git clone https://github.com/data61/PSL

Automating Induction for Isabelle/HOL with DSLs

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Yutaka Ng
yutakang

Block or report user

CVUT, CTU, CIIRC

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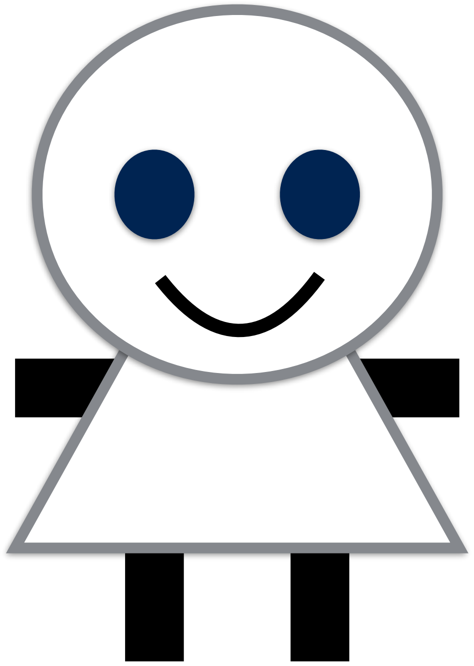
Interactive theorem proving with Isabelle/HOL



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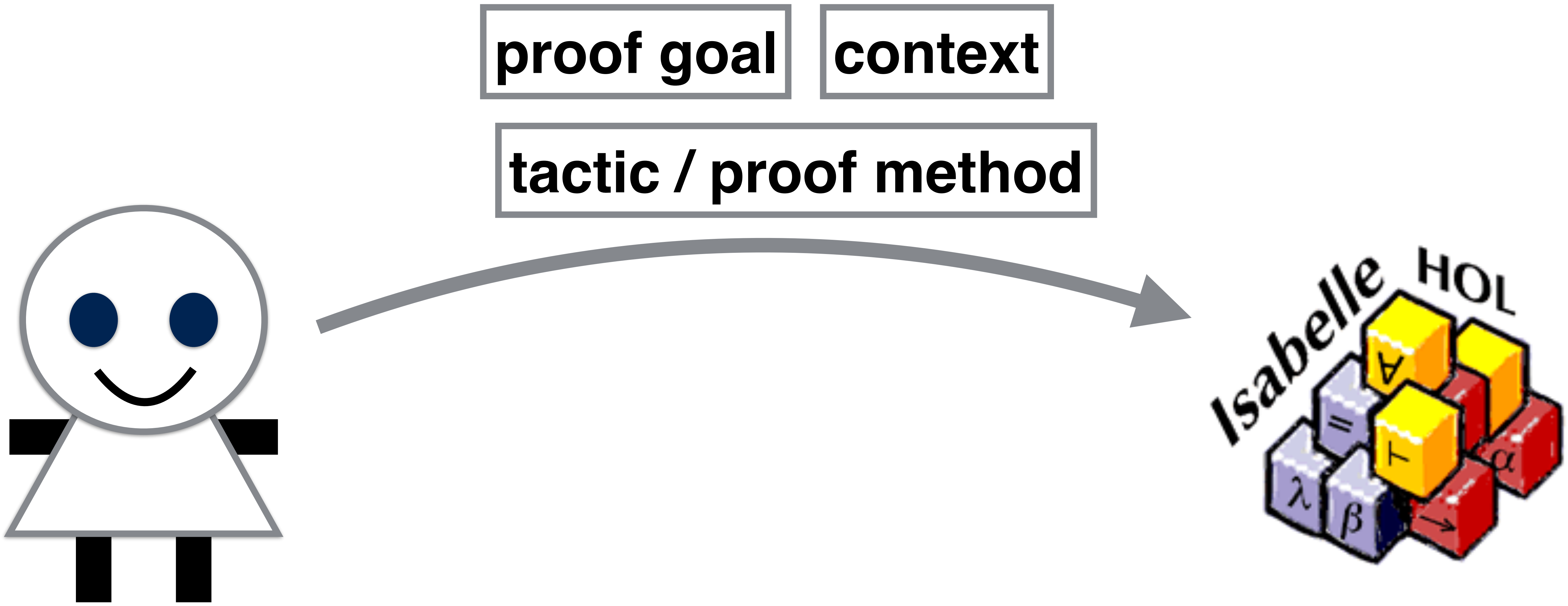
Interactive theorem proving with Isabelle/HOL



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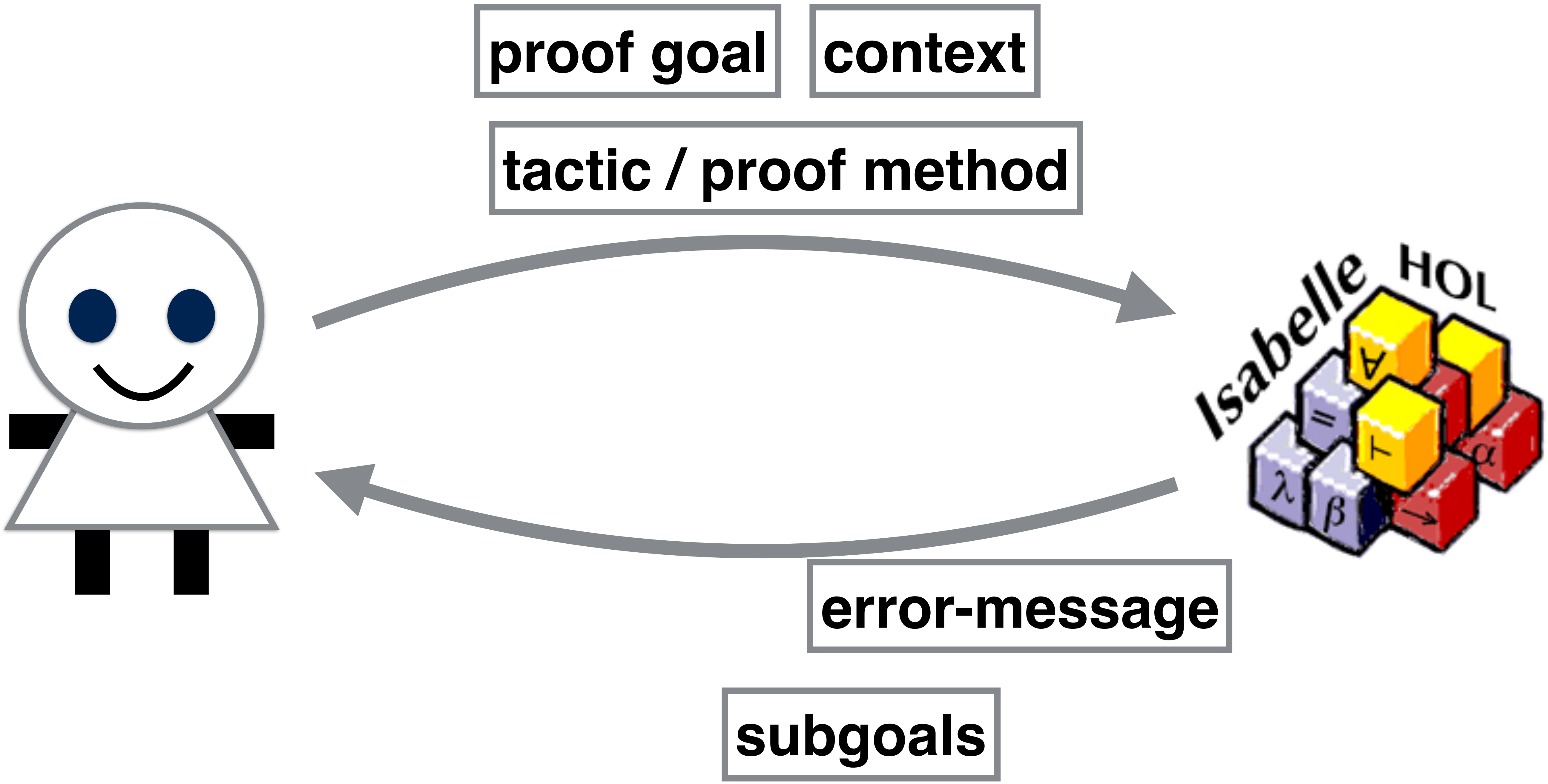

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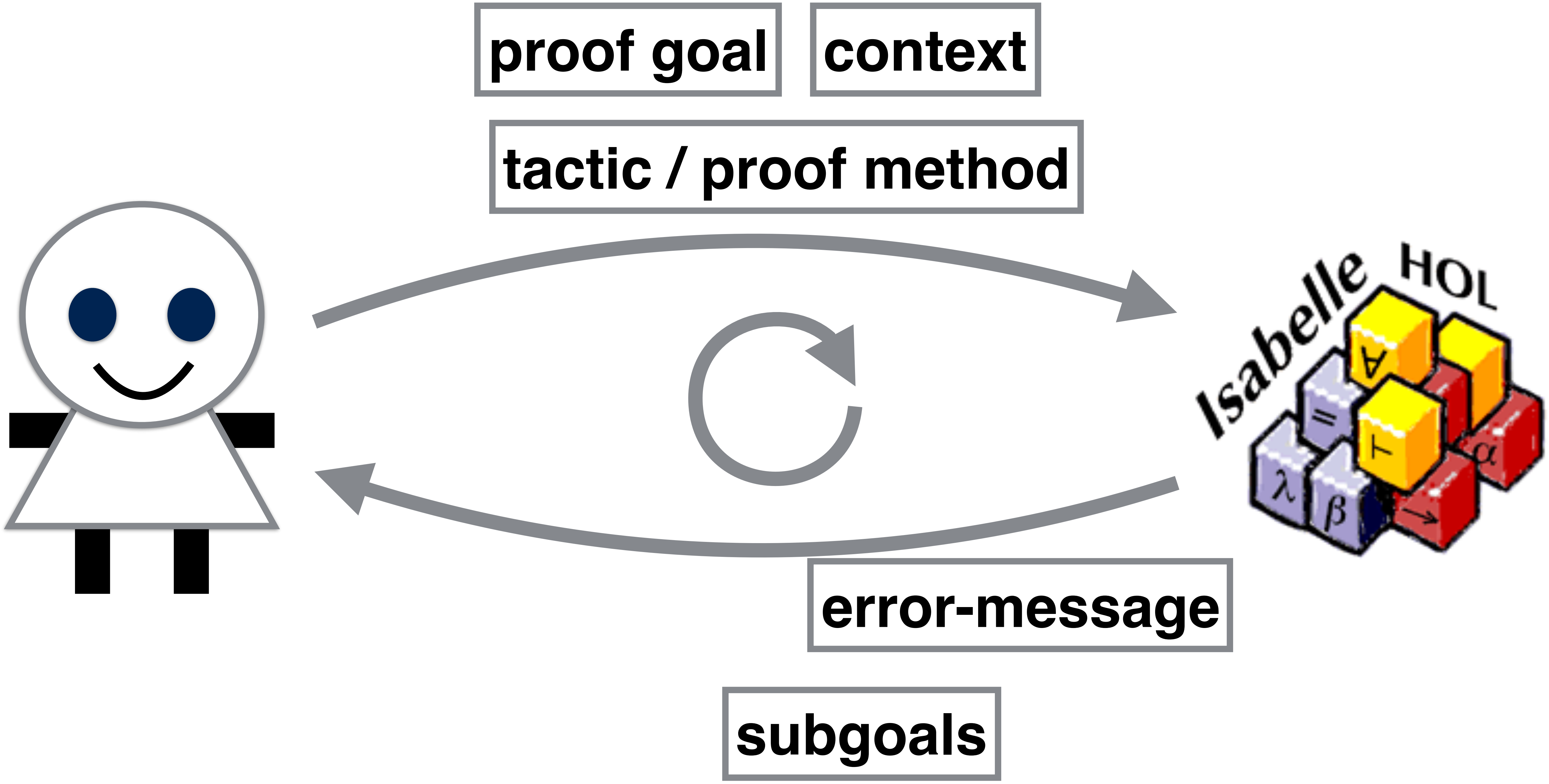
Interactive theorem proving with Isabelle/HOL



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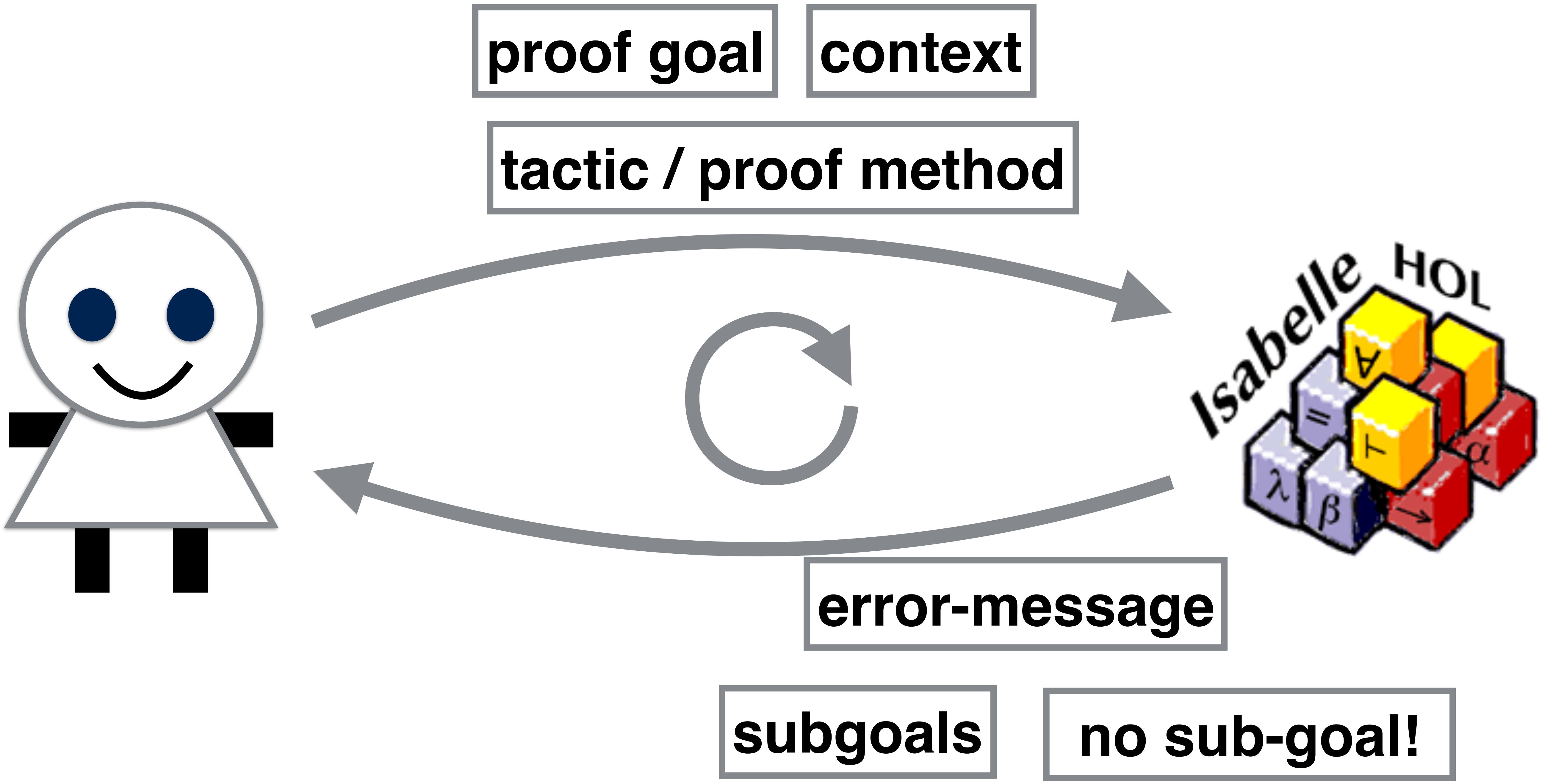
Interactive theorem proving with Isabelle/HOL



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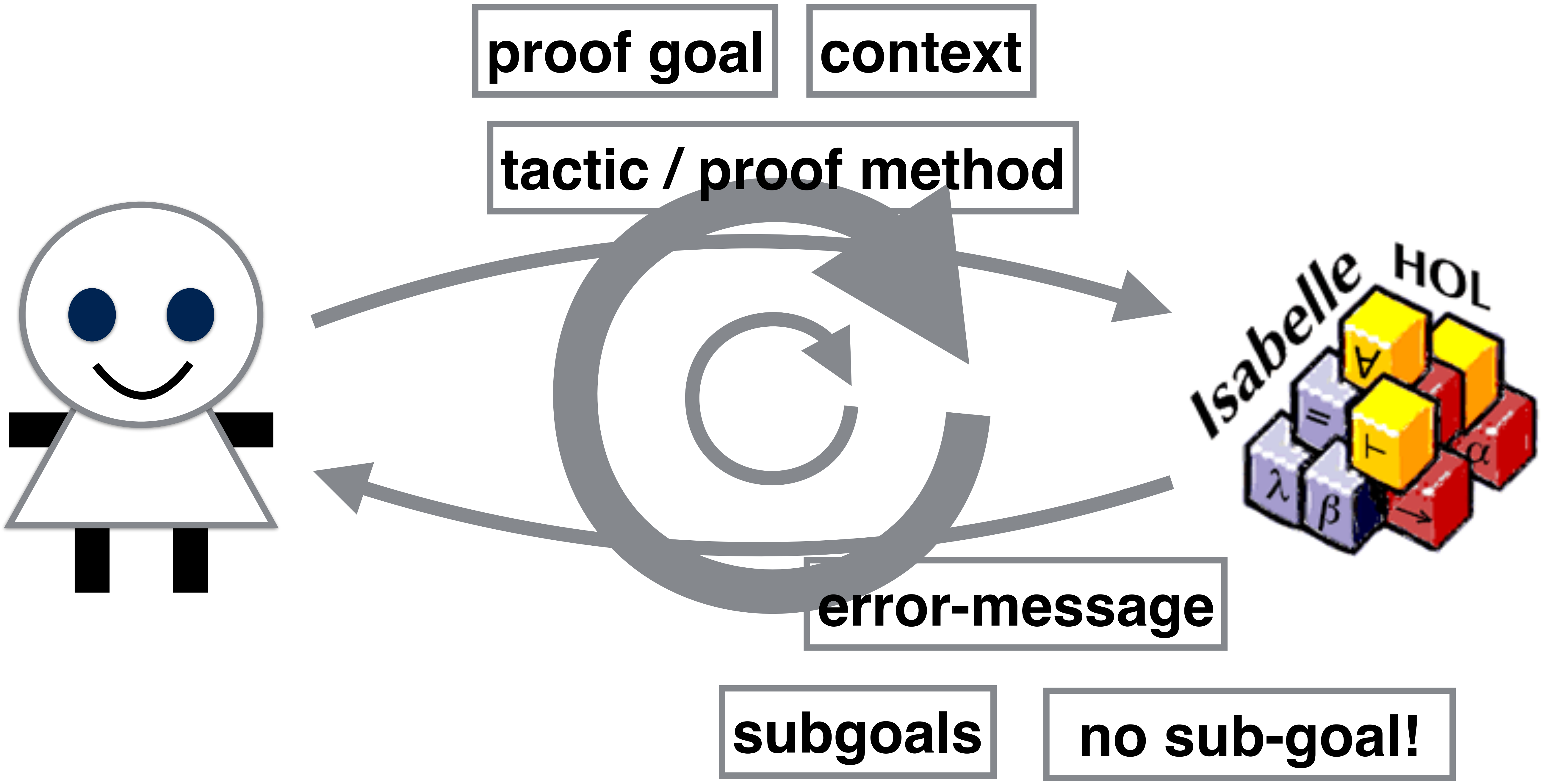
Interactive theorem proving with Isabelle/HOL



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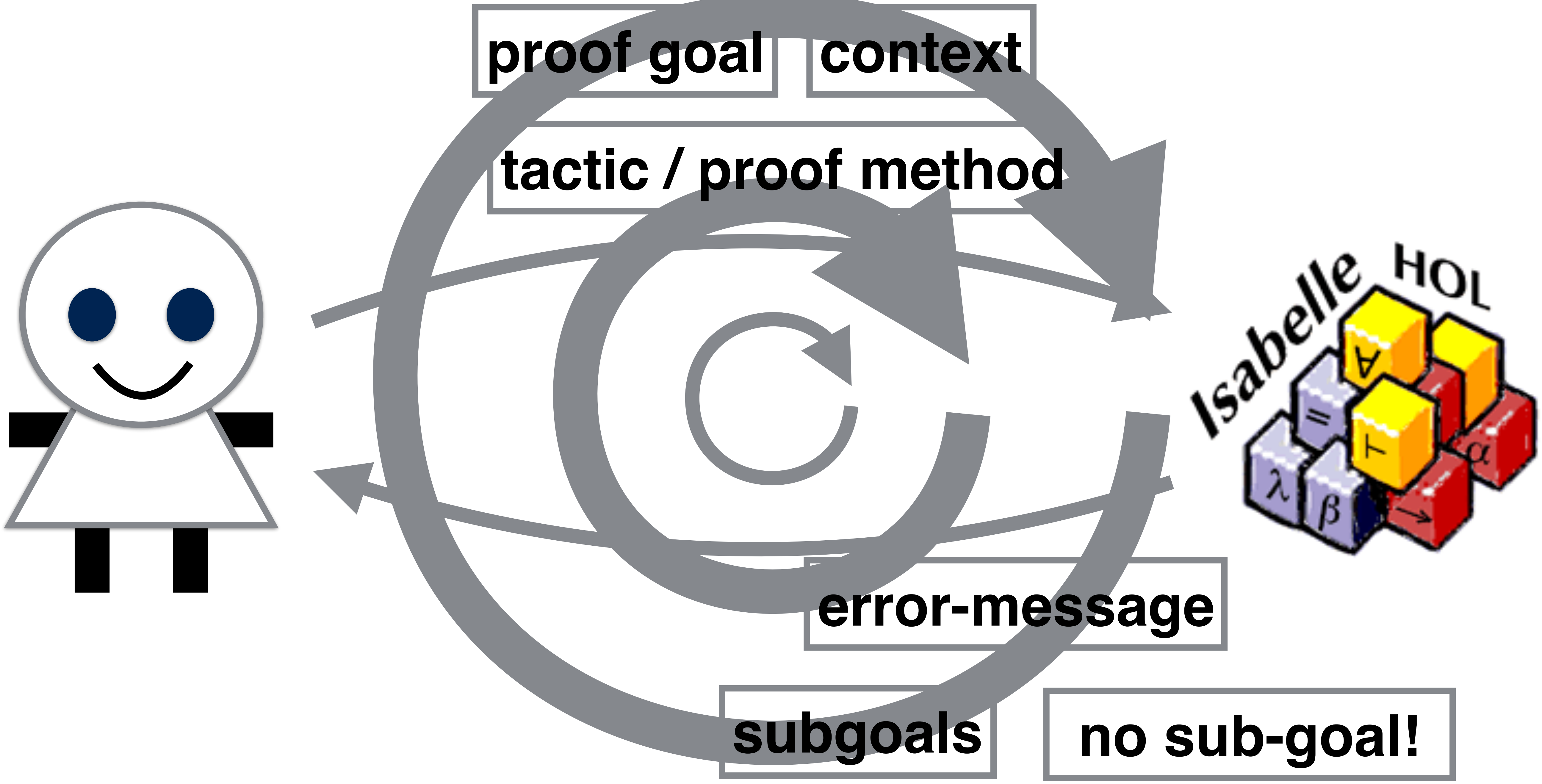
Interactive theorem proving with Isabelle/HOL



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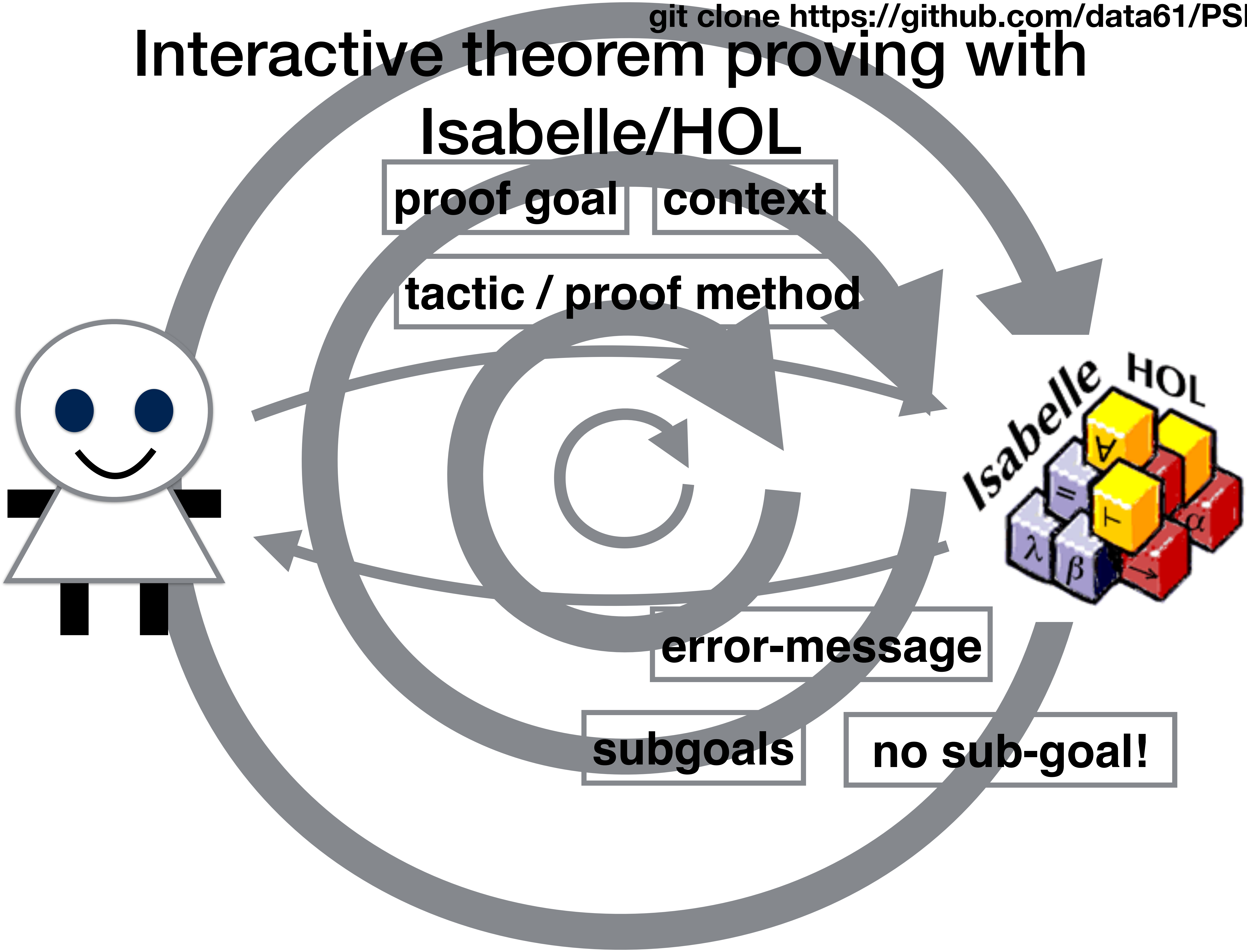
Interactive theorem proving with Isabelle/HOL



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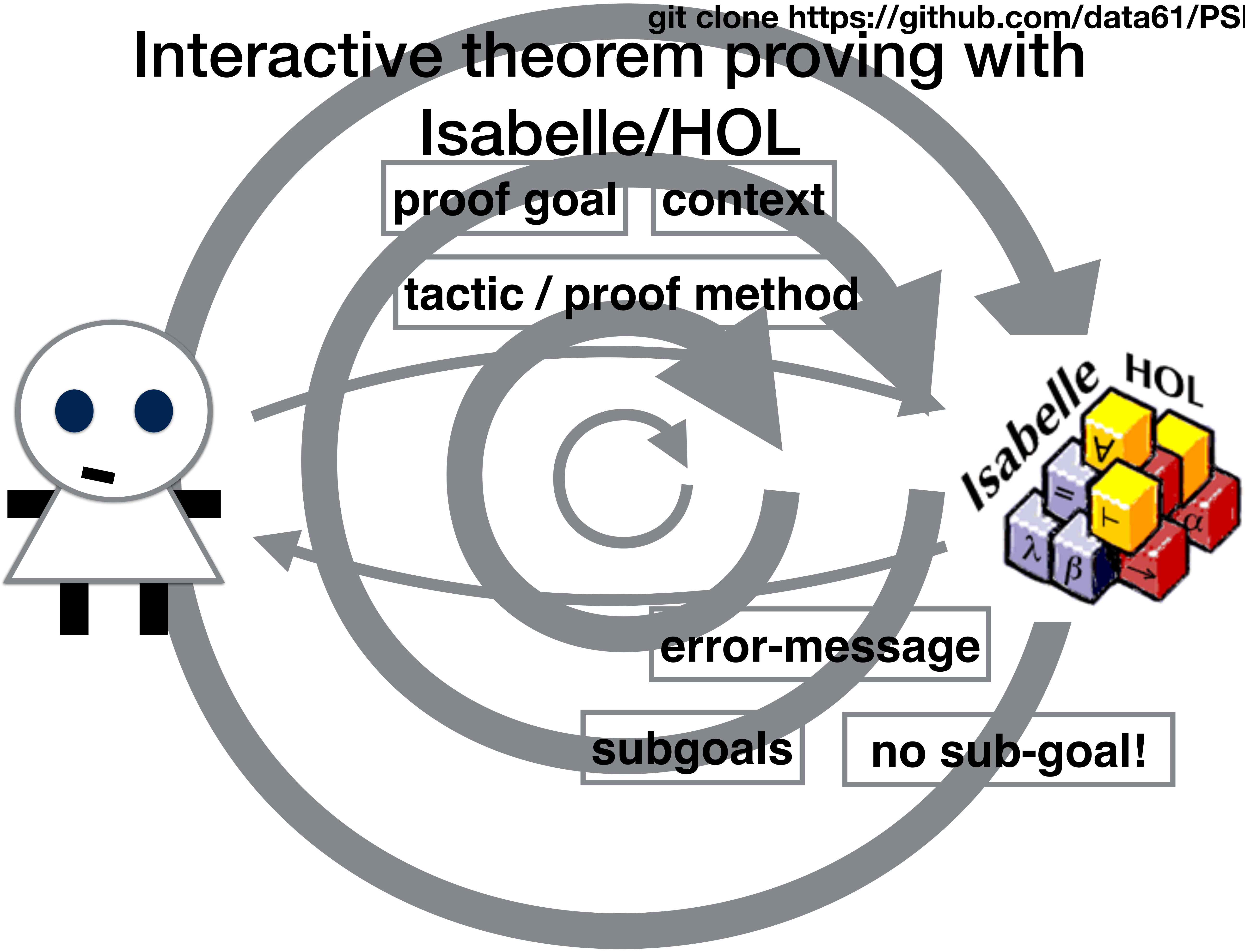
Interactive theorem proving with Isabelle/HOL



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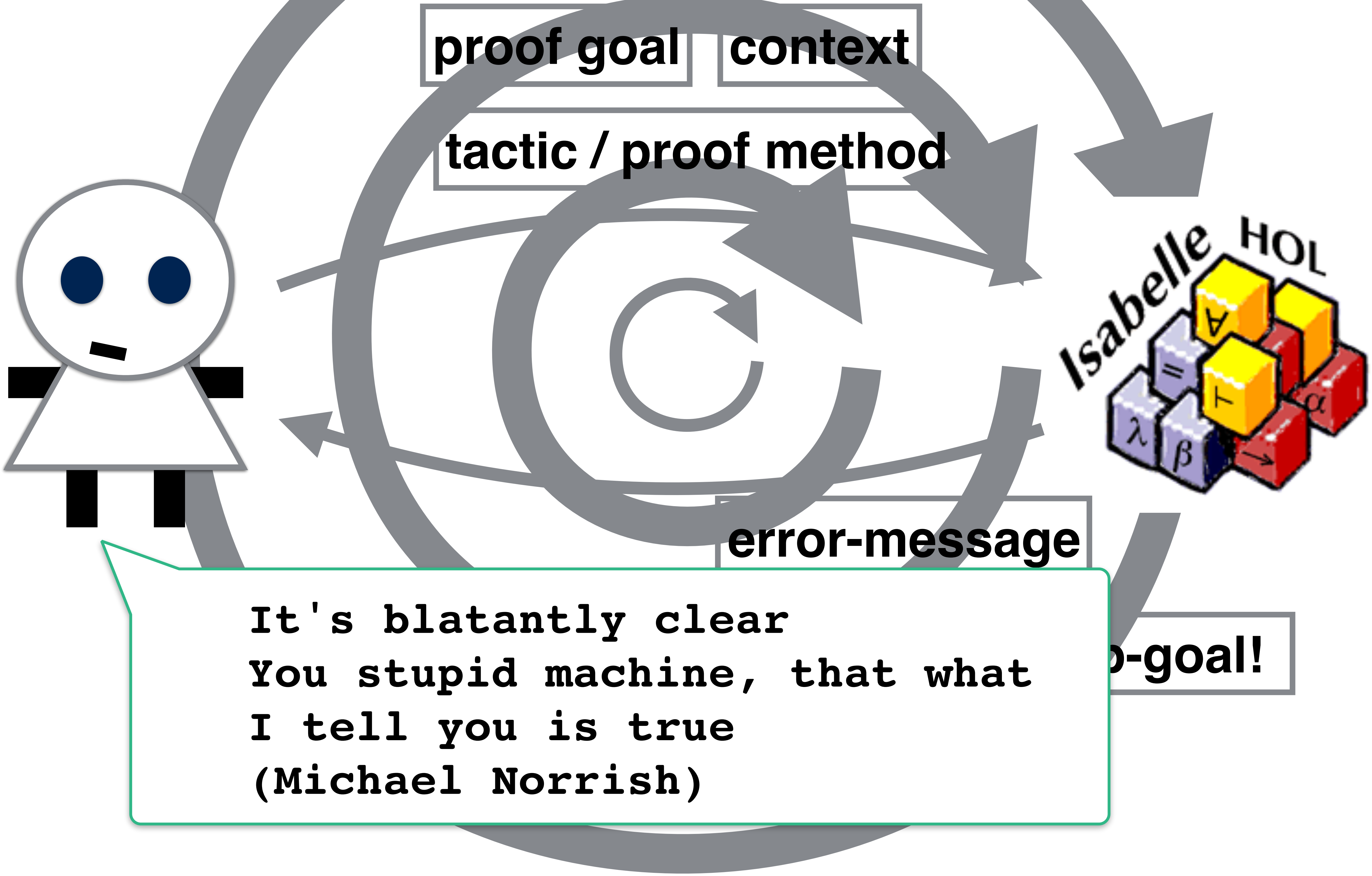
Interactive theorem proving with Isabelle/HOL


proof goal
context
tactic / proof method
Isabelle HOL
error-message
-goal!
It's blatantly clear You stupid machine, that what I tell you is true (Michael Norrish)

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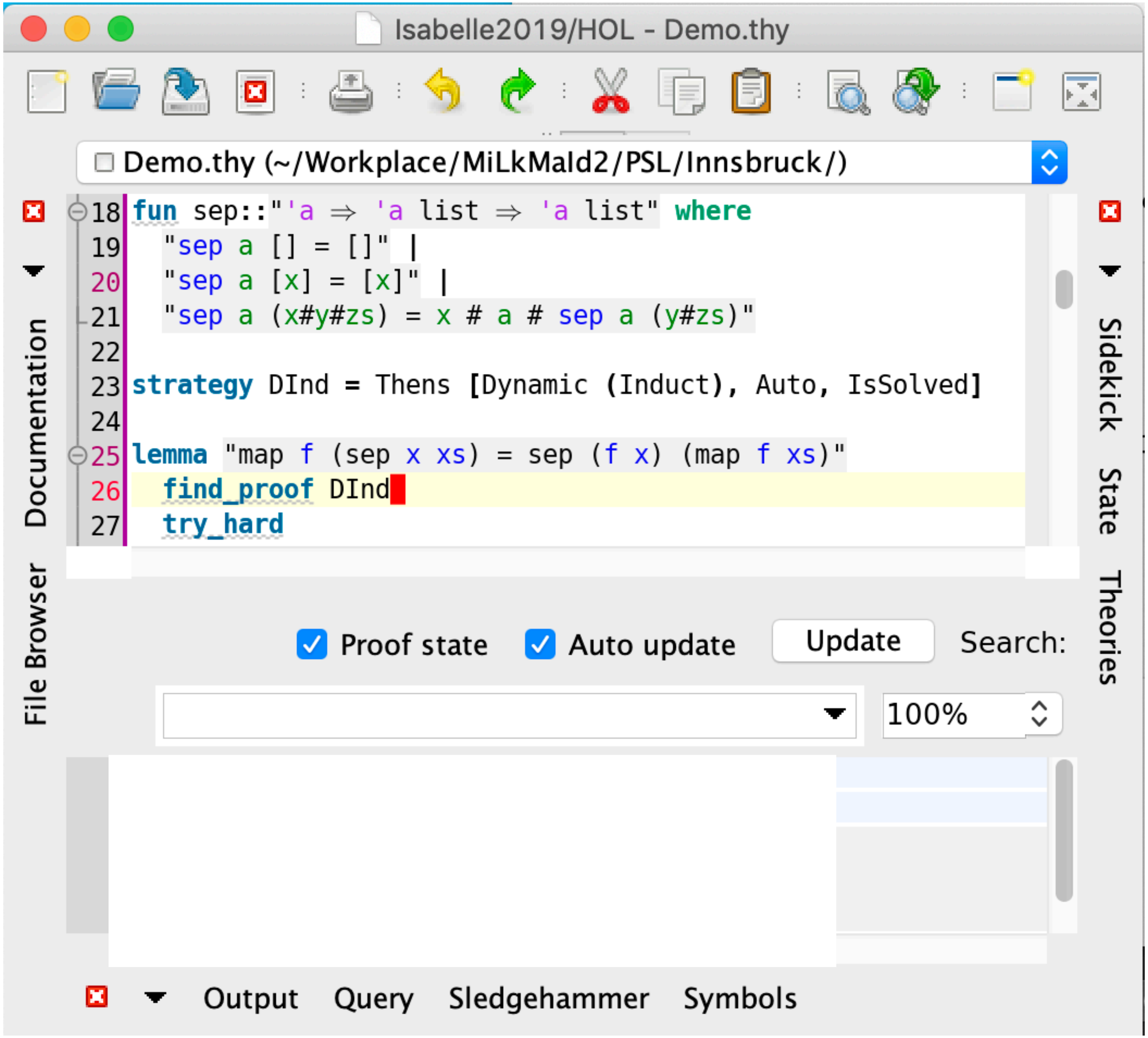

```
fun sep::"'a ⇒ 'a list ⇒ 'a list" where
  "sep a [] = []" |
  "sep a [x] = [x]" |
  "sep a (x#y#zs) = x # a # sep a (y#zs)"

strategy DInd = Thens [Dynamic (Induct), Auto, IsSolved]

lemma "map f (sep x xs) = sep (f x) (map f xs)"
  find_proof DInd
  try_hard
```

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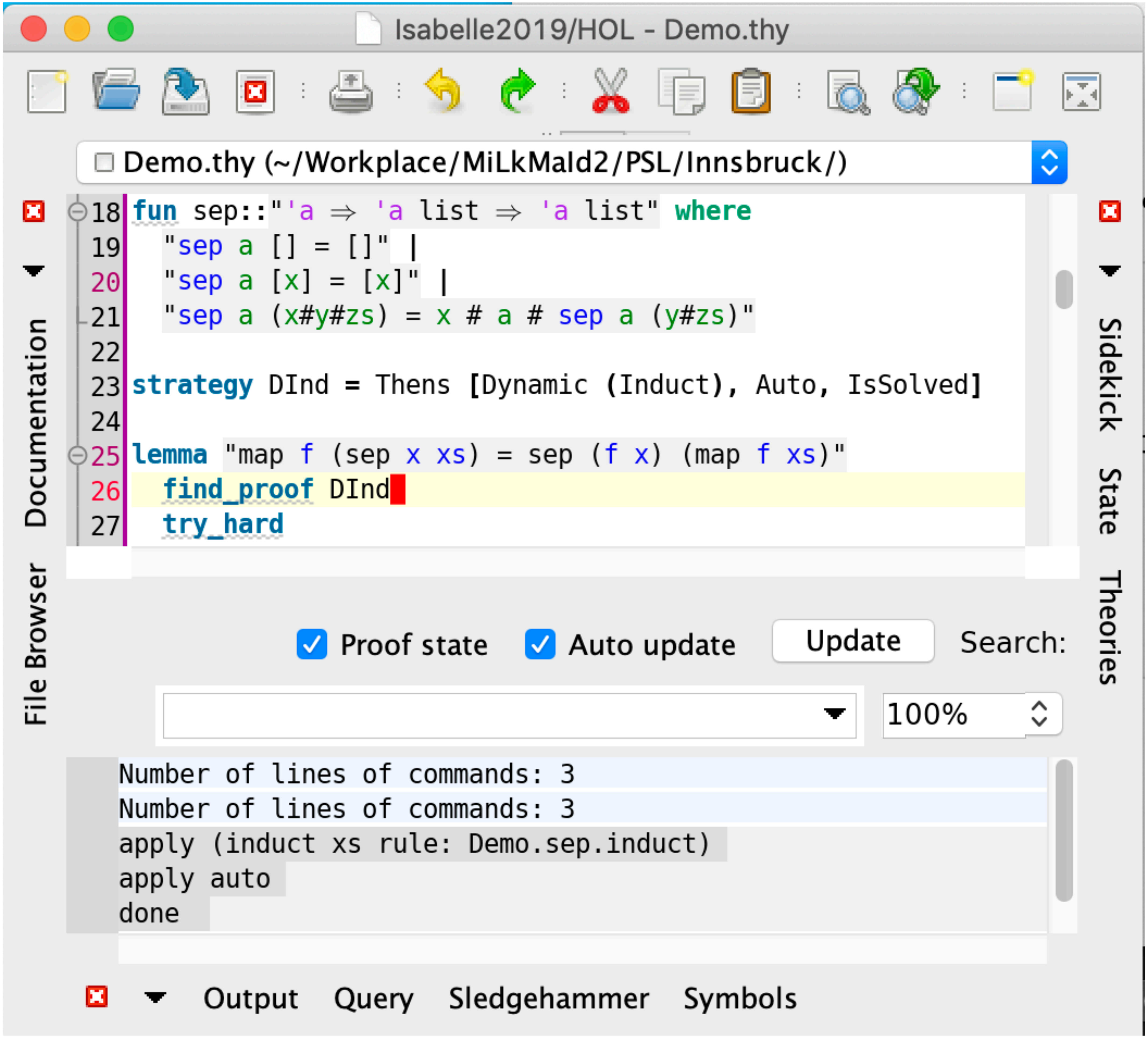

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lemma "map f (sep x xs) = sep (f x) (map f xs)"
  find_proof DInd
  try_hard
```

```
Number of lines of commands: 3
Number of lines of commands: 3
apply (induct xs rule: Demo.sep.induct)
apply auto
done
```

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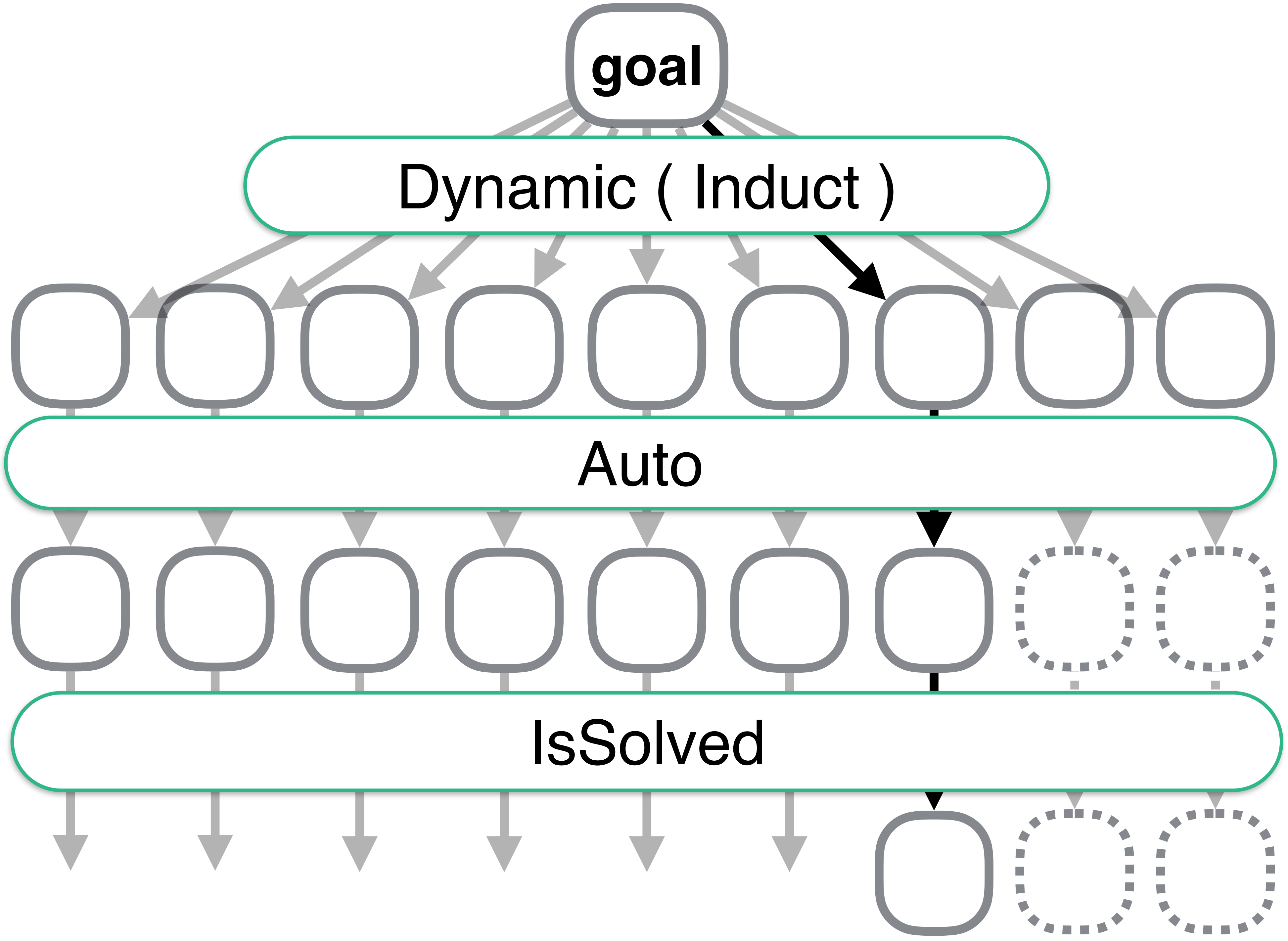
```
lemma "map f (sep x xs) = sep (f x) (map f xs)"
find_proof DInd(*= Thens [Dynamic (Induct), Auto, IsSolved]*)
```



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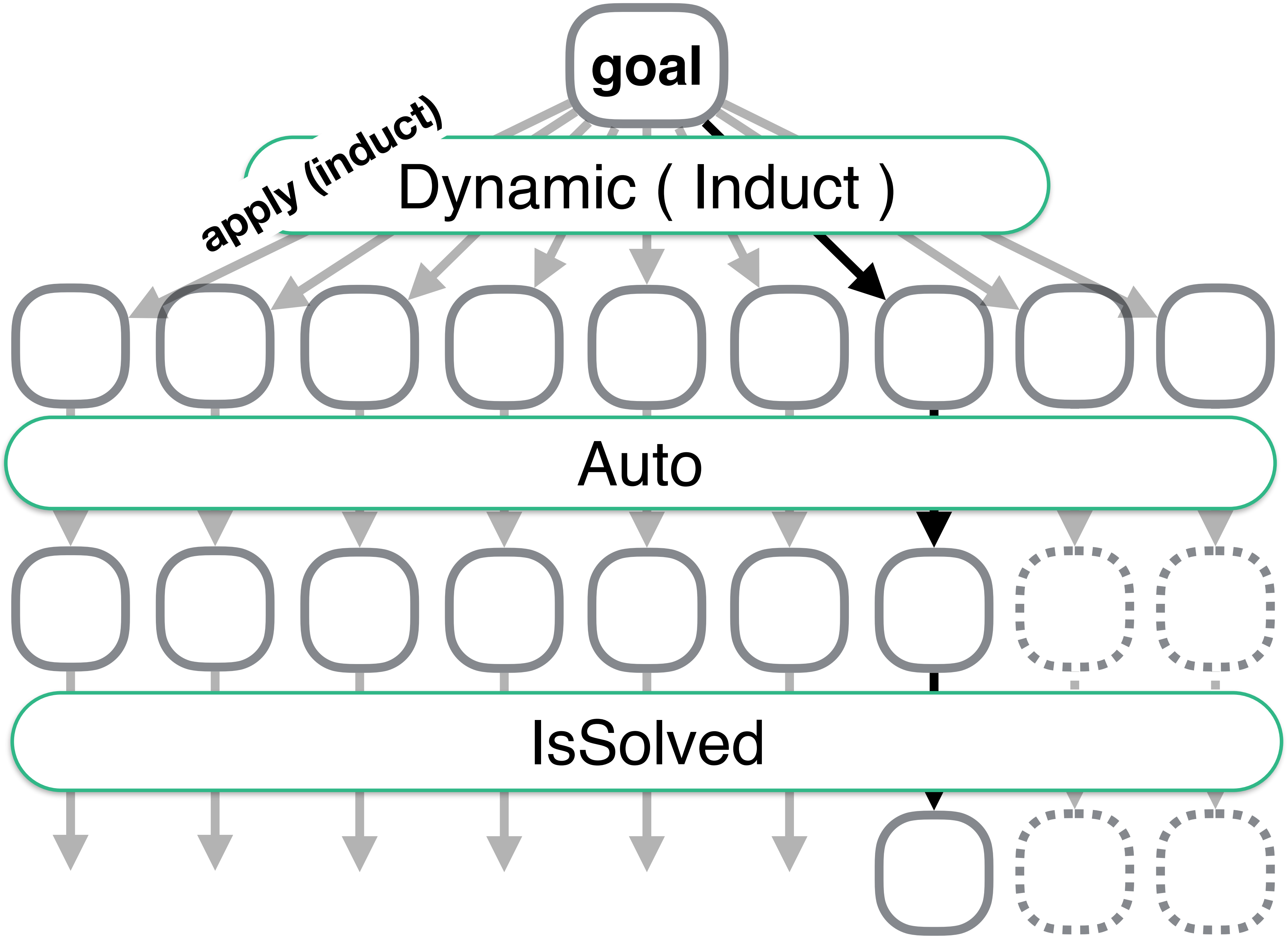

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```

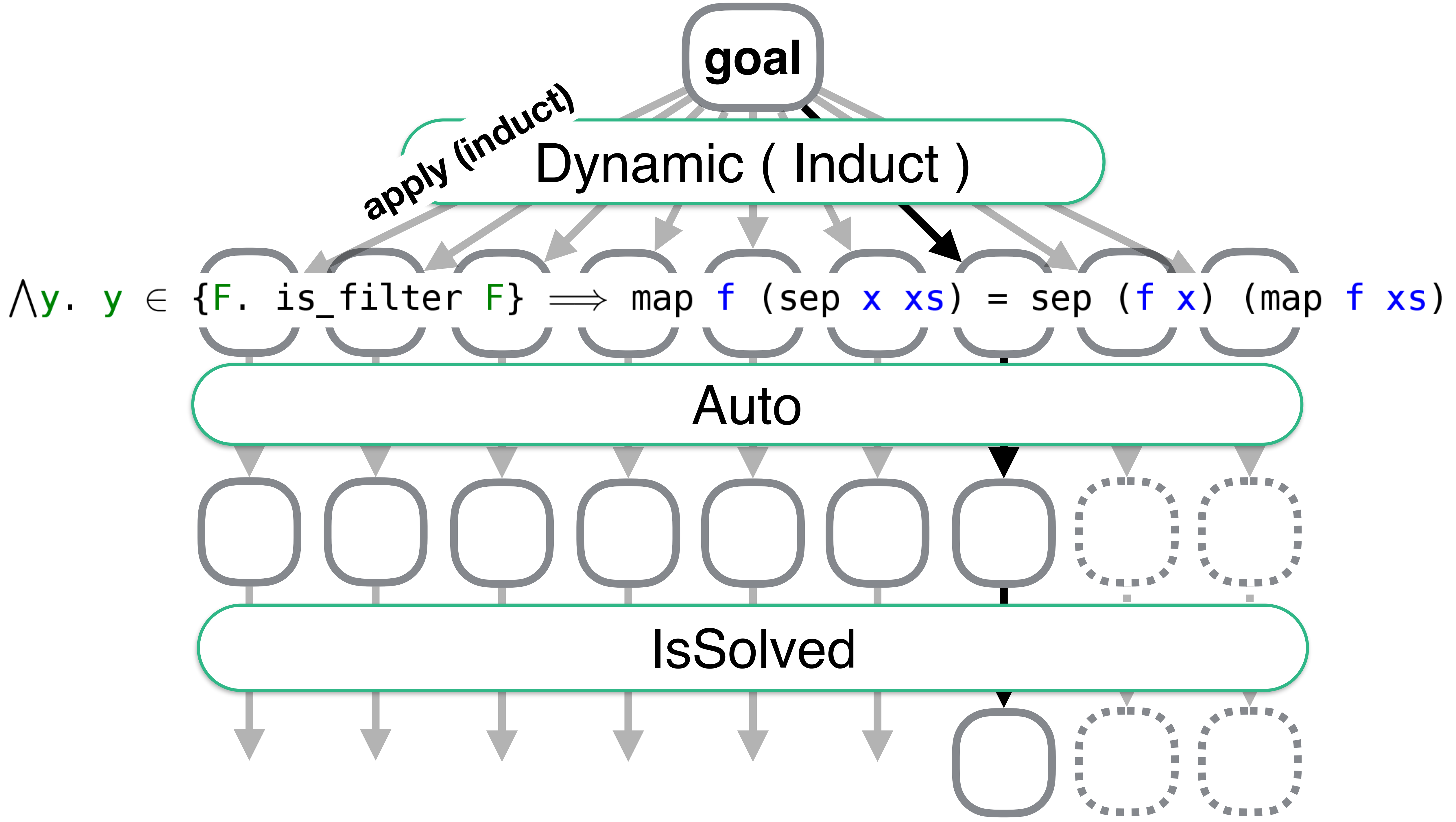
goal
apply (induct)
Dynamic (Induct)
⋀y. y ∈ {F. is_filter F} ⟹ map f (sep x xs) = sep (f x) (map f xs)
Auto
IsSolved

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lemma "map f (sep x xs) = sep (f x) (map f xs)"
find_proof DInd(*= Thens [Dynamic (Induct), Auto, IsSolved]*)
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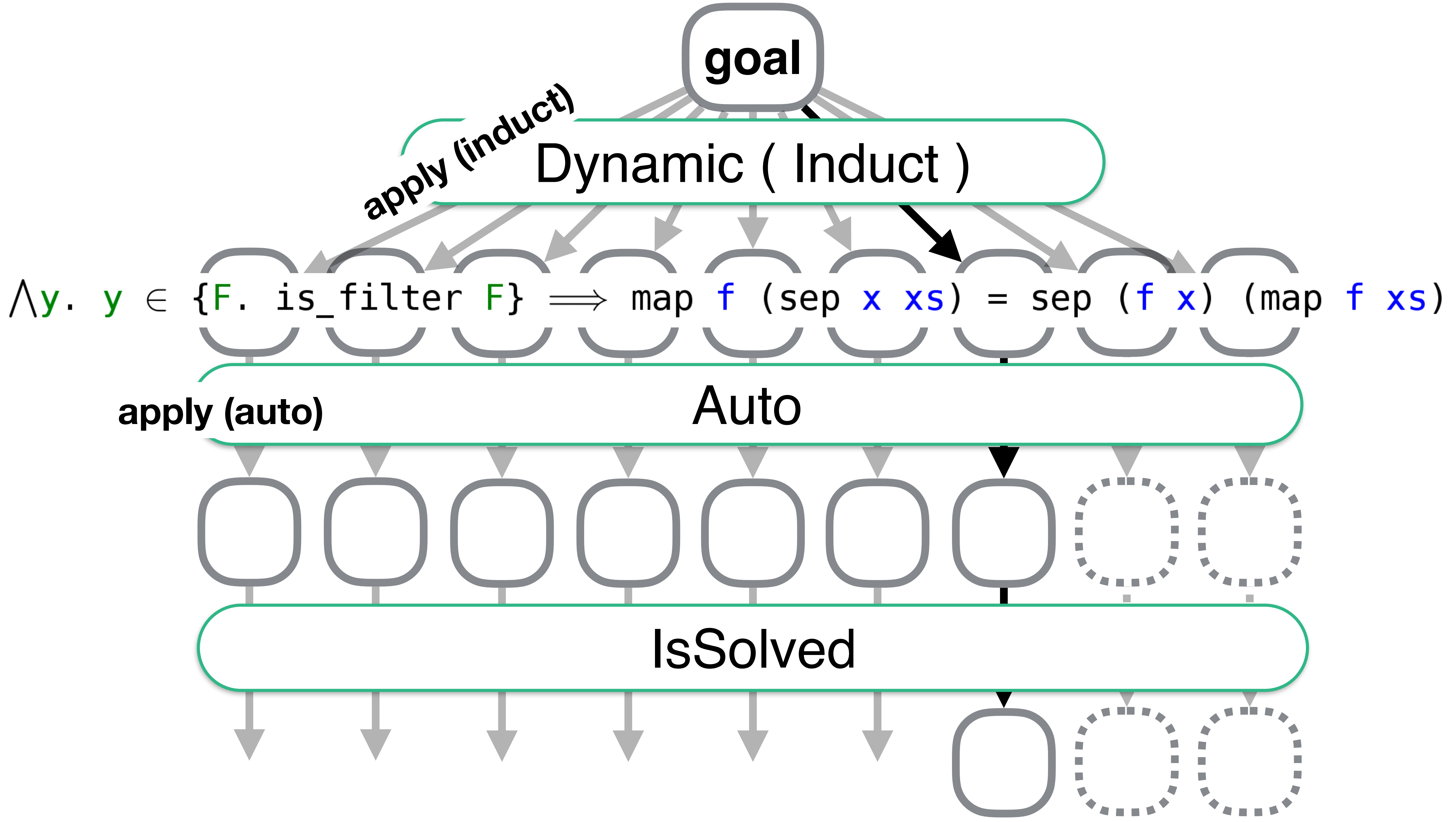
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Dynamic (Induct)
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apply (auto)
Auto
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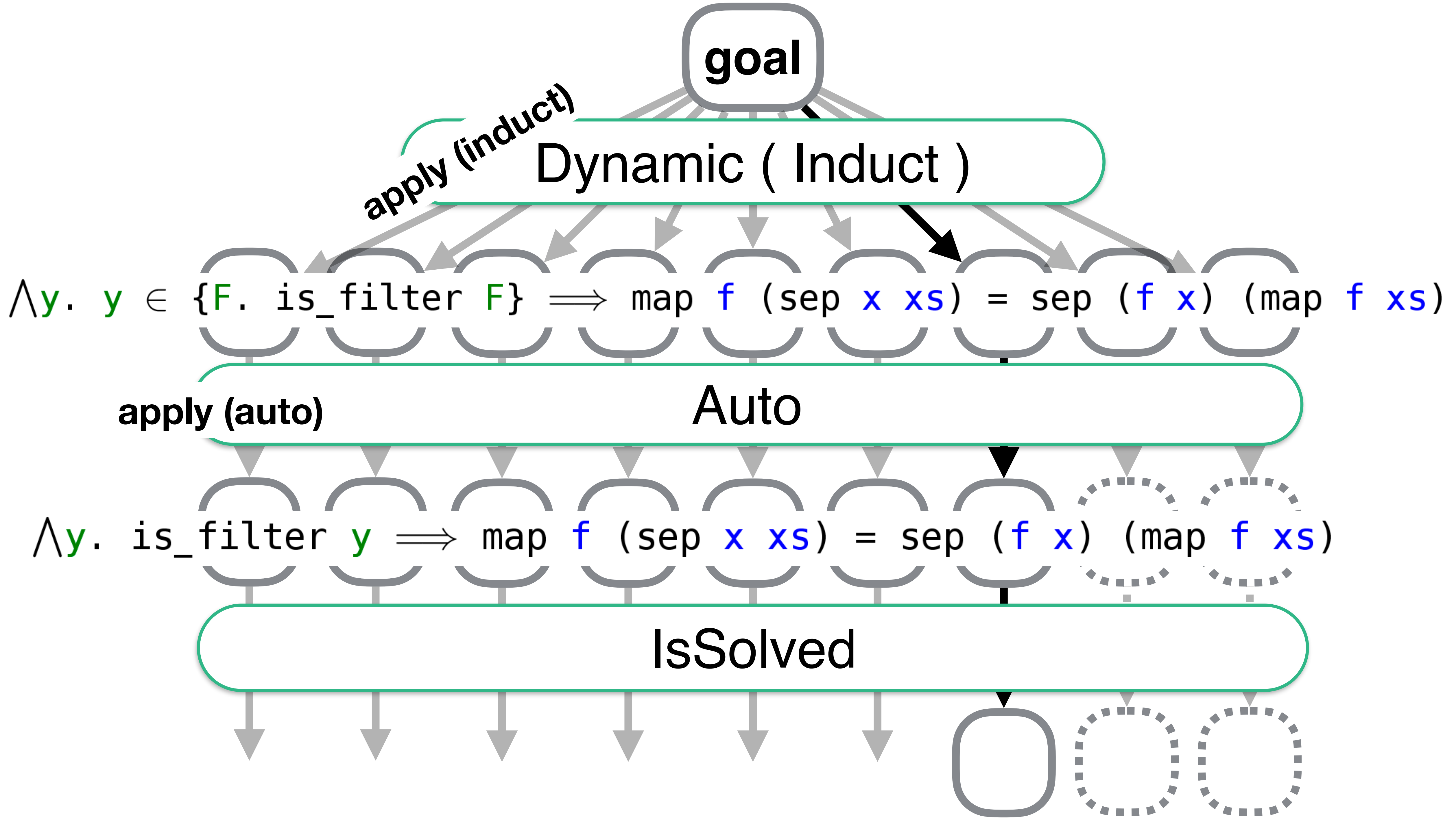
find_proof DInd(*= Thens [Dynamic (Induct), Auto, IsSolved]*)
```



goal
apply (induct)
Dynamic (Induct)
⋀y. y ∈ {F. is_filter F} ⟹ map f (sep x xs) = sep (f x) (map f xs)
apply (auto)
Auto
⋀y. is_filter y ⟹ map f (sep x xs) = sep (f x) (map f xs)
IsSolved

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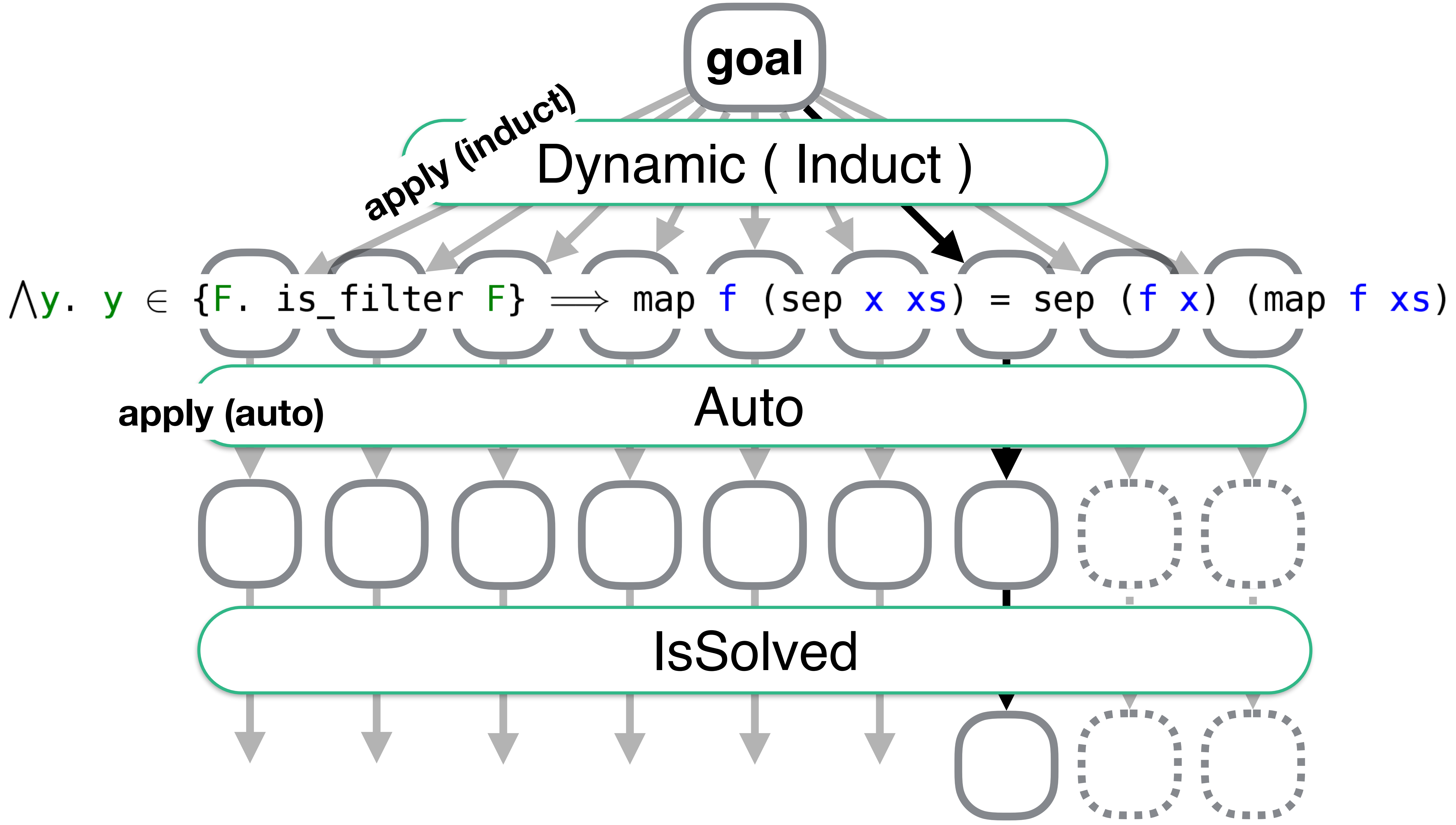
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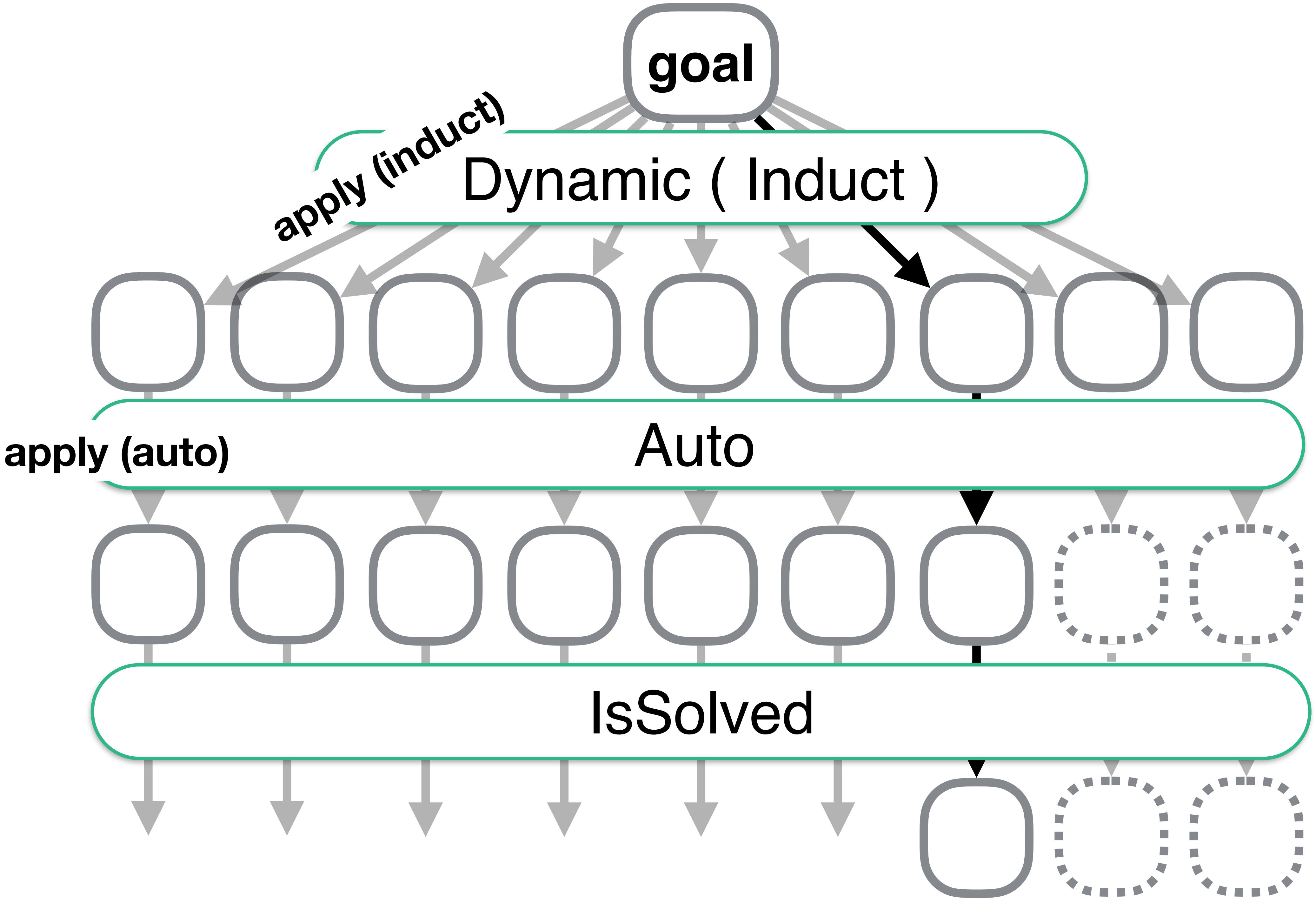

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lemma "map f (sep x xs) = sep (f x) (map f xs)"

find_proof DInd(*= Thens [Dynamic (Induct), Auto, IsSolved]*)
```

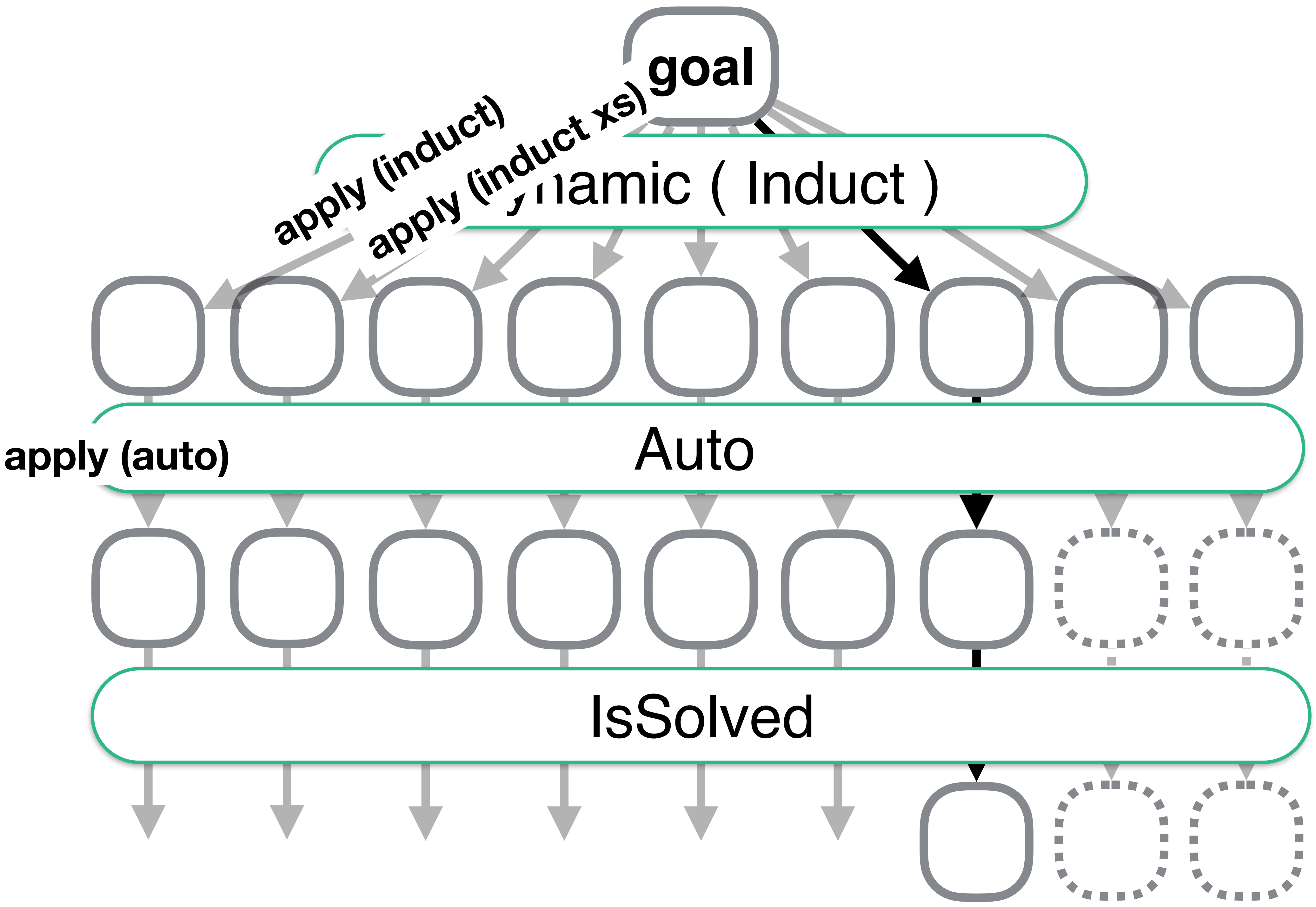

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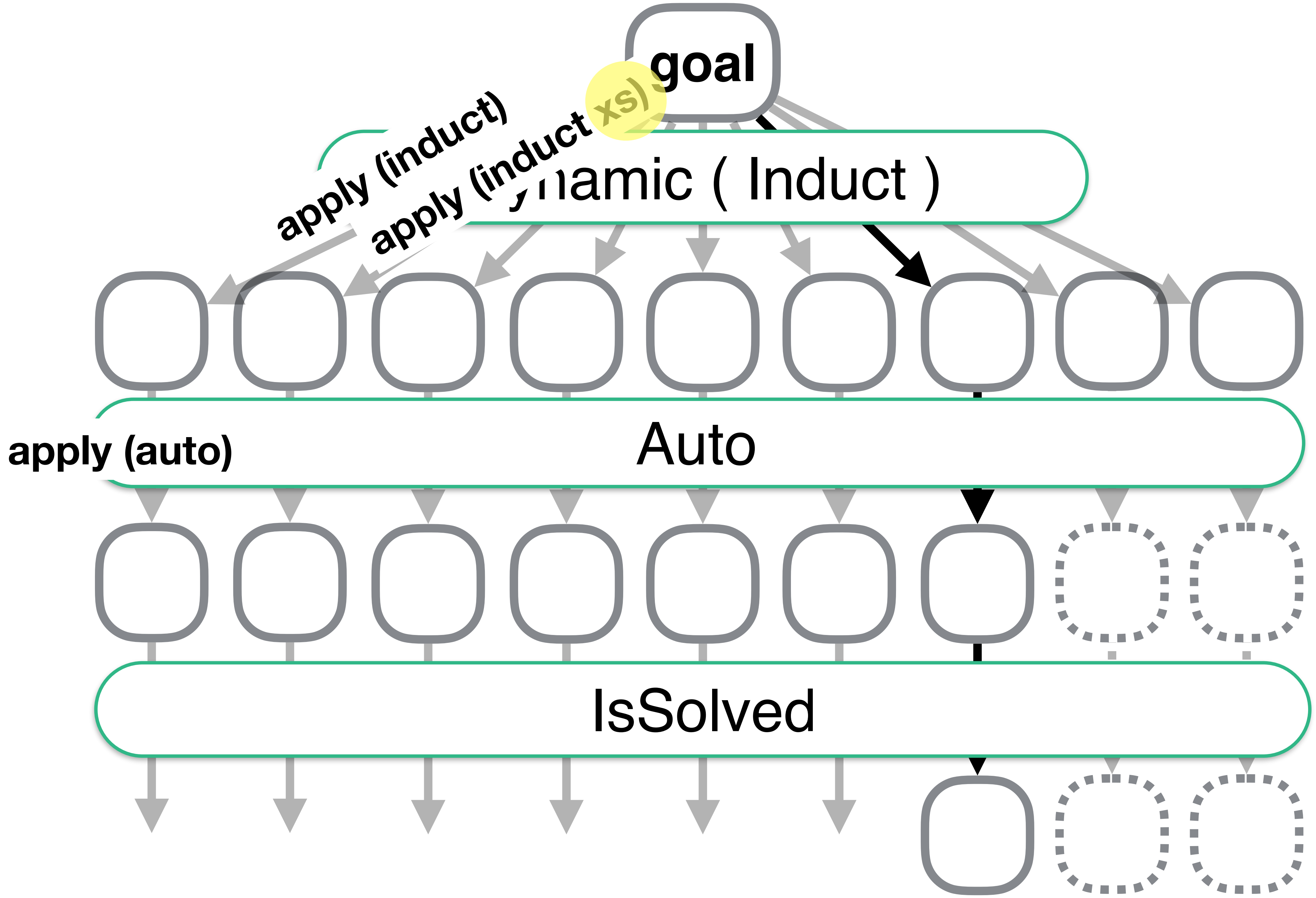

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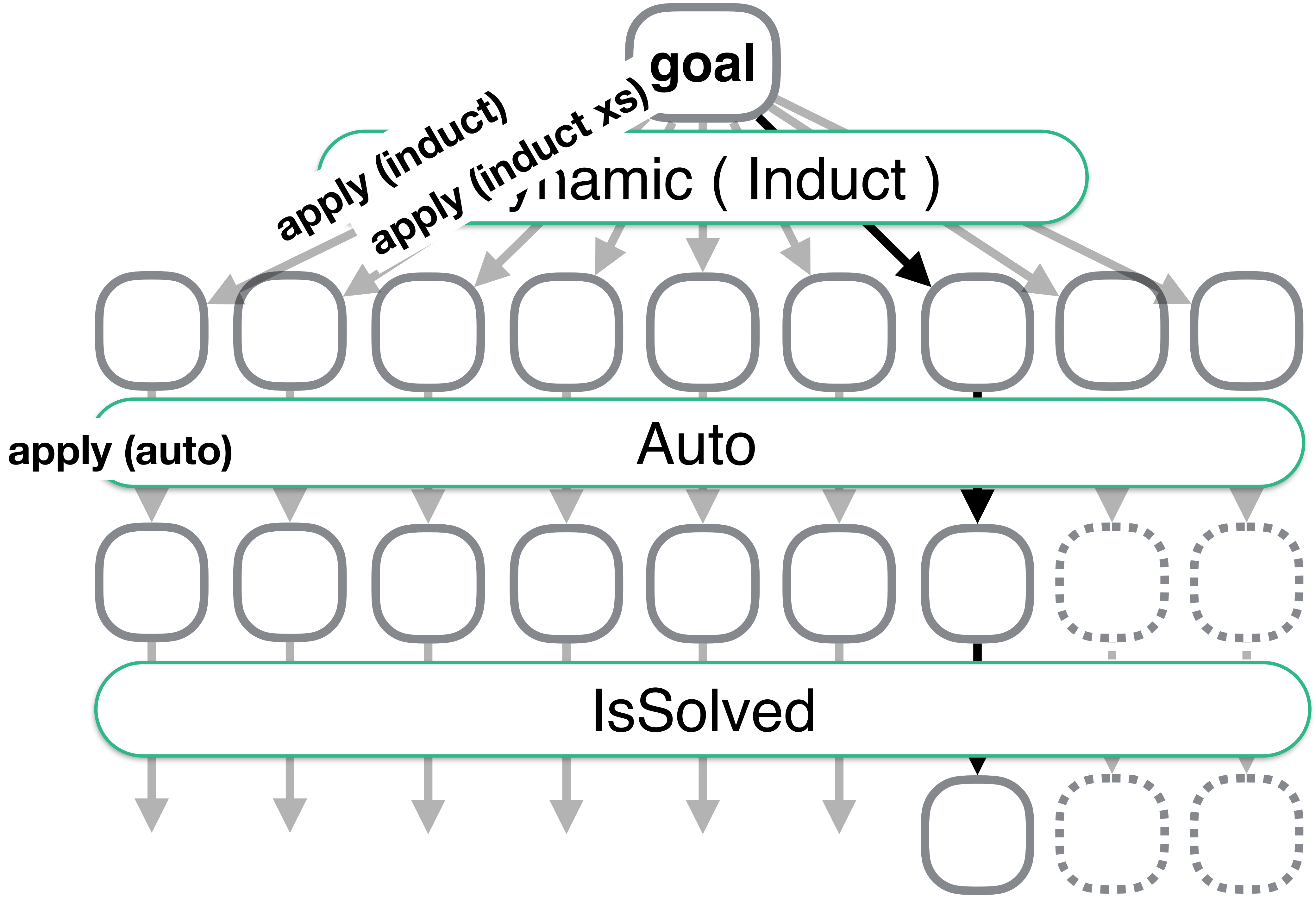

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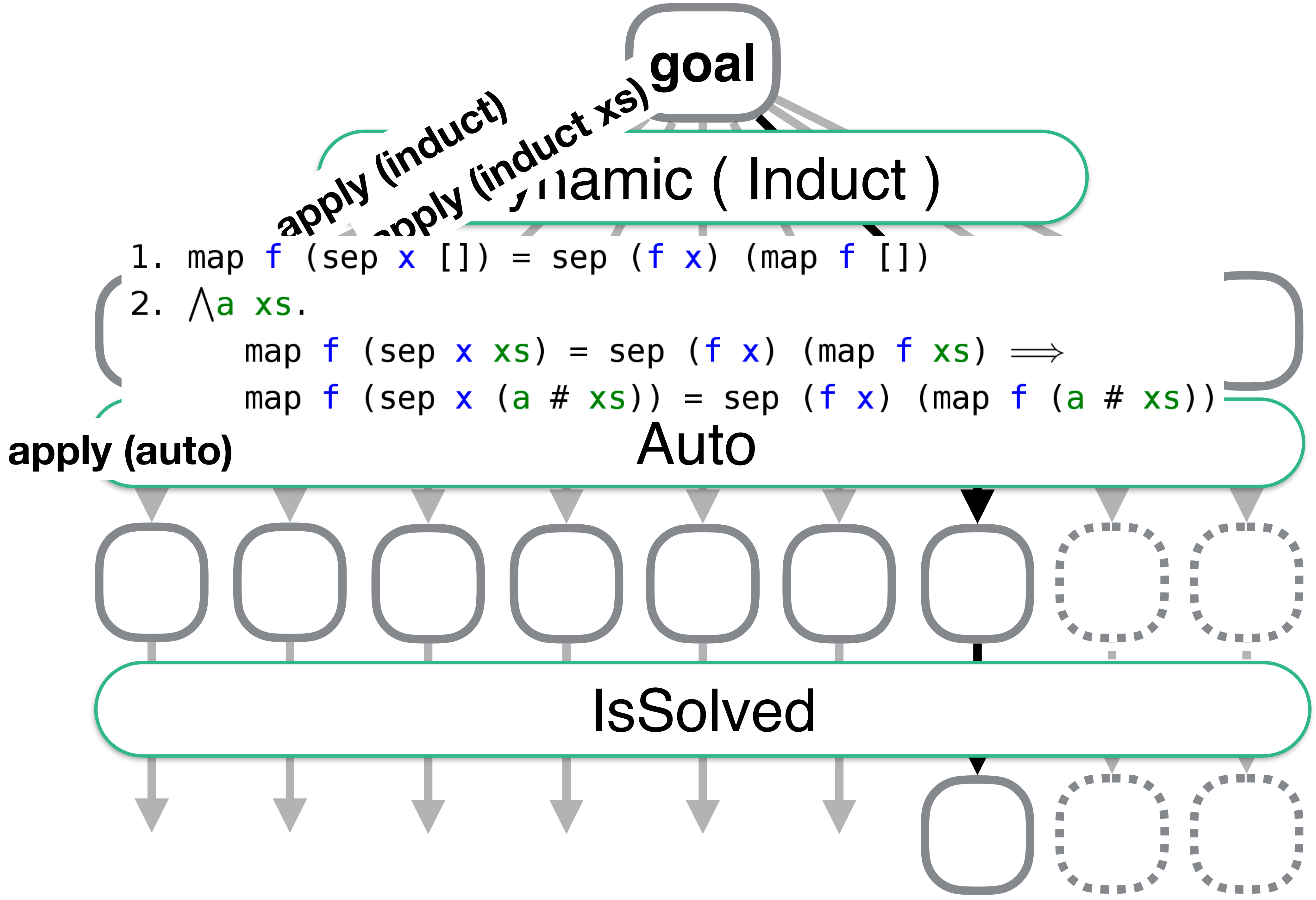

```
1. map f (sep x []) = sep (f x) (map f [])
2. ⋀a xs.
      map f (sep x xs) = sep (f x) (map f xs) ⟹
      map f (sep x (a # xs)) = sep (f x) (map f (a # xs))
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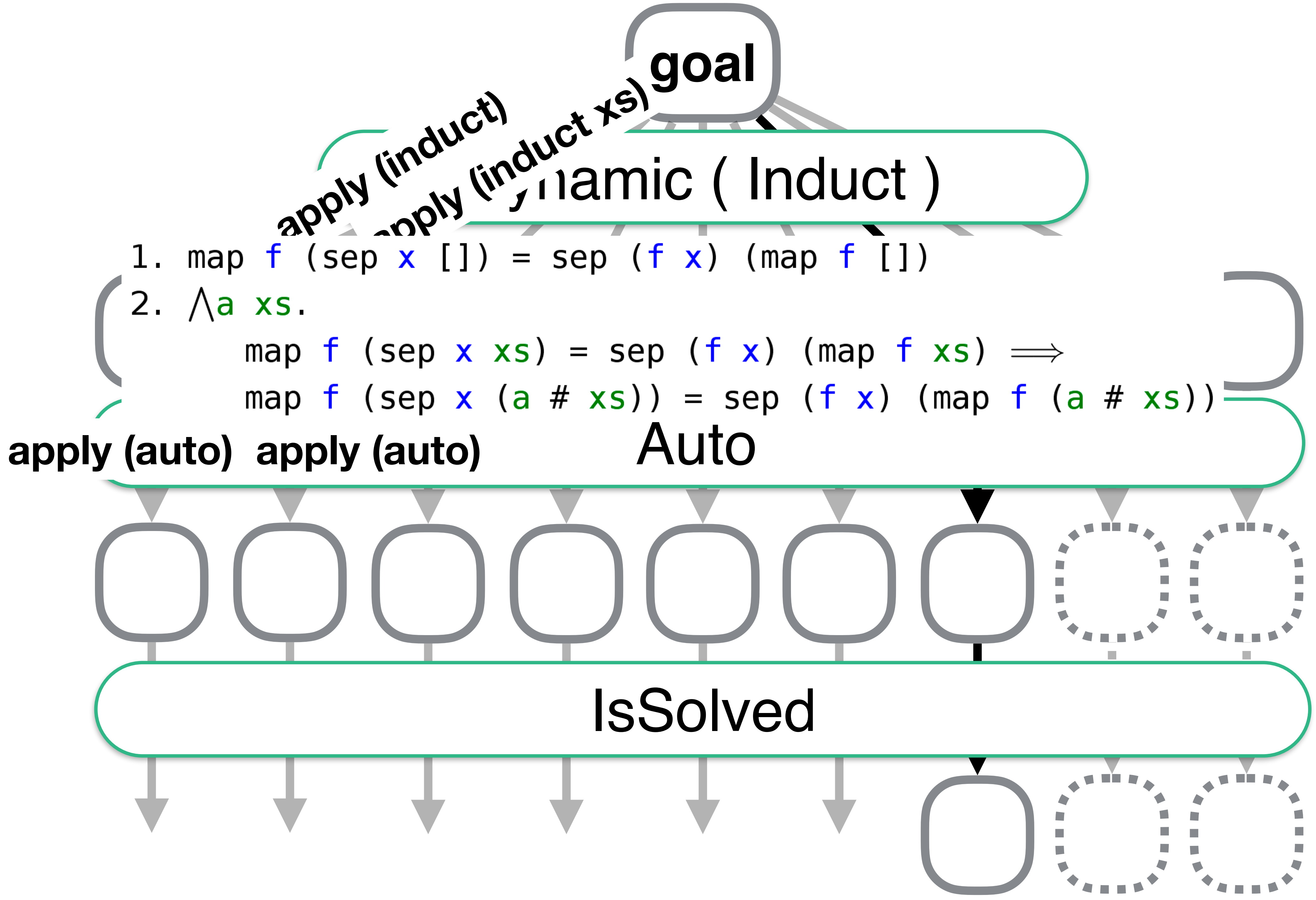

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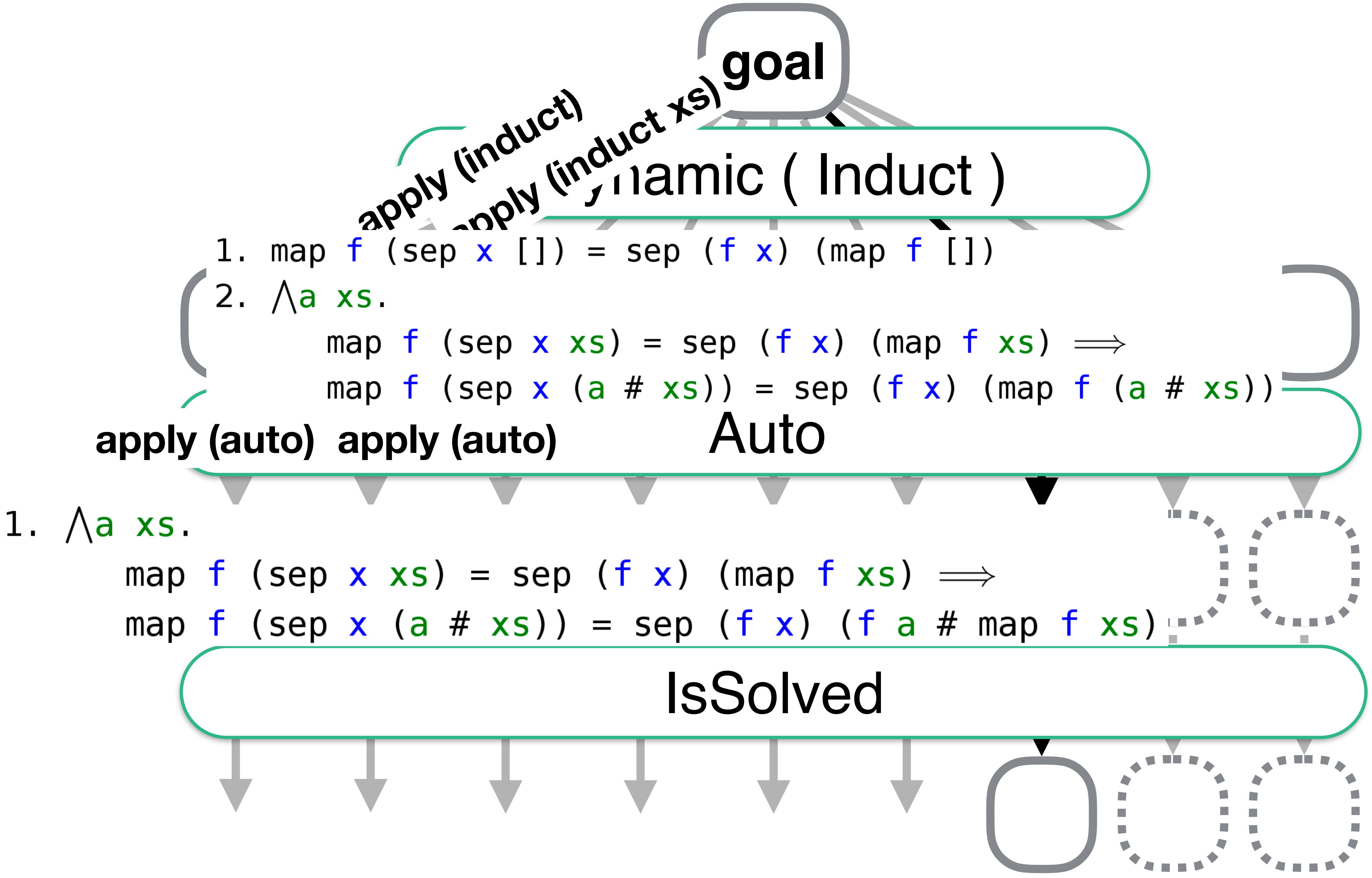

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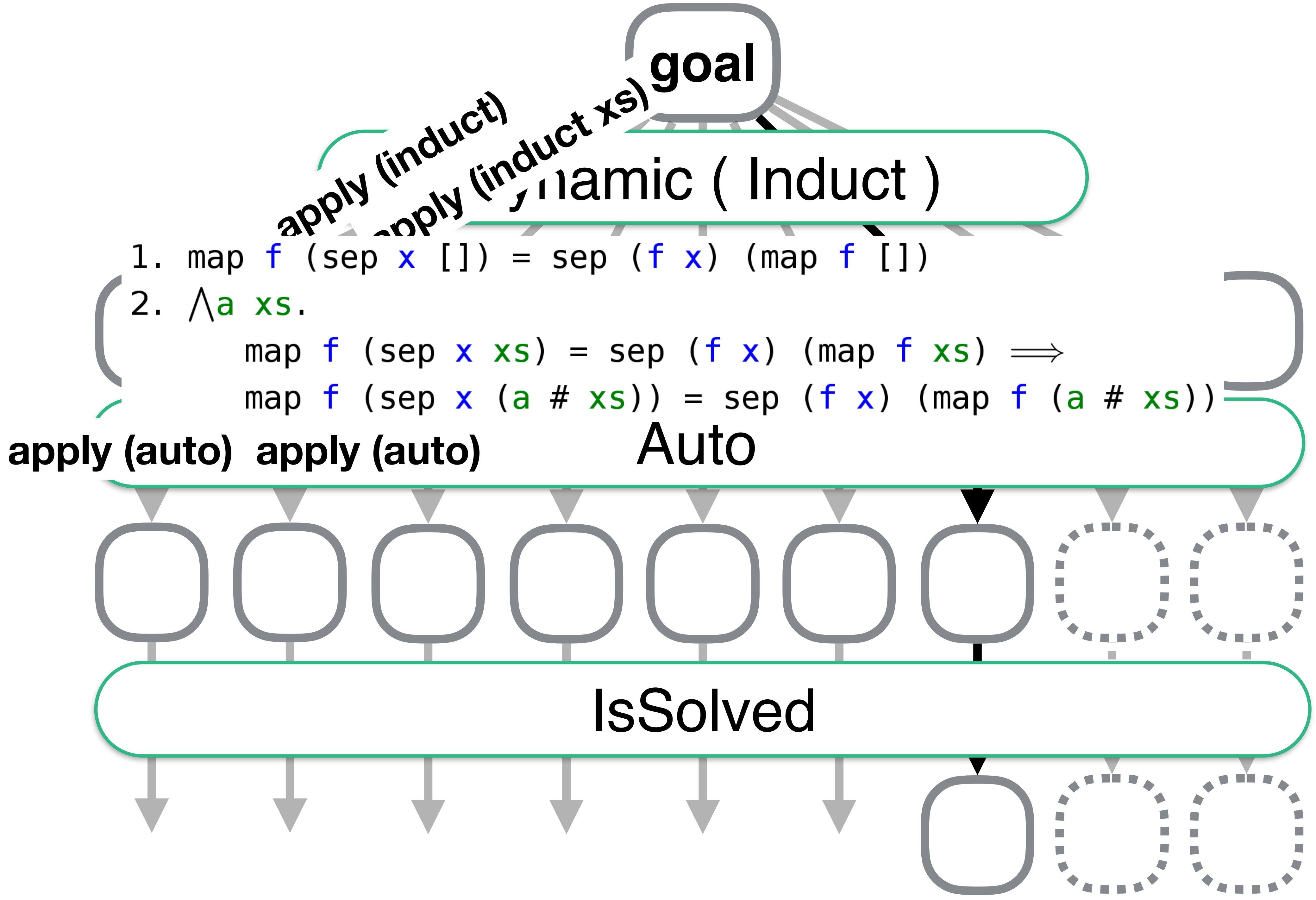

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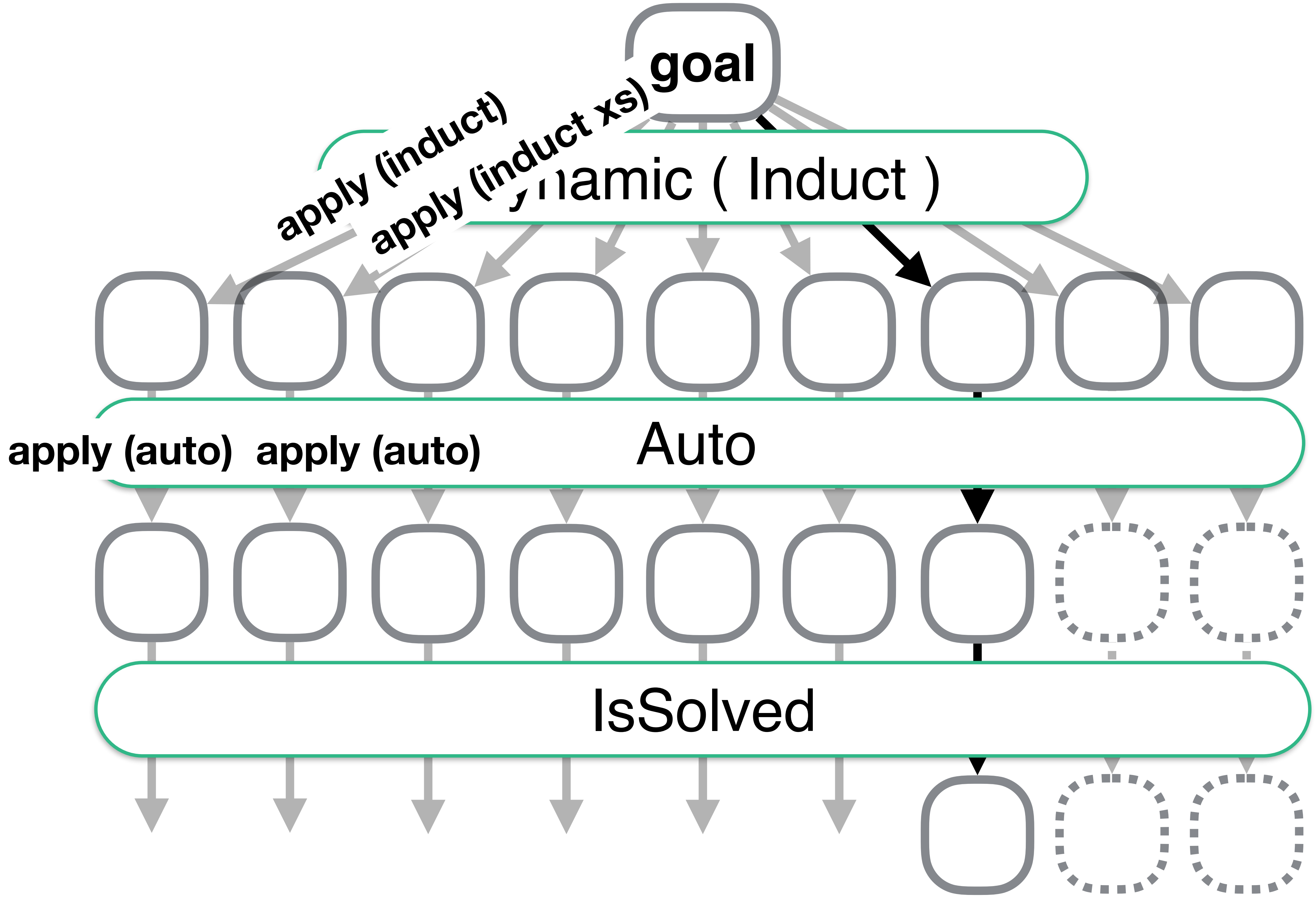

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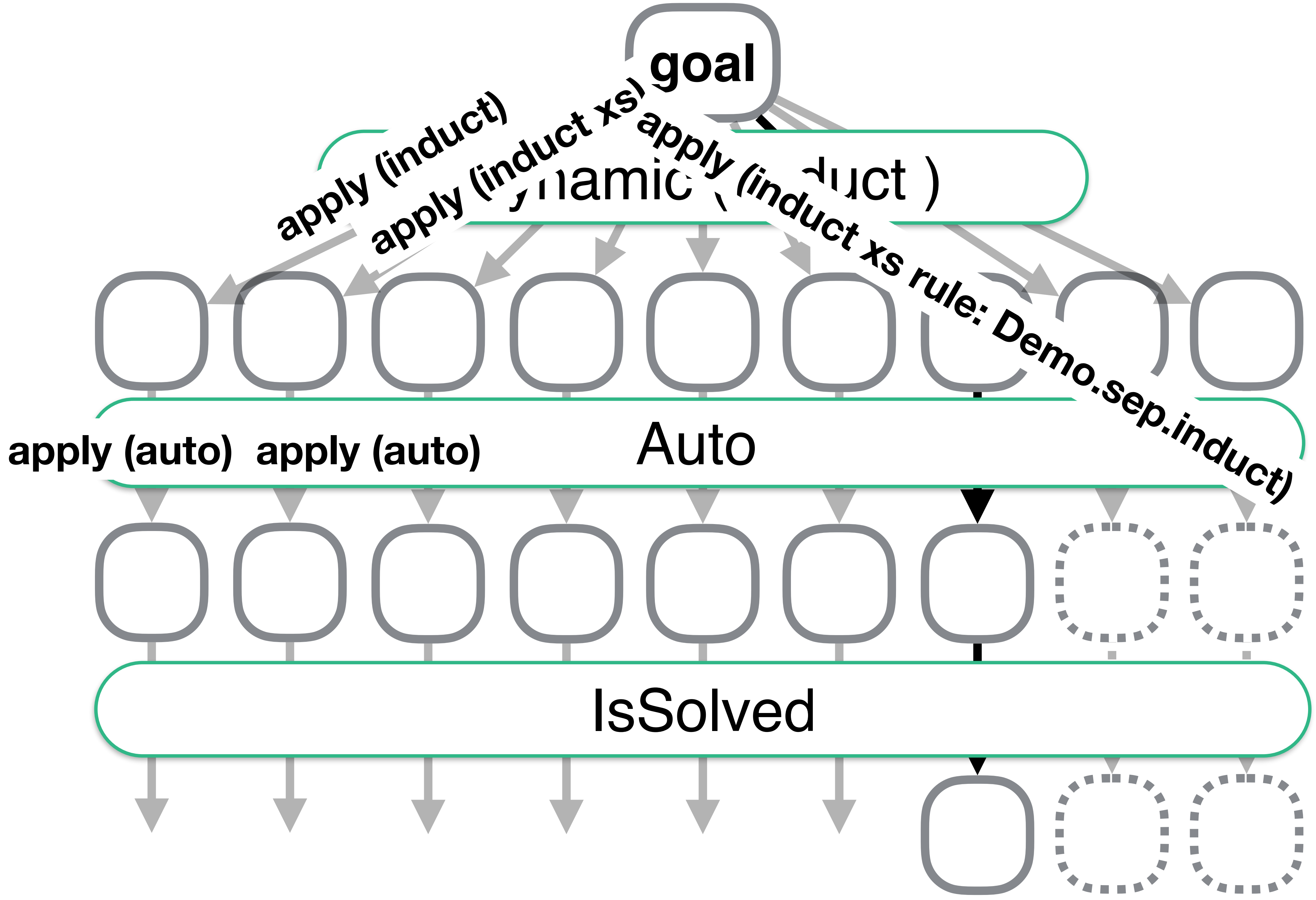

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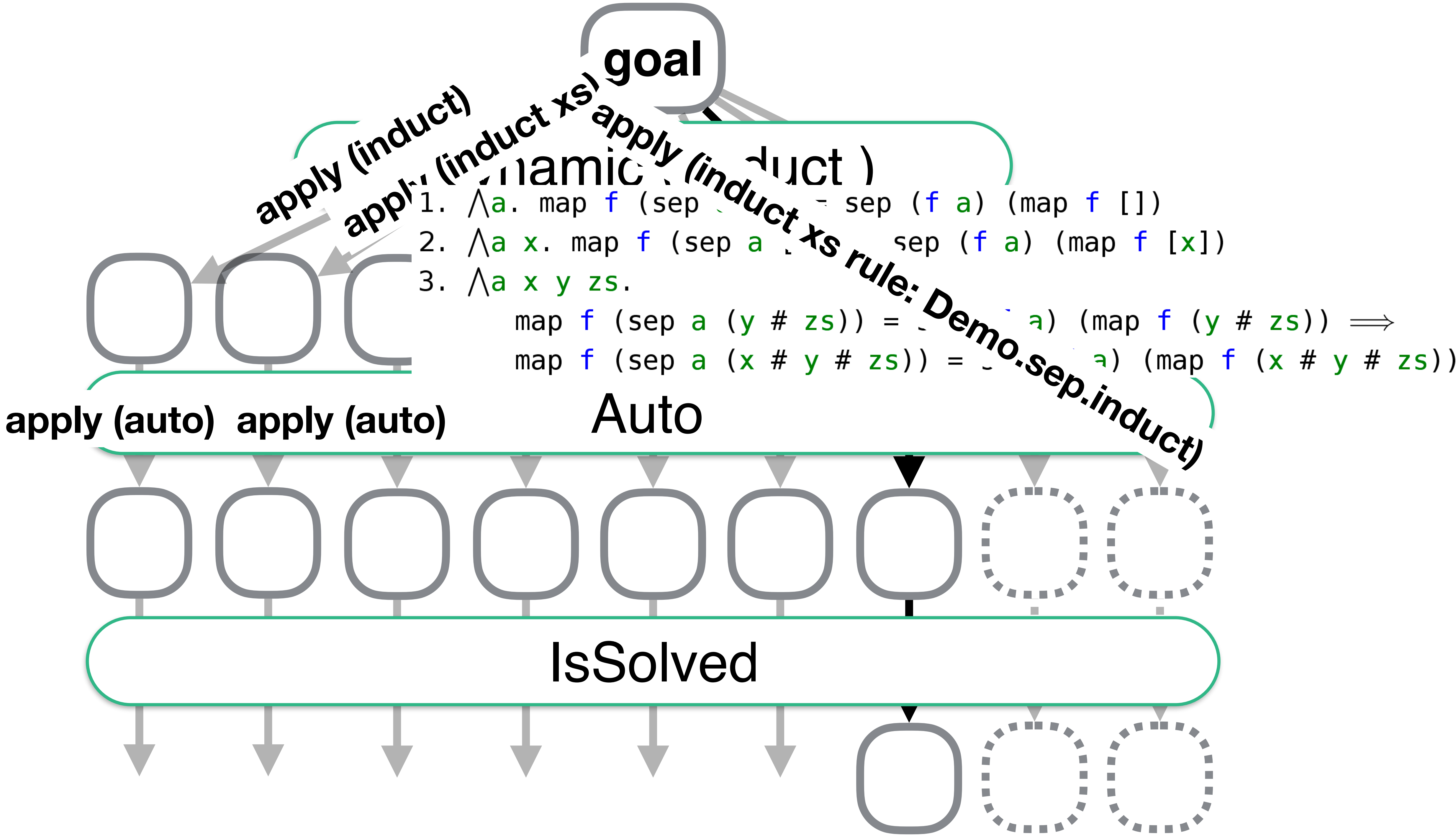

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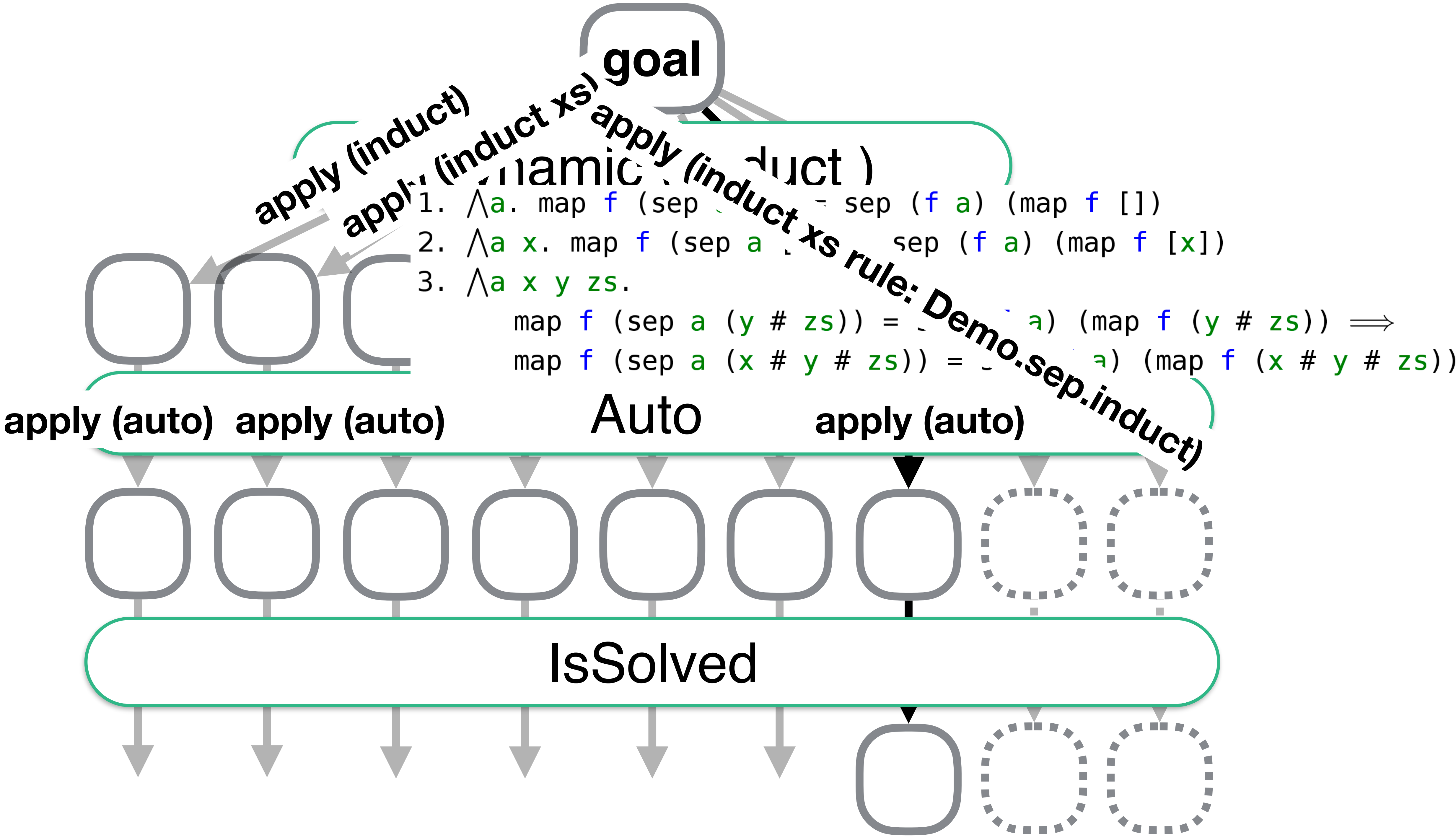

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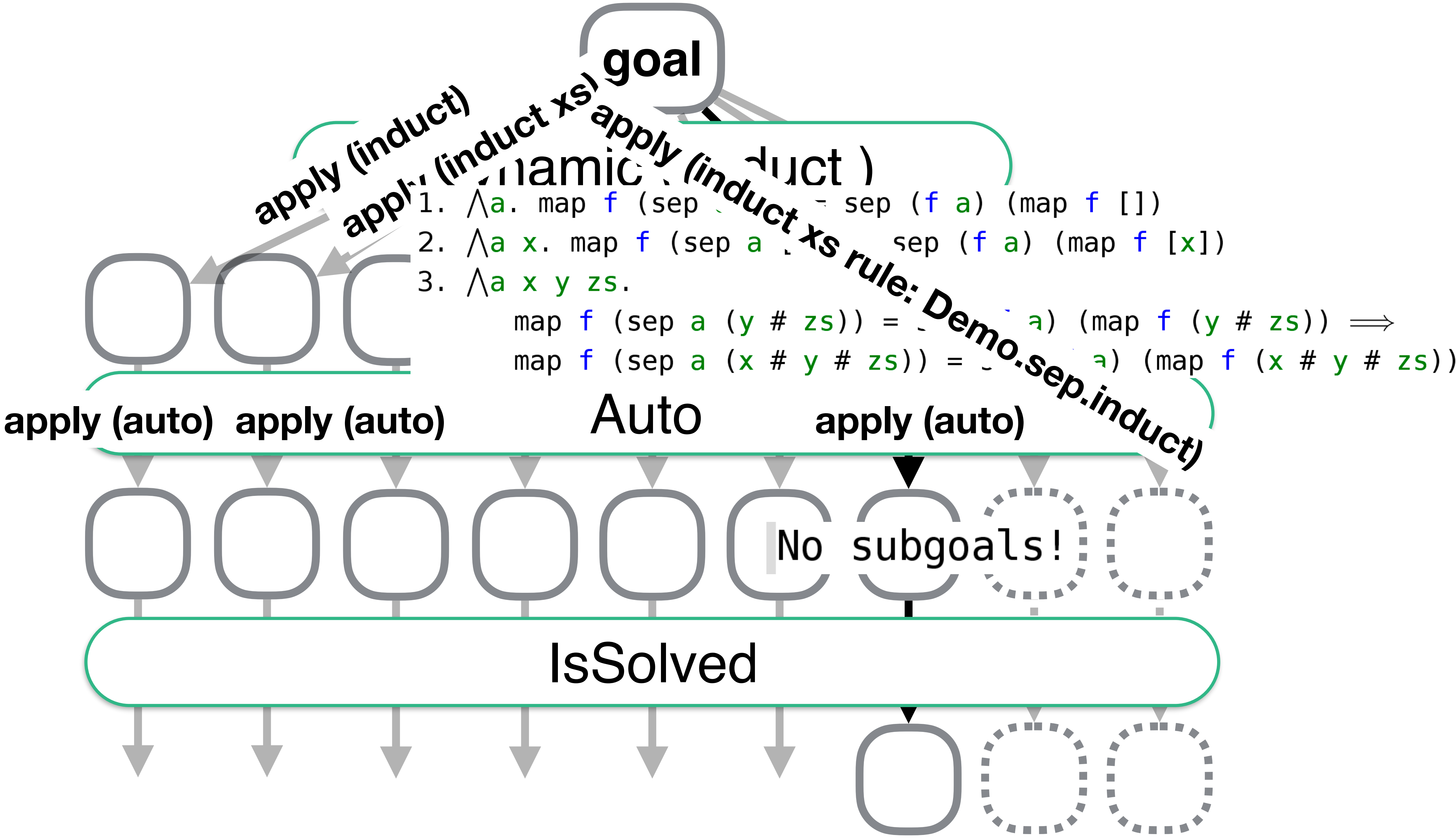

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```

```
No subgoals!
```

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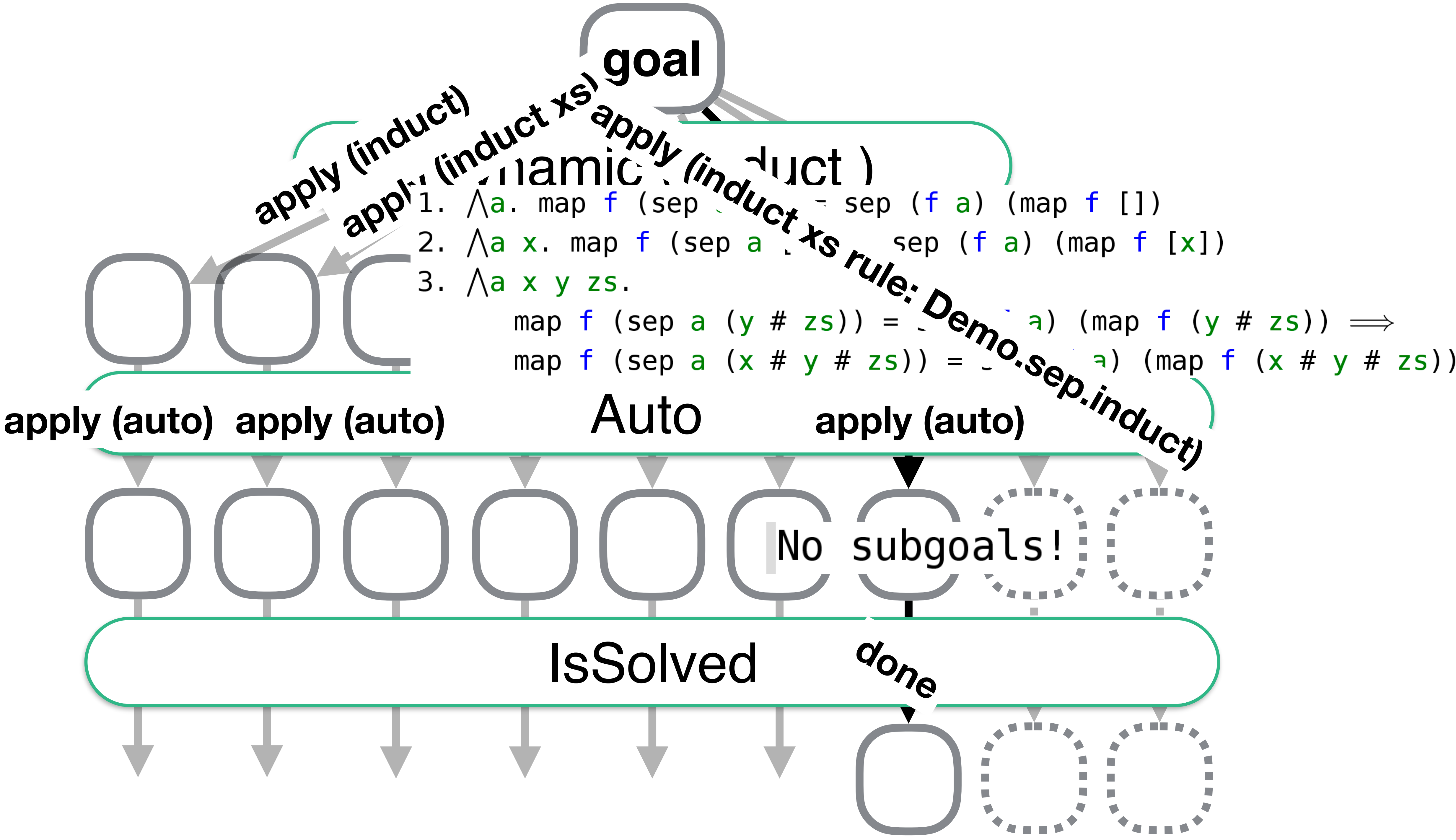

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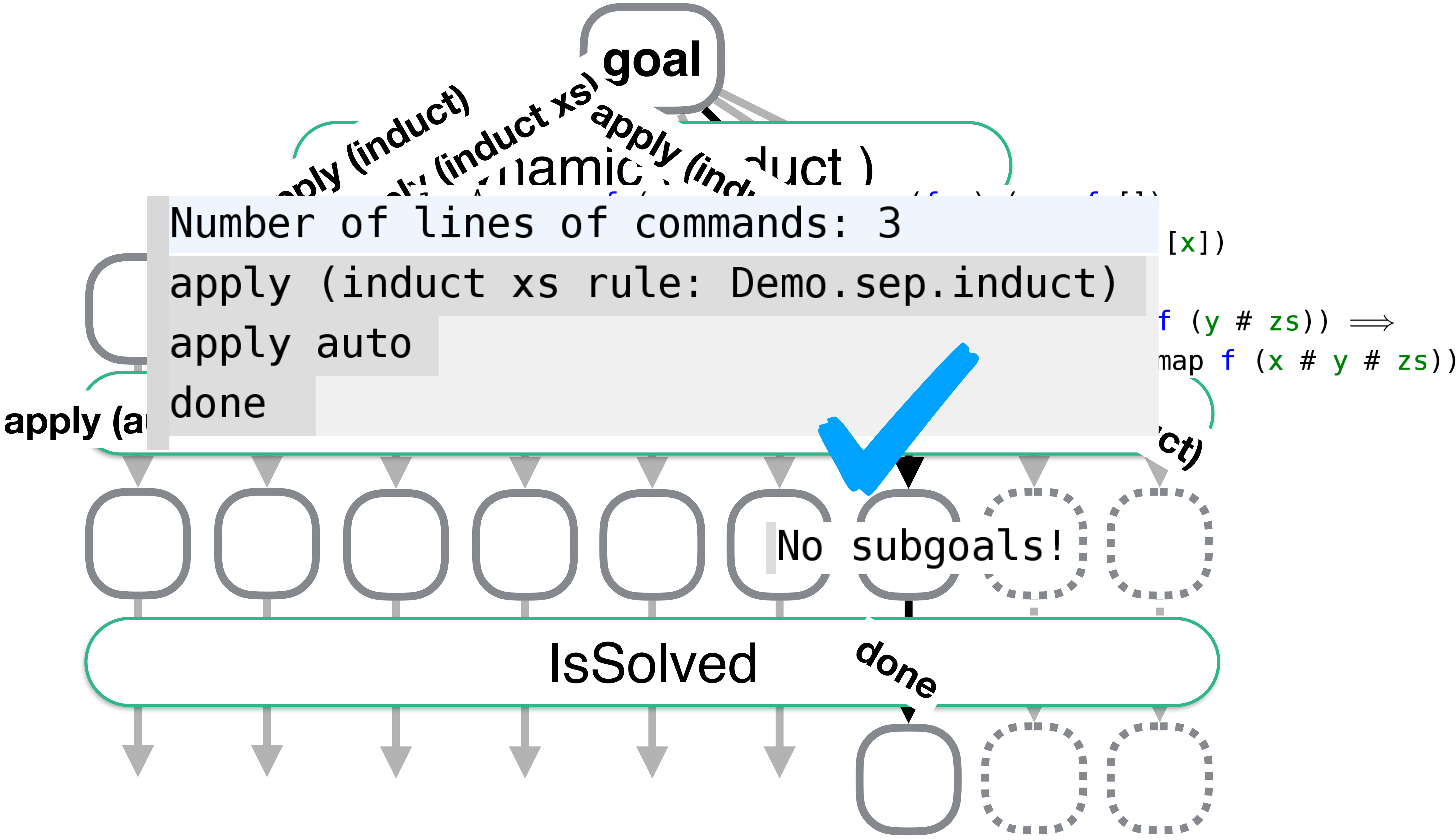

```
Number of lines of commands: 3
apply (induct xs rule: Demo.sep.induct)
apply auto
done
```

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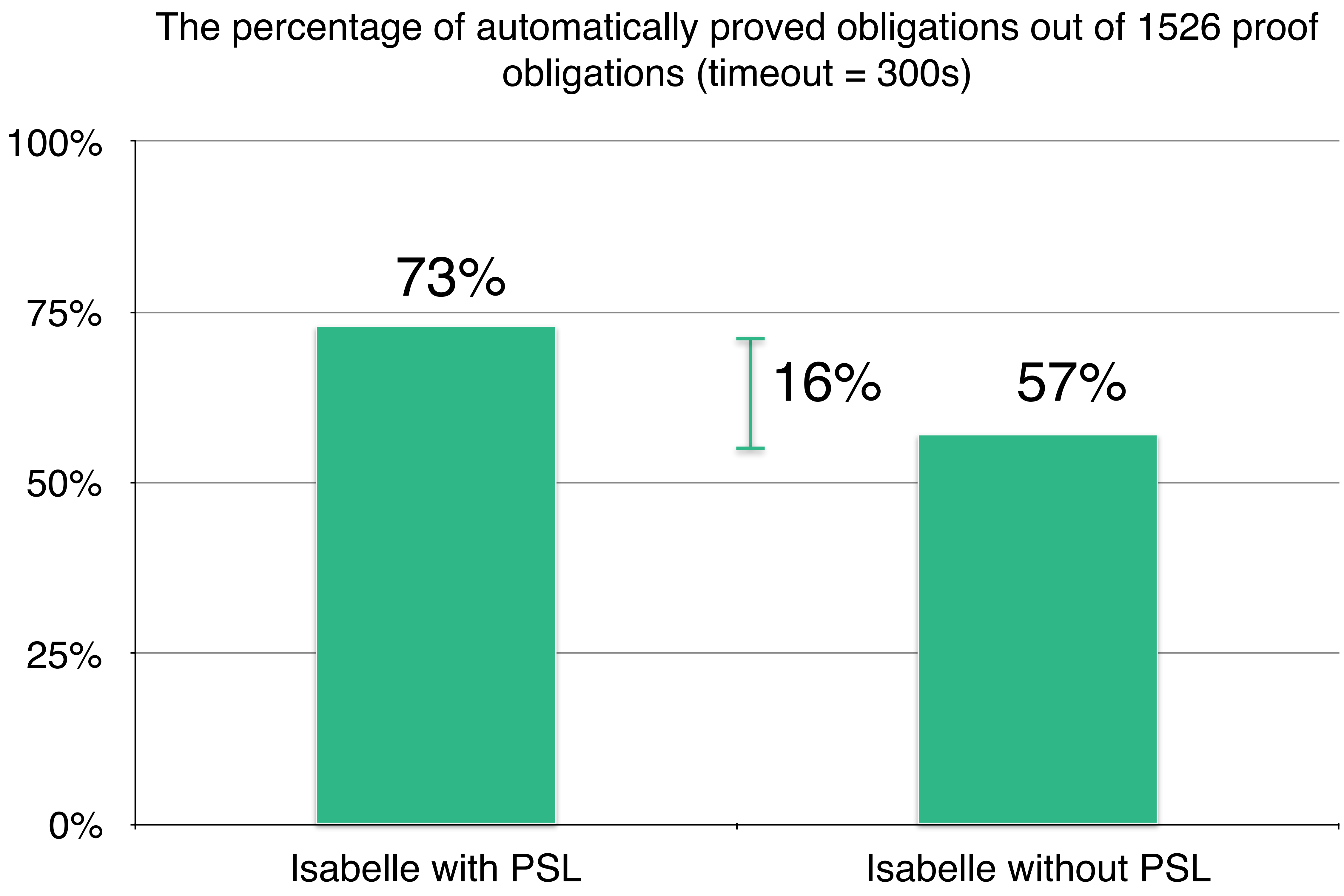
Evaluation



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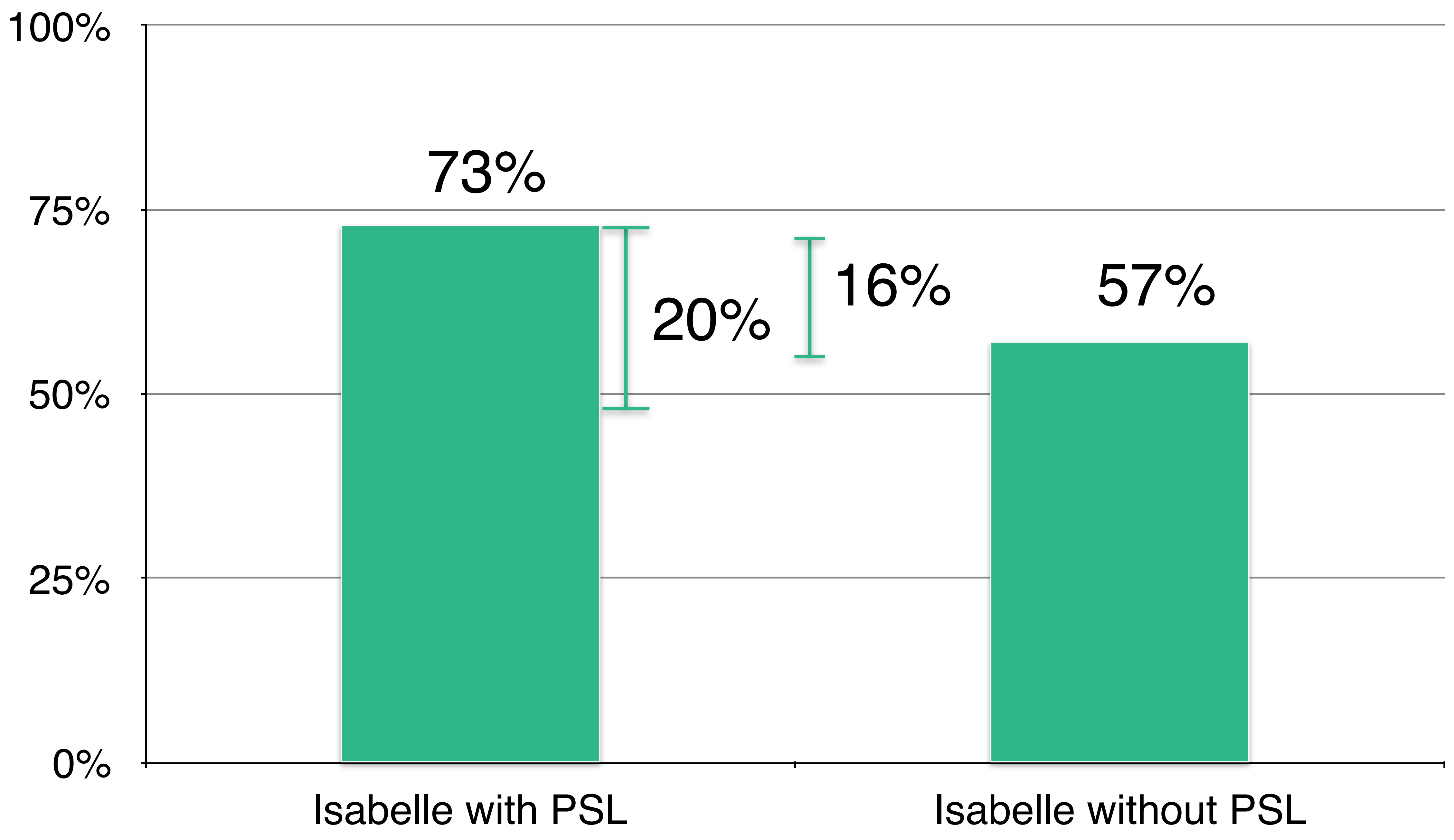
Evaluation

The percentage of automatically proved obligations out of 1526 proof obligations (timeout = 300s)



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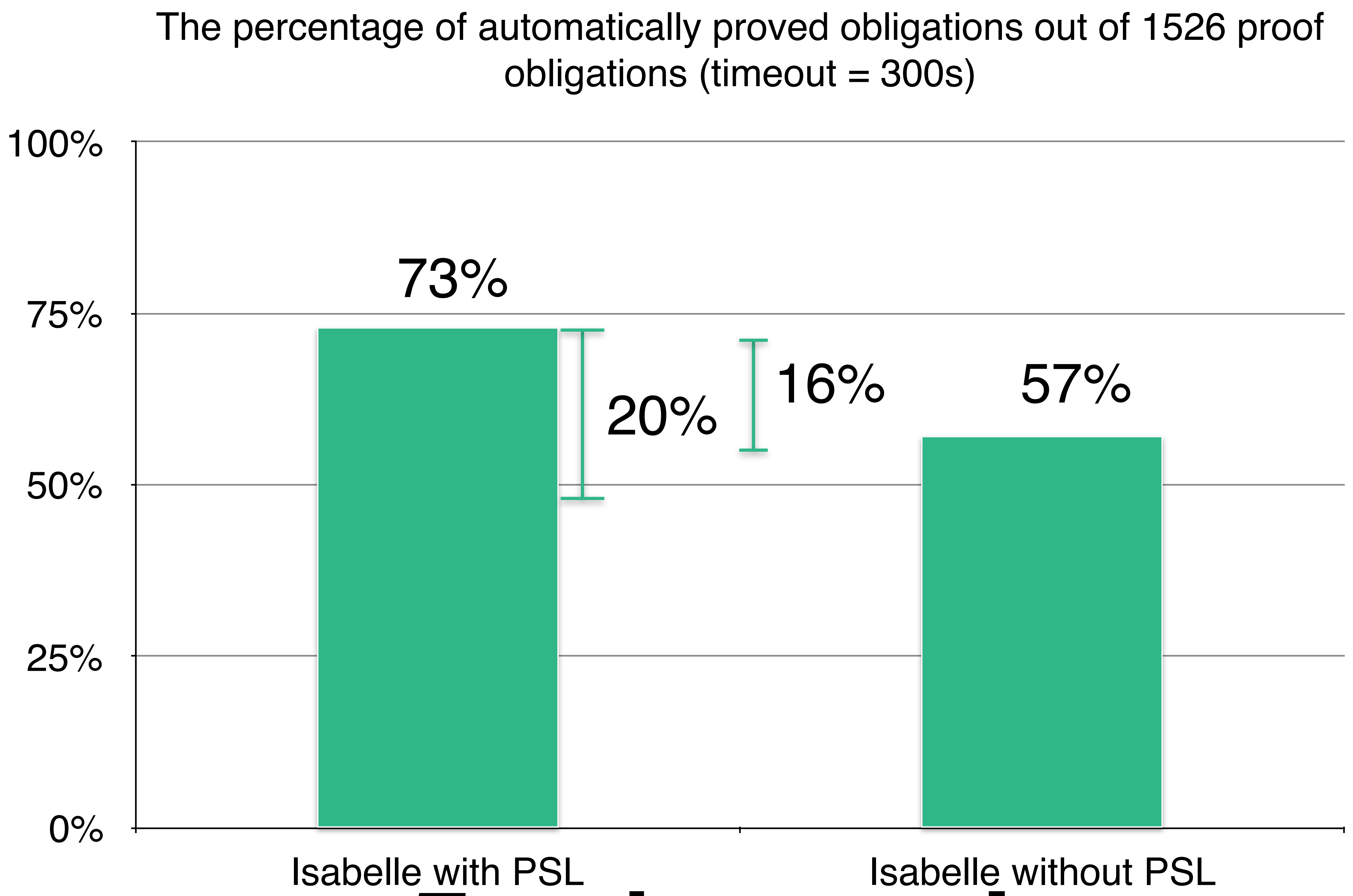
Evaluation



5 minutes!