

# *Automatic Guidance for Refinement based Formal Methods*

Maria Teresa Llano<sup>1</sup>  
Andrew Ireland<sup>1</sup>   Gudmund Grov<sup>2</sup>

<sup>1</sup>School of Mathematical and Computer Sciences  
Heriot-Watt University

<sup>2</sup>School of Informatics  
University of Edinburgh

AFM'10, July, 2010

- The rigour of formal methods brings a lot of benefits to the development of systems.
- However, it is still not widely used outside specific domains.
- A major problem is the need for expertise in:
  - Formal modelling.
  - Theorem proving.
  - Understanding the close relationship between proof and modelling.

**Goal: To abstract away from the complexities of proof obligations, providing high-level modelling guidance.**

Here, with a focus on refinement.

Proof planning uses proof patterns to automate the search for proofs.

- Automatic proof failure analysis and patching.
- Reusability of proof strategies.

An approach that provides high-level modelling guidance by combining proof and modelling patterns.

- Currently the ideas of reasoned modelling are being developed in Event-B.
- We are working in *Refinement Plans*, a type of reasoned modelling method that focus on refinement.

- A formalism that supports modelling and reasoning of discrete event systems.
- Uses a *posit-and-prove* style of modelling.
- Promotes the evolution of models through *refinement*.
- Uses mathematical proof to verify the consistency between refinement levels.

# Event-B Example: A logging system

Records whether a resource is allocated or unallocated.

Context
Sets <i>RESOURCES</i>
Machine
<b>Variables:</b> <i>resources, allocated, unallocated</i>
<b>Invariants:</b> $resources \in \mathbb{P}(RESOURCES)$ $allocated \subseteq resources$ $unallocated \subseteq resources$ $allocated \cap unallocated = \emptyset$ $allocated \cup unallocated = resources$
<b>Events ...</b>
<b>Event</b> <i>unallocate</i> $\hat{=}$
<b>any</b> <i>r</i>
<b>where</b> $r \in allocated$
<b>then</b> $allocated = allocated \setminus \{r\}$ $unallocated = unallocated \cup \{r\}$
<b>end</b>

Handle the complexity of large systems through the use of abstraction.

Refinement in Event-B:

- Existing events can be split, merged or modified.
- New events can be added.
- Variables can be added and/or removed.
- Gluing invariants: relate the state of the abstract model with the state of the concrete model.

All refinement steps are verified through proof obligations.

- Classify common patterns of refinement at the level of models.
- Combine modelling and proof knowledge.
- Detect partial matches of known patterns of refinement in a development.
- Provide user guidance in terms of modelling decisions.



# *Roles of refinement plans*

- **Correcting refinements:** modifications to flawed refinements.
- **Layering refinements:** introduction of intermediate layers of abstraction in complex refinements.
- **Abstracting refinements:** reduction of the initial complexity of a development.
- **Suggesting refinements:** suggesting alternative refinement steps.
- **Increasing proof automation:** associated proof patterns.

**refinement plan = refinement method  
+ proof methods  
+ critics**

- Describes a common pattern of refinement (abstract model, concrete model and gluing invariant).
- Represented with declarative preconditions.
- These preconditions identify partial or complete instances of the pattern within a user's refinement.

# Refinement method example: *sets-to-function*

An instance of this pattern requires the:

- abstract model to contain a partition of sets (many variables),
- concrete model to replace the partition with a function (one variable)

Abstract model	Concrete model
$state1 \subseteq Elements$ $state2 \subseteq Elements$ $state1 \cap state2 = \emptyset$ $state1 \cup state2 = Elements$	$Status = \{STATE1, STATE2\}$ $fStatus \in Elements \rightarrow Status$ $state1 = fStatus^{-1}[\{STATE1\}]$ <u><math>state2 = fStatus^{-1}[\{STATE2\}]</math></u>

- Reasoning patterns associated to a refinement method.
- Proof methods are used when a user's refinement fully matches with a pattern but it has a set of unproven POs.
- The use of proof methods and the analysis of partial success at the level of proof planning represents future work.

---

<sup>1</sup>Taken from Computational Logic [Bundy, 1991]

- Exception handling mechanism.
- If a partial instance of a refinement method is found, critics are applied.
- Represented with declarative preconditions.
- All preconditions must succeed for a critic to be applicable.
- Modelling guidance to overcome the failure is automatic generated (e.g. change guard/action/(gluing) invariants).
- The decision of applying/choosing guidance is left to the user.

## Example: Logging system applied to the sets-to-function refinement method

Precondition: There must exist a set of gluing invariants with the pattern:  
**stateVariable** = **concreteFunction**<sup>-1</sup>[**{stateConstant}**]

Abstract model	Concrete model
$allocated \subseteq Resources$ $unallocated \subseteq Resources$ $unallocated \cap unallocated = \emptyset$ $unallocated \cup unallocated = Resources$	$Status = \{ALLOCATED, UNALLOCATED\}$ $rStatus \in Resources \rightarrow Status$  <i>Missing gluing invariants</i>

There exists a partial instance of the refinement method!

- 1 There exists a failed guard strengthening PO in the concrete model with the form:

$$\begin{aligned} \exists \text{failed\_po} \in \{ \langle -, -, -, PO \rangle \in \text{POs} \mid \text{failed\_proof}(PO) \}. \\ \text{failed\_po} = \langle M, E, - / \text{GRD}, \underbrace{(\Delta, \text{stateFunction}(x) = Y)}_{\text{concrete guard}} \underbrace{\vdash x \in \{z\}}_{\text{abstract guard}} \rangle \end{aligned}$$

- 2 That by adding the gluing invariant pattern to the set of hypotheses the failed PO is provable:

$$\text{provable } (\Delta, \text{stateFunction}(x) = Y, \underline{\{z\} = \text{stateFunction}^{-1}[\{Y\}]} \vdash x \in \{z\})$$



# Instantiation of the critic: logging example

- ❶ Failed POs with the form “ $\Delta, stateFunction(x) = Y \vdash x \in \{z\}$ ”: ✓

...,  $rStatus(r) = ALLOCATED \vdash r \in \{allocated\}$

...,  $rStatus(r) = UNALLOCATED \vdash r \in \{unallocated\}$

- ❷ The addition of the gluing invariant discharges the POs: ✓

$provable (... , rStatus(r) = ALLOCATED,$   
 $\underline{\{allocated\} = rStatus^{-1}[\{ALLOCATED\}]} \vdash r \in \{allocated\})$

(A similar instantiation is given for state unallocated)

All preconditions succeed, then the guidance is the addition of the gluing invariants in the concrete model.

# *The REMO<sup>2</sup> tool*

---

<sup>2</sup>The REMO acronym follows from REasoned MOdelling

- Development of more refinement plans and their evaluation through case studies.
- Explore the role of refinement plans for:
  - Guiding users in their initial choice of refinement.
  - Suggesting intermediate refinement steps.
- Tool implementation: *REMO*.
- Development of the proof planning mechanism to exploit proof methods.

- Refinement plans aim at providing modelling guidance by automatically analysing specifications that lie just outside a known pattern of refinement.
- While the analysis of failure and generation of guidance is automatic, the decision as to whether or not to take the guidance on offer will be left to the user.
- We believe that this approach will enable us to turn low-level proof-failures into high-level modelling guidance.

# Refinement plan and critic schemas

PLAN(*Name*)

INPUTS:

PO\_SET {*POs*}

MODELS {*AM*, *CM*}

REFINEMENT METHOD:

1. *Precondition*

....

*I. Precondition*

PROOF METHODS:

1. *Proof\_Method*

....

*J. Proof\_Method*

CRITICS:

1. *Critic\_Name*

....

*K. Critic\_Name*

CRITIC(*Name*)

INPUTS:

PO\_SET {*POs*}

MODELS {*AM*, *CM*}

R\_INSTANCES {*Instances*}

P\_INSTANCES {*Instances*}

PRECONDITIONS:

1. *Precondition*

....

*L. Precondition*

OUTPUTS:

PATCH *patch description*

GUIDE *guidance description*