Objective Measures of Apparent Order of Point Patterns

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ABSTRACT
Patterns of points in the plane can look more or less ordered. An objective measure of the degree of order would be useful in developmental biology. Candidate measures of order were assessed for their agreement with human judgements. The judgement data was collected using a 2AFC task using comparison between pairs of point patterns. No measure agreed with subject data as often as subjects agreed with each other.

Categories and Subject Descriptors
H.1.2 [**Information Systems**]: User/Machine Systems – Human information processing

General Terms
Experimentation, Human Factors

Keywords
Order, regularity, psychophysics, point patterns

1. INTRODUCTION
There is a considerable body of mathematics to describe the ways in which patterns of points in the plane can be ordered [8]. These formulations characterize, using the language of Groups and Transformations, types of order which are *exact*. However, there is no agreed mathematical formulation of points being in *approximate* order, nor of the *degree* of order of such points. Introspection suggests that one can make assessments of degree of approximate order without great difficulty, and such assessments are frequently used in diverse areas of science. One area is in developmental biology, where the arrangement of cells in a developing tissue becomes more ordered over time [4] and the high degree of order eventually attained is crucial to the effective functioning of the organism [14]. Biologists studying such phenomenon would benefit from objective measures of order, so that their time course could be plotted and hypotheses about the disruptive effect of mutations could be rigorously tested [5]. A particular motivating example for the research reported here is work on the development of the regular pattern of mechanosensory bristles on the notum of drosophila (see figure 1). There is a real need in this research for a quantitative measure of the approximate order of a pattern of points [4].

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There has been substantial research on the perception of symmetry and periodicity of 2D patterns and textures [2] [13] [6] [3] as well as on the perception of glass patterns (random dot stimuli that generate a percept of global structure) [11]. However, despite some interesting findings on the after-effect of perceived regularity [10] and texture regularity [9], little work has been done on identifying a measure of approximate order that correlates with human perception [12]. In this paper we report findings from an initial study into this. Our ultimate goal is to discover a measure that agrees with the majority judgement of subjects on which patterns are more ordered. An intermediate goal is for a measure that agrees with the judgements of subjects as often as different subjects agree with each other.

2. METHODS
We first made a list of candidate measures of order of point patterns based on features of the patterns such as point-to-point distances, nearest neighbour distances, numbers of nearest neighbours, spacing around points and angular regularity. To identify nearest neighbours we used the Delaunay Triangulation [7]. Next, we collected data on apparent order using a two-alternative forced choice task in which subjects indicate which of a pair of point patterns is the apparently more ordered. Finally, we assessed how well our candidate measures of order agreed with the judgements of our subjects.

There is an unbounded variety of point patterns which could be used in such an experiment. We have limited ourselves to patterns similar to those encountered in the specific biological problem that motivated this work. Specifically, we constructed patterns in a multi-step process. 1. Choose a base lattice which was triangular, square or hexagonal. 2. Stretch the lattice by a random amount in a random direction. 3. Randomly perturb the position of the points. 4. Remove a fraction of points or add additional points at random position. Finally, we isotropically scale the pattern so that a fixed number (180) of points lies within a particular circle, so that all patterns have the same density of points.

We generated a large set of patterns, using the above method, and then selected twenty of them. We used twenty patterns as this was the largest number such that every subject could view every pair of patterns within an acceptable time scale. The set of twenty was chosen so that it ranged from highly ordered to disordered, was well spaced in apparent order, and was diverse in terms of the base lattice used, and the amounts of perturbation, deletion and addition. Three example patterns are shown in figure 2.

For viewing the patterns we printed them onto circular discs (r=6.2cm) of stiff matt white paper. Points were printed as 1mm diameter solid black dots. Subjects viewed the patterns under comfortable internal illumination, seated at a table whose top was covered in dark grey card.

In each trial the subject took patterns from the top of two stacks, turned them over and compared them. The subjects were free to put them onto the table top for better viewing but typically just held them in their hands. The stacks were shuffled so that random pairs were presented, with each pattern at a random orientation. We are unaware of whether absolute orientation affects perceived order, so in order to negate any possible effects, we used circular cards presented with random orientation and freely rotatable by the subjects. Subjects were instructed to choose the pattern which 'appeared the more ordered' to them. Each subject performed 100 comparisons. Subjects were free to control the pace of the experiment, but all took 10-12 minutes. Twenty subjects participated, 10 male and 10 female.

3. RESULTS

A total of 2000 comparisons were made, but 106 of these were between two presentations of the same pattern, leaving 1894. There are 190 (=20×19/2) pairs of patterns, hence an average of 10 comparisons were made per pair. During the trials, a subject could be presented with the same pattern pair more than once; this occurred 354 times in total. We estimated the *intra-agreement rate* which is the probability that a random subject would choose the same pattern both times when faced with the same random trial twice and which expresses how consistent subjects are, the *inter-agreement rate* which expresses how often two subjects will agree which pattern is the more ordered, and the *oracle rate* which expressed how often subjects agree with an optimal ranking of the patterns (see Table 1).

We computed the value of each of our proposed measures of pattern order on each of the twenty patterns. So that these measures would not be distorted by border effects, the computations focused on the 100 points (out of 180) that were closest to the centre of the containing circle, though the other points were not ignored if relevant to the calculation (e.g. for nearest neighbour distances).
Each measure leads to a specific ranking of the patterns and the fraction of the experimental data which agrees with this ranking defines the performance of the measure. All the candidate measures, along with their performance, are presented in Table 2.

4. DISCUSSION

The 95% confidence intervals for Table 2 are ±2%, so all of our measures perform significantly better than chance, but the performance (73%) of even our best measure falls short of the inter-agreement rate (79%), which was our intermediate goal, let alone the oracle rate (83%) which is our ultimate goal.

Even though the performance of our measures fall short of our goals, we may consider whether the variation in their scores offers any clues towards a better performing measure. The only pattern observable is that the better performing measures all make use of nearest neighbour concepts.

Setting aside our candidate measures, agreed before the experiment, one can attempt to discover post hoc measures of order that better agree with the experimental data. A measure we have found that is both simple and performs well is: the minimum distance between two points in the pattern. This performs at 78%, very close to our intermediate goal of the inter-agreement rate (79%). However, the good performance of this post hoc measure seems to be a case of over-fitting. This is shown by constructing pairs of patterns that easily could have been in our set of twenty, but were not, for which the measure makes obviously poor predictions (see figure 3).

We currently plan a repeat of the experiment using an extended set of candidate measures, including ones arrived at from post hoc analysis of the first experiment (e.g. the minimum distance measure). We will also include patterns specifically targeted at the shortcomings of our proposals (e.g. figure 3).

5. CONCLUSIONS

Analysis of the experimental data confirms a degree of consensus about what is perceived as order. All of the measures of order examined agreed with subjects' judgements of order significantly more often than chance but none as often as subjects agreed with each other.

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7. REFERENCES

[1] Bar-Hillel, M. and Wagenaar, W.A. 1991. The perception of randomness. *Advances in Applied Mathematics*. 12, 4 (Dec. 1991), 428–454.

[2] Baylis, G.C. and Driver, J. 2001. Perception of symmetry and repetition within and across visual shapes: Part- descriptions and object-based attention. *Visual Cognition*. 8, 2 (Mar. 2001), 163–196.

[3] Bonneh, Y., Reisfeld, D. and Yeshurun, Y. Quantification of Local Symmetry: Application for Texture Discrimination. *Spatial Vision*. 8, 515–530.

[4] Cohen, M., Georgiou, M., Stevenson, N.L., Miodownik, M. and Baum, B. 2010. Dynamic Filopodia Transmit Intermittent Delta-Notch Signaling to Drive Pattern Refinement during Lateral Inhibition. *Developmental Cell*. 19, 1 (Jul. 2010), 78–89.

[5] Cordero, J., Jassim, O., Bao, S. and Cagan, R. 2004. A role for wingless in an early pupal cell death event that contributes to patterning the Drosophila eye. *Mechanisms of Development*. 121, 12 (Dec. 2004), 1523–1530.

[6] Dakin, S.C. and Watt, R.J. 1994. Detection of bilateral symmetry using spatial filters. *Spatial Vision*. 8, 4 (1994), 393–413.

[7] Delaunay, B. 1934. Sur la sphère vide. *Izvestia Akademii Nauk SSSR, Otdelenie Matematicheskikh i Estestvennykh Nauk*. 7, (1934), 793–800.

[8] Griffin, L.D. 2009. Symmetries of 2-D Images: Cases without Periodic Translations. *J. Math. Imaging Vis.* 34, 3 (Jul. 2009), 259–269.

[9] Ngan, H.Y.. and Pang, G.K.. 2009. Regularity Analysis for Patterned Texture Inspection. *IEEE Transactions on Automation Science and Engineering*. 6, 1 (Jan. 2009), 131–144.

[10] Ouhnana, M., Bell, J., Morgan, M.J., Solomon, J.A. and Kingdom, F.A.A. 2012. After-effect of perceived regularity. *2012 Vision Sciences Society Meeting* (Florida, May. 2012).

[11] Prazdny, K. 1984. On the perception of Glass patterns. *Perception*. 13, 4 (1984), 469 – 478.

[12] Preiss, A.K. and Vickers, D. 2005. An ecological approach to perceptual organization: Nearest neighbour categorization of dot displays ranging from tightly clustered to highly regular.

[13] Sally, S. and Gurnsey, R. 2001. Symmetry detection across the visual field. *Spatial Vision*. 14, 2 (Jun. 2001), 217–234.

[14] Tilney, L.G. and Saunders, J.C. 1983. Actin filaments, stereocilia, and hair cells of the bird cochlea. I. Length, number, width, and distribution of stereocilia of each hair cell are related to the position of the hair cell on the cochlea. *The Journal of Cell Biology*. 96, 3 (Mar. 1983), 807–821.

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Table 1. Rates for intra-agreement, inter-agreement and oracle

Intra-agreement rate	89%
Inter-agreement rate	79%
Oracle rate	83%

Table 2. Pre-group of candidate measures and their performance

Index	Candidate Measure	Performance (%)
1	Mean distance (over i) between the center of mass of the convex hull of a set of points A_i and the center of mass of the set of points A_i, when A_i is the progressingly inwards convex hull of the whole set of points at step i	58
2	Sum of the reciprocal of distance for all point pairs	65
3	Variance of the area of the cells in the Voronoi diagram	69
4	Variance of the medium distance in the Delaunay triangulation	70
5	Variance of the minimum angle in the Delaunay triangulation	70
6	Variance of the minimum distance in the Delaunay triangulation	71
7	Variance of the maximum angle in the Delaunay triangulation	71
8	Variance of the number of neighbours in the Voronoi diagram	72
9	Variance of the medium angle in the Delaunay triangulation	72
10	Variance of the maximum distance in the Delaunay triangulation	73

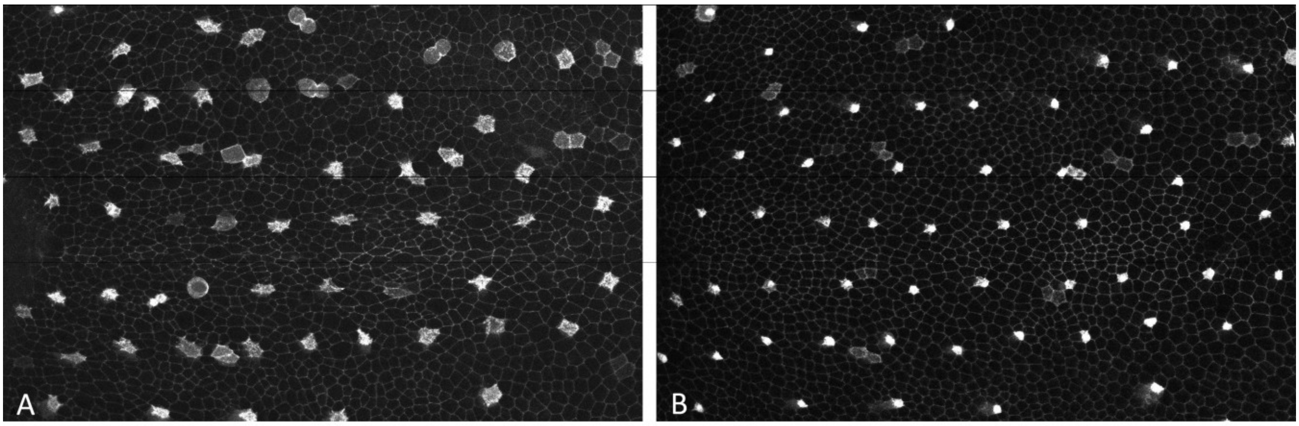

Figure 1. Green fluorescent protein marks cell outlines (E-Cadherin-GFP) and bristle precursor cells (Neu:GFP) in the dorsal thorax of a living fly pupa as it undergoes tissue refinement in the final hours prior to hatching (images from the Baum Lab, UCL). Note how cell organisation and bristle precursor spacing becomes more ordered as development proceeds. (A: previous stage of development, B: final stage of development)

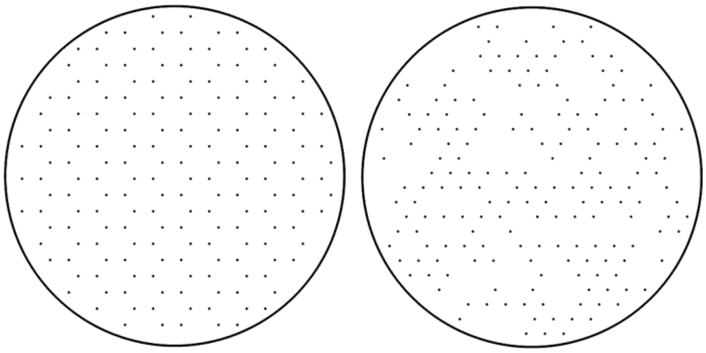

Figure 2. Examples of the patterns used in the experiment. The left pattern is 25% into the sequence of patterns when ordered from more to less ordered, the middle pattern is halfway, and the right pattern is at 75%. The choice of the removed points is completely random; however, the patterns perceived by the remaining points can often give the impression of a non-random selection for the deleted points [1].

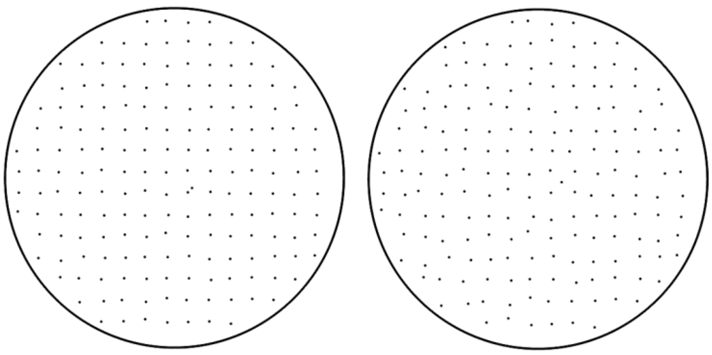

Figure 3. Example of a pair of patterns for which the post hoc identified minimum-distance measure fails. To the authors, the pattern on the left clearly appears to be more ordered than the pattern on the right, but the measure says otherwise.