Open Games: Compositional Game Theory

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Game theory

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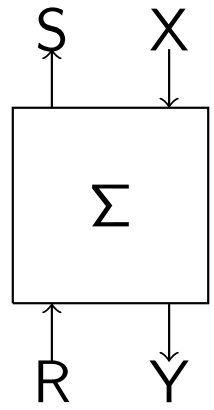
- Does not scale well
- No compositionality
- Computationally hard

What is an open game?



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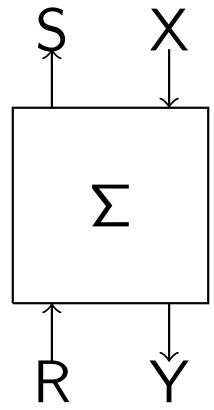

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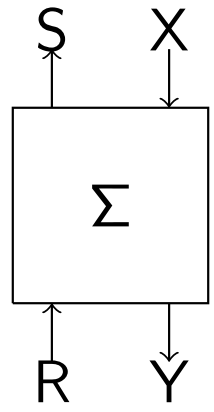

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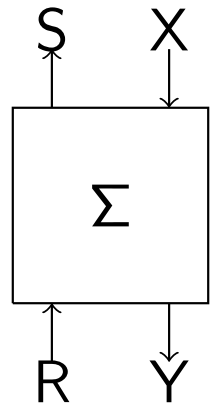

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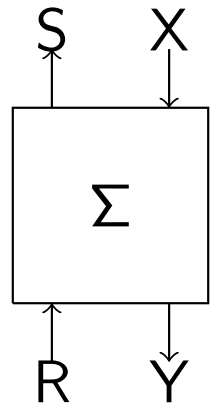

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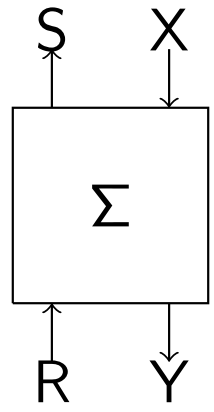

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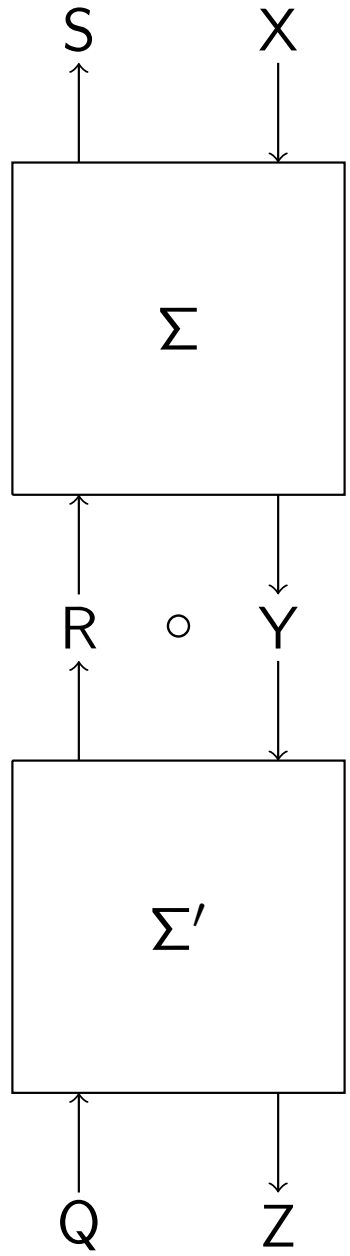
- an *equilibrium function* $E_{\mathscr{G}} : X \times (Y \to R) \to \mathscr{P}(\Sigma_{\mathscr{G}})$.

Sequential composition



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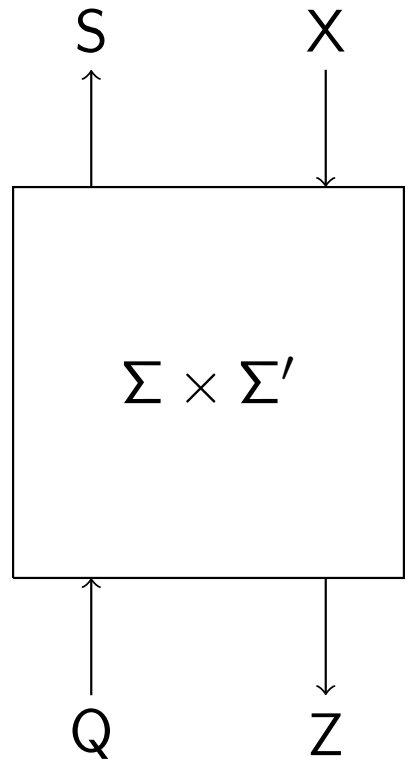

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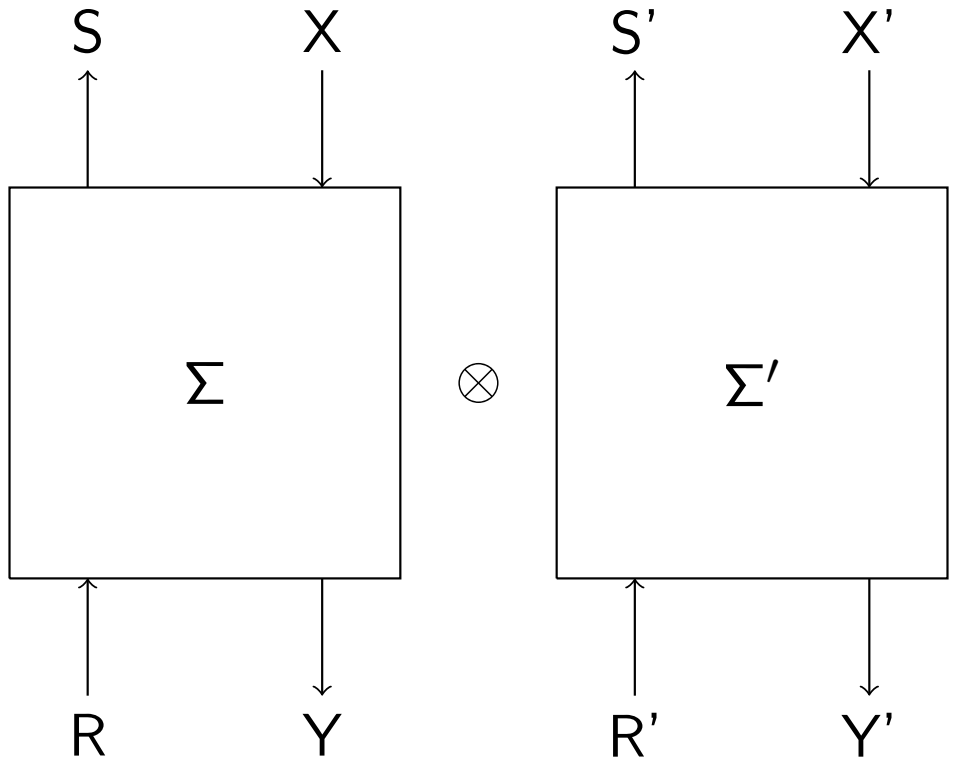

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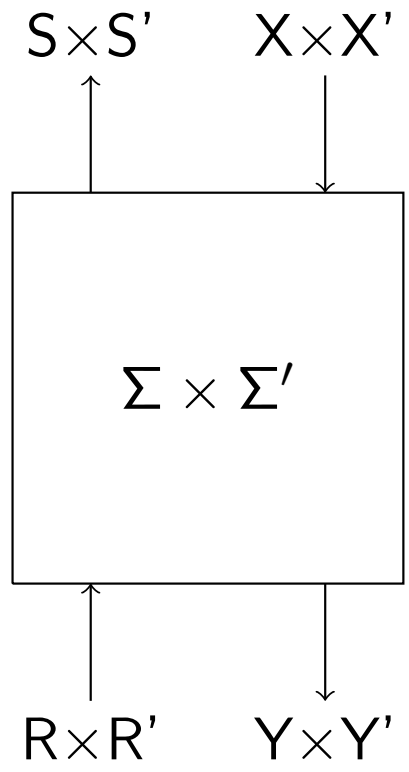

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Theorem

The collection of pairs of sets, with open games $\mathscr{G} : (X, S) \to (Y, R)$*, as their morphisms forms a symmetric monoidal category.*

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- Monads and distributive laws for probabilistic strategies

Thanks!