

# Visual and Material Identity in Natural Scenes: Predicting How Often Indistinguishable Surfaces Become Distinguishable

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## ABSTRACT

If surfaces from a scene are visually indistinguishable under one light, they may become distinguishable under another. The aim here was to test whether the frequency of such metamerism can be predicted by a statistical property of the colours of a scene, namely their conditional entropy. Simulations were based on 50 hyperspectral images of natural scenes under sunlight and north skylight. The correlation between the logarithm of the conditional frequency of metamerism and the conditional entropy of colours was strong, with  $r = 0.80$ – $0.87$ . Additionally, the more likely that indistinguishable surfaces were distinguishable under a different daylight, the more reliable the prediction by conditional entropy.

## Categories and Subject Descriptors

D.3.3 H.1.2 User/Machine Systems: Human information processing

## General Terms

Measurement, Experimentation, Human Factors.

## Keywords

Metamerism, entropy, surface colour, colour differences.

## 1. INTRODUCTION

Colour vision helps inform us of the natural environment. Yet as with any sensing system, the information provided by colour is susceptible to error, not just because of noise, but more fundamentally because complex spectra are reduced to a simple trichromatic description by the cone receptors of the eye (or sensors of a camera). The error reveals itself as metamerism, the phenomenon whereby surfaces that are visually indistinguishable under one light are distinguishable under another [1].

For practical applications, it is useful to express the degree of metamerism in relation to a threshold for distinguishability. Given such a threshold, the frequency of metamerism in a scene (i.e. how often it occurs) is measured by the number of pairs of surfaces whose colour differences are less than threshold under one light and greater than threshold by a certain multiple—the criterion degree of metamerism—under another light [2].

By this measure, metamerism in natural scenes under sunlight and skylight is rare. Estimates of its relative frequency, i.e. as a proportion of all pairs of surfaces, range from about  $10^{-6}$  to  $10^{-4}$  across daylights with correlated colour temperatures (CCTs) of 4000 K and 25,000 K [2]. But these relative frequencies refer to surfaces chosen at random, which, in a chromatically diverse scene, are unlikely to be visually indistinguishable under any daylight. More practically, then, how often do surfaces that are

indistinguishable under one daylight become distinguishable under another daylight? This conditional relative frequency is much higher, ranging from about  $10^{-2}$  to  $10^{-1}$ , enough to affect visual inferences about material identity [2].

Unfortunately, estimating relative frequencies or conditional relative frequencies of metamerism by theoretical methods is difficult not least because of the complexity of the spectra involved. Instead, explicit numerical simulations need to be made in which many pairs of surface colours are tested for metamerism [2]. As an alternative to using numerical simulations for prediction, it is possible to exploit a statistical property of the colours of a scene, specifically their uncertainty or entropy, in the sense of Shannon [3]. Thus, in a previous study [4], when the logarithm of the relative frequency of metamerism across a pair of daylights with CCTs of 4000 K and 25,000 K was regressed on combinations of the estimated differential entropy and conditional differential entropy [3] of colours, the correlation was found to be very strong over scenes, with  $r = 0.84$ – $0.97$ , the smaller values of  $r$  associated with larger criterion degrees of metamerism, i.e. with more extreme colour differences [4].

The aim of this study was to test whether the conditional relative frequency of metamerism could, analogously, be predicted solely by the conditional differential entropy of colours.

## 2. METHODS

The approach followed that in [2, 4, 5]. Spectral-reflectance data were taken from 50 hyperspectral images of natural scenes [2] (eight examples in Fig. 2) and daylight illuminants were simulated from those described by the CIE [6], each defined over 400–720 nm. The colour signal at each point in a scene under a daylight illuminant with CCT of either 4000 K or 25,000 K was represented by its 1931 CIE XYZ tristimulus values [1], which were then transformed into the corresponding tristimulus values for a reference daylight illuminant with CCT of 6500 K according to the CIECAM02 specification with default values [7]. These corresponding tristimulus values were then transformed into CIECAM02 Cartesian coordinates  $J$ ,  $a_c$ ,  $b_c$  [7]. Because of its approximate uniformity, colour differences between points in CIECAM02 were calculated with the Euclidean metric [8, 9].

The conditional relative frequency of metamerism was estimated as in [2]. Under spatially uniform random sampling, 50,000 points (about 15% of the total available) were chosen from each scene, yielding  $N \approx 1.25 \times 10^9$  pairs. From this set, the subset of  $N_0$  pairs with colour differences less than a nominal threshold  $\Delta E^{\text{thr}} = 0.5$  (or  $\Delta E^{\text{thr}} = 1.0$ ) were determined under the first daylight illuminant with CCT of 4000 K. From this subset of  $N_0$  pairs, the  $N_1$  pairs with colour differences greater than  $n\Delta E^{\text{thr}}$ , with  $n = 1, 2, 3, 4$ , were then determined under the second daylight illuminant with CCT of 25,000 K. The conditional relative frequency of metamerism in the scene was estimated by  $N_1/N_0$ .

Because they were sampled randomly, the triplets  $(J, a_c, b_c)$  may be treated as values of a random variable,  $U$  say, with a probability density function  $f$ . The Shannon differential entropy  $h(U)$  of  $U$  is defined [3] as

$$h(U) = -\int f(u) \log f(u) \, du,$$

which is measured in bits if the logarithm is to the base 2. For two continuous random variables  $U_1$  and  $U_2$ , arising from two different daylight illuminants 1 and 2, the conditional differential entropy  $h(U_2 | U_1)$  of  $U_2$  given  $U_1$  is obtained [3] by

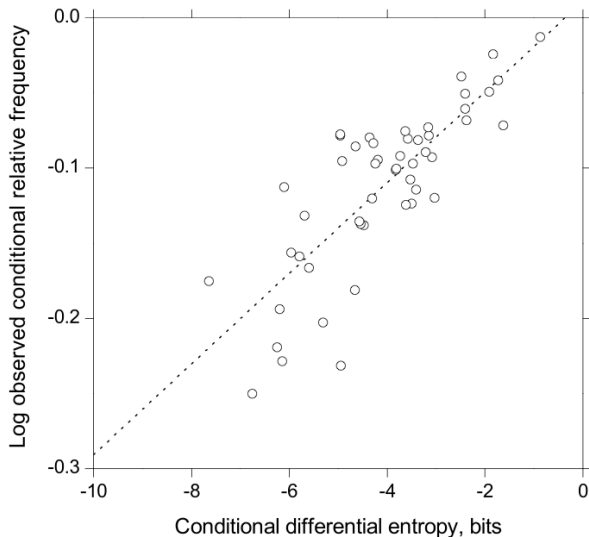
$$h(U_2 | U_1) = h(U_2, U_1) - h(U_1). \quad (1)$$

To estimate these entropies, an asymptotically bias-free,  $k$ -nearest-neighbour estimator due to Kozachenko and Leonenko [10] was used, in a computationally efficient offset form [5].

The observed value of  $\log N_j/N_0$  (logarithm to the base 10) for each scene and pair of daylight illuminants was regressed on the estimated value of  $h(U_2 | U_1)$  over the 50 scenes. Goodness of fit was summarized by  $R^2$ , the proportion of variance accounted for, adjusted for degrees of freedom in the fit [11].

### 3. RESULTS AND COMMENT

Figure 1 shows the logarithm of the observed conditional relative frequency of metamerism plotted against the estimated conditional differential entropy of colours across daylight illuminants with CCTs of 4000 K and 25,000 K. Each point represents data from one of the 50 natural scenes. The nominal colour-difference threshold  $\Delta E^{\text{thr}} = 0.5$  and the criterion degree of metamerism  $n = 1$ . For this condition, adjusted  $R^2 = 0.64$ , giving  $r = 0.80$ . As the criterion degree of metamerism  $n$  increased,  $R^2$  increased, so that with  $n = 4$ , adjusted  $R^2 = 0.75$ , giving  $r = 0.87$  (data not shown). Results were similar with  $\Delta E^{\text{thr}} = 1.0$ .



**Figure 1. Predicting how often an indistinguishable pair of surfaces under sunlight (CCT 4000 K) becomes distinguishable under north skylight (CCT 25,000 K). The logarithm of the observed conditional relative frequency of metamerism is plotted against estimated conditional differential entropy. The dotted line is a linear regression.**

### 4. CONCLUSION

The frequency of failures of visual and material identity in natural scenes can be predicted. For the 50 natural scenes considered here, illuminated in turn by simulated sunlight and north skylight, the correlation between the logarithm of the conditional relative frequency of metamerism and the conditional entropy of colours was strong, with  $r = 0.80$ – $0.87$ , the larger values of  $r$  associated with more extreme colour differences (cf. [4]).

The variations in the logarithm of the observed frequencies about their predicted values were large for small and intermediate conditional entropies (Fig. 1, centre), but they decreased for larger conditional entropies (upper right). Thus, although it is not possible to predict perfectly for a given scene how often indistinguishable surfaces become distinguishable under a different daylight, the more likely that this is to happen, the more reliable the prediction.

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**Figure 2. Eight of the 50 natural scenes used in this study (adapted from Fig. 2 of [2]). Examples of other natural scenes can be found in the references cited in [4].**