On the use of gradient space eigenvalues for rotation invariant texture classification

M.J. Chantler and G. McGunnigle
Department of Computing and Electrical Engineering, Heriot-Watt University, Edinburgh, Scotland
M.J.Chantler@hw.ac.uk, http://www.cee.hw.ac.uk/~mjc

Abstract

Many image-rotation invariant texture classification approaches have been presented previously. This paper proposes a novel scheme that is surface-rotation invariant. It uses the eigenvalues of a surface's gradient-space distribution as its features. Unlike the partial derivatives, from which they are computed, these eigenvalue features are invariant to surface rotation. First we show that a simple classifier using a single isotropic feature (grey-level standard deviation) is not invariant to surface rotation. Then a practical surface rotation invariant classifier that uses photometric stereo to estimate surface derivatives is developed. Results for both classifiers are presented.

1. Introduction

Many texture classification schemes have been presented that are invariant to *image* rotation [1,2,3]. They either employ isotropic features that are *insensitive* to directional information, or they use some relative directional characteristics that are *invariant* to image rotation. In either case they derive their features directly from a single image and are tested using rotated *images*. If the image texture results solely from albedo variation rather that surface relief, or if the illumination is not directional or immediately overhead; then these schemes are sufficient for surface rotation as well.

However, in many cases rotation of a textured surface produces images that differ radically from those provided by pure image rotation. This is mainly due to the directional filtering effect of imaging using side-lighting [4].

In this paper we use a model of the surface-to-image transfer function [5,6] to predict that the output of a simple isotropic feature (grey level standard deviation) is not invariant to surface rotation unless the surface is isotropic.

Following this we present our classification scheme that uses surface relief characteristics directly. The surface's partial derivatives are estimated using photometric stereo. These features are not surface-rotation variant, therefore we compute the gradient-space eigenvalues which are.

Results for the isotropic and the eigenvalue-based classifiers are presented.

2. Image variance as a function of image rotation

This section presents a frequency domain image model and uses it to develop an expression for image variance as a function of relative illuminant tilt angle.

A. Imaging Geometry

The imaging geometry assumptions are as follows. The test surface is mounted in the *x*-*y* plane and is perpendicular to the camera axis (the *z*-axis). It is illuminated by a point source located at infinity, i.e. the incident vector field is uniform in magnitude and direction throughout the test area. The direction of the illuminant with respect to the texture is defined by two polar co-ordinates: slant and tilt. Slant (σ) is the angle between the camera axis and the illuminant, while tilt (τ) is the angle that the illuminant direction makes with the *x*-axis in the *x*-*y* plane. Surface rotation is measured in the *x*-*y* plane using the variable φ. For a directional surface it is taken as the angle between the *x*-axis and the angle of the most dominant surface directionality. It is the effect of surface rotation (i.e. variation of φ) that is of most interest here.

B. Image Model

Kube and Pentland presented a frequency domain model of the formation of the image from the surface [5]. The most significant assumptions are that: the surface reflectance is Lambertian, there is no self or cast shadowing, the camera model is orthographic and that surface slope angles are small. Chantler [6] modified and generalised this to a form similar to that shown below (1).

$$I(\omega,\theta) = \omega^2 \cos^2(\theta-\tau)\sin^2(\sigma)S_\varphi(\omega,\theta) \tag{1}$$

where

(ω,θ)	is the polar form of spatial frequency with θ = 0° being the direction of the *x*-axis.
$I(\omega,\theta)$	is the image power spectrum.
$S_\varphi(\omega,\theta)$	is the surface power spectrum of a surface orientated at φ.

In this paper the most important aspect of (1) is the $\cos^2(\theta-\tau)$ term; this is directional filter dependent upon the tilt angle (τ) of the illuminant. It is this directional filtering effect that makes the output of an *isotropic* texture feature vary with the orientation of a directional surface, as is shown in the next section.

C. Predicting the Variation of Image Variance with Surface Rotation

From (1) the isotropic texture feature: image variance, may be expressed as a function of surface rotation:

$$f_1(\varphi) = \int_0^{\infty} \omega^2 \sin^2(\sigma).2\int_0^{\pi} \cos^2(\theta - \tau).S_{\varphi}(\omega,\theta).d\theta.d\omega \qquad (2)$$

Now choosing a new set of axes aligned with the direction of the texture such that $\theta_{new} = \theta - \varphi$, (2) becomes:

$$f_1(\varphi) = \int_0^{\infty} \omega^2 \sin^2(\sigma).2\int_{-\varphi}^{\pi-\varphi} \cos^2(\theta_{new} + \varphi - \tau).S(\omega,\theta_{new}).d\theta_{new}.d\omega \qquad (3)$$

where $S(\omega,\theta_{new})$ is the surface power spectrum of the surface in the new axes system. Now, considering the inner integral of (3)

$$= 2\int_{-\varphi}^{\pi-\varphi} \cos^2(\theta_{new} + \varphi - \tau).S(\omega,\theta_{new}).d\theta_{new}$$

$$= \int_0^{\pi} (1 + \cos(2\theta_{new} + 2\varphi - 2\tau)).S(\omega,\theta_{new}).d\theta_{new}$$

$$= \int_0^{\pi} S(\omega,\theta_{new}).d\theta_{new} + \int_0^{\pi} \cos(2\theta_{new} + 2\varphi - 2\tau).S(\omega,\theta_{new}).d\theta_{new}$$

$$= \int_0^{\pi} S(\omega,\theta).d\theta + \lim_{n\to\infty} \sum_{i=0}^{i=n-1} \cos(2i\Delta\theta + 2\varphi - 2\tau).S(\omega,i\Delta\theta)\Delta\theta$$

$$where \quad \Delta\theta = \pi/n$$

With respect to surface's rotation angle φ, the above summation is simply a series of cosines of varying amplitude and phase but of constant frequency (2φ). A similar argument holds for the outer integral of (3). Hence image variance is

$$f_1(\varphi) = a + b\cos(2\varphi - 2\tau) \qquad (4)$$

where a, b are constants, and

$$a = \sin^2(\sigma)\int_0^{\infty} \omega^2 \int_0^{\pi} S(\omega,\theta).d\theta.d\omega$$

from (4) an alternative isotropic feature may be derived: the standard deviation of an image:

$$f_2(\tau) = \sqrt{a + b\cos(2\varphi - 2\tau)} \qquad (5)$$

If the surface is isotropic then the b will evaluate to zero and only the constant a term will be left. However, for directional surfaces b will be non-zero and hence image variance will be a sinusoidal function of surface rotation.

Experimental verification

Figure 1 shows images of two directional textures obtained at three angles of rotation.

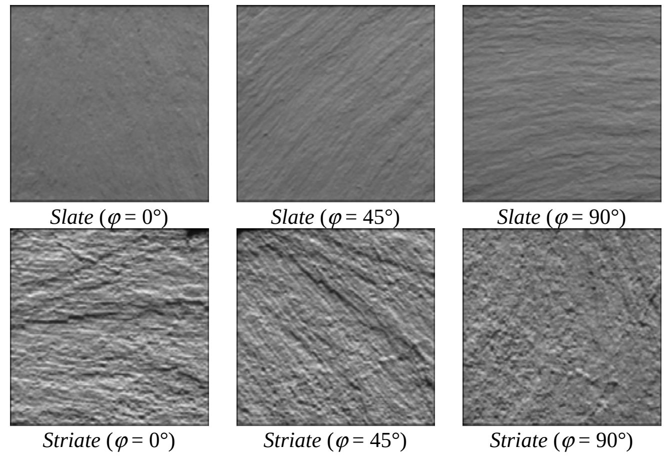


Figure 1 –Striate and Slate textures at three rotation angles

It can be seen that these images are very different from those that would result from i*mage* rotation alone.

Figure 2 shows the variation of standard deviation that occurs when these surfaces are rotated. It can be seen that for these two directional textures, image variance (which is an isotropic feature) is certainly not invariant to surface rotation, and that the plots are sinusoidal in nature.

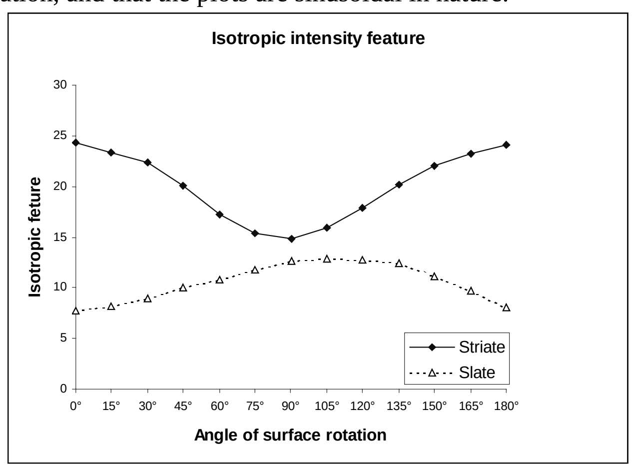
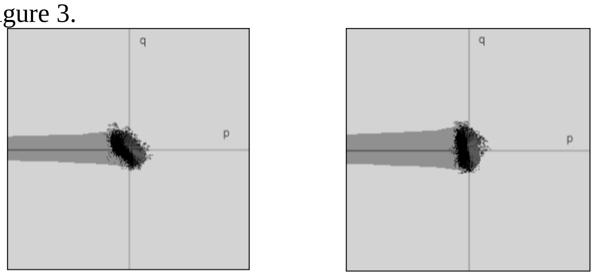


Figure 2 - Variation of standard deviation with surface rotation

3. A surface rotation invariant classification scheme

If a classifier is to be robust to surface rotation then it would be better to use features that are a function of topography rather than image intensity. Photometric stereo [7,8] provides a means of estimating such information. It uses multiple images of the same scene obtained under different illumination orientations. When applied to Lambertian surfaces it provides estimates of three quantities for each facet: the facet albedo (ρ), and the facet's gradient in terms of its partial derivatives

$p = \partial z/\partial x, q = \partial z/\partial y$. Here we will assume constant albedo and only use partial derivative information. The actual scheme that we use is fully described in [9]. Unfortunately the partial derivatives are not invariant to rotation of the surface. Rotation of the surface results in an equivalent rotation of gradient (*p/q*) space as is shown in Figure 3.



Slate ($\varphi = 45°$) *Slate* ($\varphi = 90°$)

Figure 3 - Scatter plots of *p* and *q* partial derivatives estimated from Slate at rotations of 45° and 90°

We therefore adopt the following approach: at each pixel, a local estimate of the bivariate (*p*, *q*) gradient distribution is made. The eigenvalues (λ_1 and λ_2) of this distribution are calculated and these form the basis for classification. The approach is illustrated in **Figure 4** and [10]. Although the distributions may rotate the eigenvalues will be constant. A classifier based on this approach, which we shall call a λ or *eigenvalue classifier*, should therefore be rotation invariant.

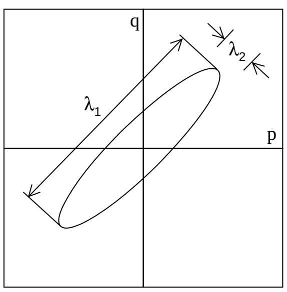
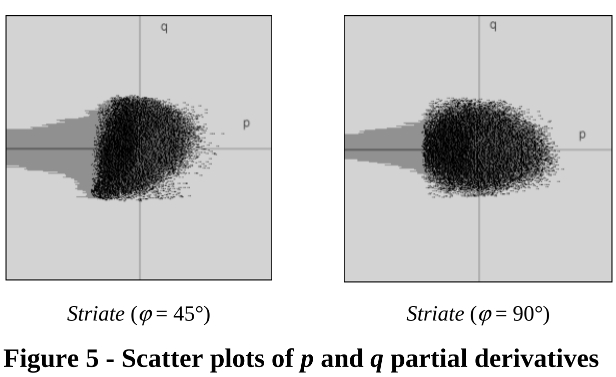


Figure 4 – Eigenvalues shown in gradient-space

Note that the absolute values of the eigenvalues give a measure of the roughness of the surface, while their relative values give an indication of the degree of the surface directionality. Compare, for instance, the gradient distributions of *Striate* (Figure 5) and *Slate* (Figure 3).



Striate ($\varphi = 45°$) *Striate* ($\varphi = 90°$)

Figure 5 - Scatter plots of *p* and *q* partial derivatives estimated from Striate at rotations of 45° and 90°

Results

This section presents two types of results. The first are simply the behaviours of the isotropic and eigenvalue features. The second set concerns classification performance. Here the isotropic and eigenvalue features are used to construct two classifiers. Each classifier uses similar post processing: a Gaussian smoothing filter is used to reduce the variance of the features and the results are fed into a linear discriminant which is derived from the class feature statistics [9].

Section 2 showed that an isotropic texture feature, grey level standard deviation, *is* sensitive to surface rotation. In contrast the Eigenvalue classifier's features have been designed to be robust to such variations. Plots of the first eigenvalue against surface rotation are shown in **Figure 6**. They show that it is much more stable than the standard deviation feature (compare against Figure 2).

The stability of the eigenvalue features vs. image variance is apparent in the classification results shown in Figure 7 and Figure 8. The intensity classifier rapidly fails as the surfaces are rotated. However, the λ (eigenvalue) classifier does not.

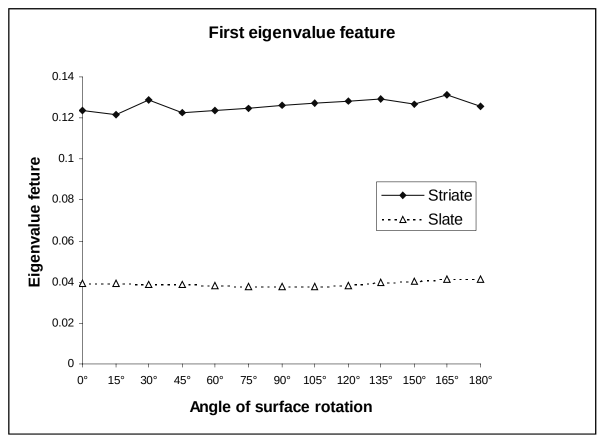


Figure 6 - The variation with rotation of the first λ feature mean for *Striate* and *Slate* surfaces

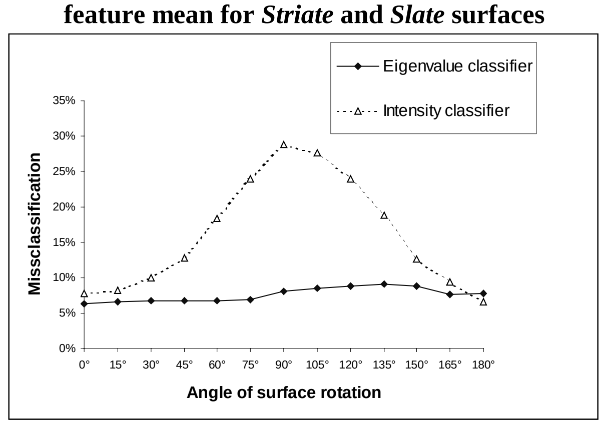


Figure 7 - Comparison of classifiers for rotation of *Striate* and *Slate* surfaces

4. Conclusions

This paper has shown that an intensity-based isotropic texture feature (i.e. grey level standard deviation) is not surface rotation invariant when used on anisotropic surfaces.

Furthermore we have presented a novel texture classifier that uses photometric stereo to obtain surface gradient data. This data, which is rotation sensitive, is used to provide estimates of the gradient-space eigenvalues. The result is a classifier which is *surface rotation invariant.*

References

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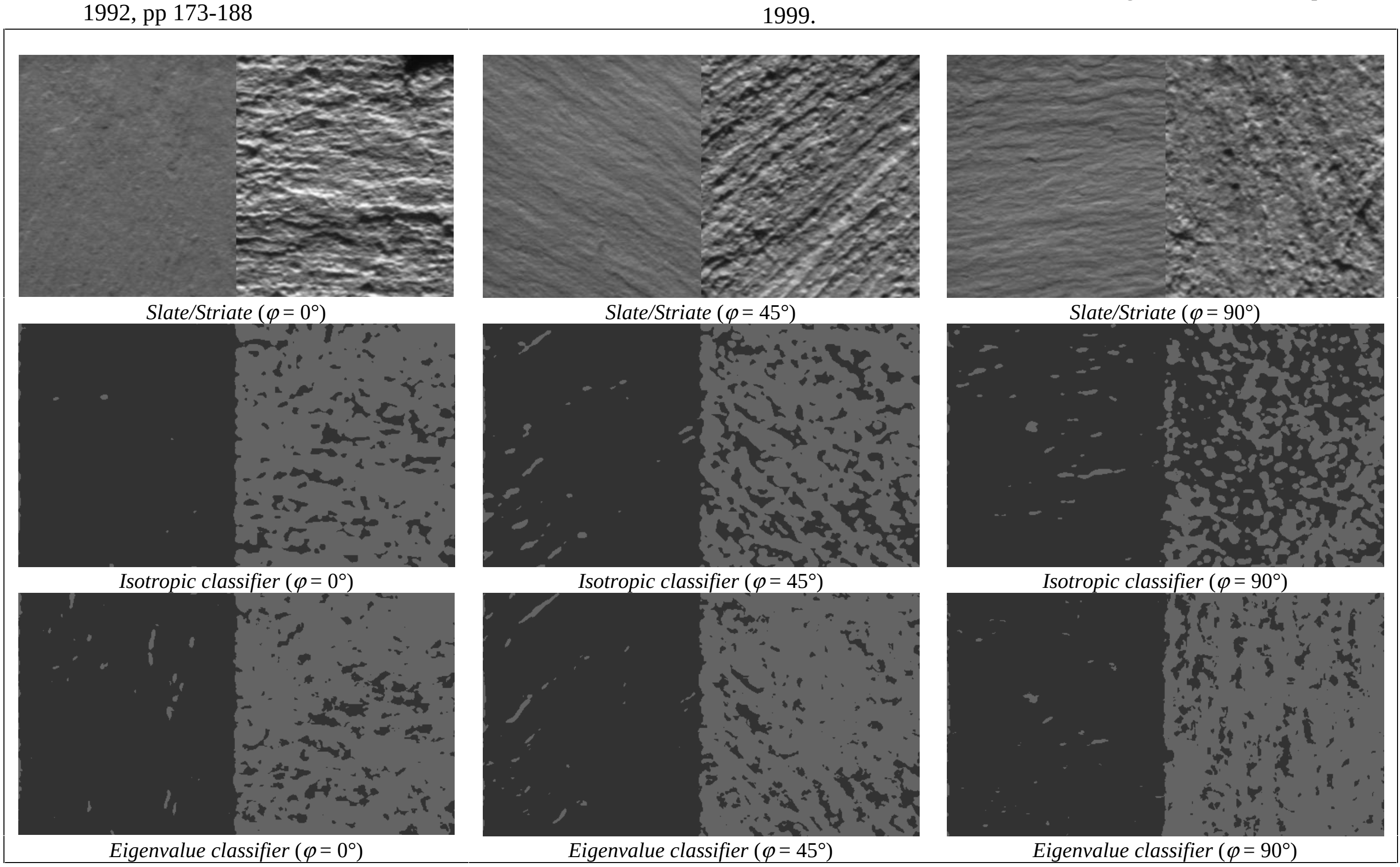

Figure 8 – Original montages, and sample results from Isotropic and Eigenvalue classifiers shown at three angles of surface rotation