adsorption and the parameter “r” how fast the adsorption
reaction occurs. For instance, a high value of parameter “k” could be counter
balanced by a low value of parameter “r”. Although, these two parameters
seemed to be correlated, it is not clear, as there are other parameters that
influence the simulation results such as residence time, chemical concentration,
etc. As shown before, the correlation was only clear for very low misfit values.

Finally, it is likely that more than one good solution exists, i.e. a solution that
shows a low misfit error value. A series of simulations based on two good
solutions were shown, where a number of treatment parameters were varied.
The results showed that the predictions were slightly different, when the
treatment was not varied significantly, resulting in very close predictions,
concluding that they were equivalent. However, the solutions might not be
equivalent if the treatment characteristics change very significantly. Despite this
fact, these predictions in a field application are very valuable, as they are
probably the best predictions given the data.

In the next section the algorithm is applied to a series of field cases, where
the same treatment was injected several consecutive times. This data is of great
value as the pseudo-isotherm solution suggested by the algorithm from the first
treatment return profile can be used to assess how good the predictions are on
the subsequent treatments. And somehow, validate the solution.
4 Field Cases analysis

The algorithm was applied to two field applications, where the well was treated with the same scale inhibitor several consecutive times. The procedure was as follows: first derive a pseudo-adsorption isotherm from the first treatment and then compare the predictions with the subsequent treatments. The objective of these calculations is to somehow validate the suggested pseudo-isotherm, assessing how good the predictions of the other treatments are.

Based on results shown before, the 100% Train approach was adopted, as overfitting does not seem to be a significant issue and its performance was slightly better than Train/Eval approach. The parameter space was slightly modified, making the parameter “r” range slightly smaller. The limits of the parameter space selected are as follows $1 \leq k \leq 3000$, $0.01 \leq n \leq 1$ and $0.001 \leq r \leq 4$. Reducing the limits of the parameter “r” might reduce the number of local minima, since parameters k and r seem to be correlated. The parameter “k” controls the maximum level of adsorption and the parameter “r” controls the speed of the adsorption reaction, therefore it is not surprising that they could be correlated. Finally, the simulations are single phase and under kinetic conditions, which makes it a 3D problem.

Two different cases were under study; referred as Well A and Well B. The first well was treated with the same chemical package for four consecutive times, whereas the second well, Well B, was treated three consecutive times. The procedure is the same for both wells; derive an isotherm candidate from the first treatment, allowing a budget of 80 iterations assuming kinetic conditions. Then compare the predictions using the suggested solution with the subsequent treatments.

For Well A, the algorithm suggested the pseudo-adsorption isotherm is described by the trio $(k,n,r) = (500.5, 0.18, 0.78)$ for treatment 1, see Figure 4.1. The predictions for treatments 2, 3 and 4 using the suggested pseudo-isotherm, are shown in Figure 4.2, 4.3 and 4.4. The match for Treatment 1 is very good. Looking at the predictions for the subsequent treatments, they seemed reasonably good, apart for Treatment 4, which follows the trend, but it is
significantly low compared to the field profile. Generally, and considering the fact that the algorithm was allowed a budget of only 80 iterations, the results are good, the predictions although a bit low, they are reasonable.

One possible explanation for the worsening of the matching, in particular Treatment 4, could be due to changes in the conditions near the wellbore, such as temperature and salinity changes.

Figure 4.1  SI return concentration Well A, Treatment 1. Best solution trio \((k,n,r) = (500.5, 0.18, 0.78)\) assuming single phase flow.
Figure 4.2  SI return concentration Well A, Treatment 2. Best solution trio \((k,n,r) = (500.5, 0.18, 0.78)\) assuming single phase flow.

Figure 4.3  SI return concentration Well A, Treatment 3. Best solution trio \((k,n,r) = (500.5, 0.18, 0.78)\) assuming single phase flow.
Figure 4.4 SI return concentration Well A, Treatment 4. Best solution trio \((k,n,r) = (500.5, 0.18, 0.78)\) assuming single phase flow.

The procedure was applied to Well B, which was treated 3 consecutive times. The suggested pseudo-isotherm from the first treatment is described by trio \((k,n,r) = (729,0.63,1.93)\), see Figure 4.5. The predictions for treatment 2 and 3 are shown in Figure 4.6 and 4.7. The results for Well B are similar to Well A. Although, the match for the first treatment was very good, the matches for the subsequent treatments worsened slightly. As for the previous field case, one possible explanation is the changing conditions in the near wellbore area.
Figure 4.5  SI return concentration Well B, Treatment 1. Best solution trio \((k, n, r) = (729, 0.63, 1.93)\) assuming single phase flow.

Figure 4.6  SI return concentration Well B, Treatment 2. Best solution trio \((k, n, r) = (729, 0.63, 1.93)\) assuming single phase flow.
Figure 4.7  SI return concentration Well B, Treatment 3. Best solution trio (k,n,r) = (729,0.63,1.93) assuming single phase flow.
5 Final Conclusions

The applicability of the Hill Climber algorithm for the pseudo-adsorption isotherm matching for oilfield scale inhibitor squeeze treatments has been demonstrated. In many stochastic algorithms overfitting is a common feature, however for this particular problem there does not seem to be any signs of it, and if so it is very mild. Therefore overfitting was not considered to be a major concern.

The test problems demonstrated the existence of local minima in the 3D problem. Based on the analysis of the results, this may be due to the large range of the parameter “r”. This parameter controls the speed of the adsorption reaction and parameter “k” sets the maximum level of adsorption, therefore it is not surprising that they could be correlated.

It was only possible to find a clear correlation for the set of low misfit values. For the set of higher misfit values, no clear correlation was identified, this might be explained as other parameters, (mainly residence time and SI concentration), in combination with the pseudo-isotherm influence the simulation outcome. Other interesting result demonstrated that the average distance to the analytical solution of parameter “k” was significantly higher than the other two parameters.

The question if good proposed solutions, possibly local minima, were equivalent to an analytical solution was addressed. The predictions, where some squeeze treatment parameters were varied, were slightly higher using the proposed solutions. Although, slightly different, they were possibly within the prediction tolerance error. For small variations they would be considered equivalent, when more changes were applied the predictions were a bit further apart. And although, they would not possibly be equivalent, they would provide valuable information in a field application.

The algorithm was applied to two real field cases for validation. The cases under study were wells that were treated a number of consecutive times. This is of extremely value, since it is possible to evaluate the prediction of the suggested pseudo-isotherm from the first treatment. The results were good for the first treatment and the predictions were reasonable, bearing in mind that a
budget of 80 simulations was allowed. Some of the predictions, in particular the ones for treatments third and forth would have difficulties passing quality control assessments, possibly due to changing conditions in the wellbore area from the time of the first treatment.

To sum up, it has been demonstrated that there might be several local minima. Predictions based on these local minima may change when the treatment parameters are varied, as a consequence the predictions might be compromised. The algorithm proposed can be used to find some of these local minima. It has been shown that at least for one or two subsequent treatments in real field cases, the predictions were good. Providing very reasonable matches, which in most scenarios are believed they will pass a quality assessment.
6 Future recommendations

This study focused on applying stochastic optimization techniques to match the scale inhibitor return concentration profile. However, other techniques such as machine learning approaches, which use an algorithm to learn and then produced a predictive model, could provide interesting results. Due to the time limitation and scope of this project, none of these techniques were implemented.

A further recommendation would be to try some of the population based approximate search algorithms like Particle Swarm Optimization, which might overcome the local minima trapping.

Finally, although the problem was not considered to have multiple objectives, the presence of two distinctive parts in the profile is clear, peak and tail. Therefore applying evolutionary optimizers with Pareto-based ranking schemes might give interesting results.
7 References

Appendix A  Extra Comparison between Train/Eval and 100% Training

A.1 Equilibrium Test Problem, analytical solution $k = 100, n = 0.8$

Figure A.1 Distribution of best candidates for Equilibrium $(k,n)$ with 5% Noise.

Figure A.2 Distribution of best candidates for Equilibrium $(k,n)$ with 10% Noise.
Figure A.3 Distribution of best candidates for Equilibrium \((k,n)\) with 20% Noise.

**A.2 Kinetic Test Problem, analytical solution \(k=100, n=0.3, r=20\)**

Figure A.4 Distribution of best candidates for Kinetic \((k,n,r)\) with 5% Noise.
Figure A.5 Distribution of best candidates for Kinetic (k,n,r) with 10% Noise.
Figure A.6 Distribution of best candidates for Kinetic \((k,n,r)\) with 20% Noise.
Appendix B  Scatter Density plots for 100% Training and Train/Eval

Kinetic Test Problem, analytical solution $k=100$, $n=0.3$, $r=20$

Figure B.1 Scatter density plots of best candidates for Kinetic $(k,n,r)$ with with 5% Noise 100% Train.
Figure B.2 Scatter density plots of best candidates for Kinetic \((k,n,r)\) with with 10\% Noise 100\% Train.

Figure B.3 Distribution of best candidates for Kinetic \((k,n,r)\) with 20\% Noise 100\% Train.
Figure B.4 Scatter density plots of best candidates for Kinetic (k,n,r) with % Noise Train/Eval.

Figure B.5 Scatter density plots of best candidates for Kinetic (k,n,r) with % Noise Train/Eval.
Figure B.6 Scatter density plots of best candidates for Kinetic \((k,n,r)\) with 20\% Noise Train/Eval.