CHAPTER 5

Gradient Space

5.1. Introduction

Once the surface properties (p and q) have been captured using photometric stereo, it is useful to initially represent these data in an alternative form, in order to isolate the significant textural feature of interest, especially some rotation invariant features. This may readily be achieved by performing a transformation of those data from surface to the Extended Gaussian Image, further more to the gradient space in spatial domain. In this chapter, we will mainly discuss gradient space domain.

5.2. Extended Gaussian Image

The Gaussian image of a three dimensional object is obtained by mapping surface points to a unit sphere, called the Gaussian sphere, such that points on both the sphere and original object have a corresponding surface normal direction. Hence for curved convex objects, each surface point will map to a unique location on the Gaussian sphere, while objects containing, for example planar regions, will result in object areas represented by a single point on the Gaussian sphere. An alternative representation, known as *the Extended Gaussian Image (EGI)* [Horn79] additionally associated each point on the Gaussian sphere with the inverse of the Gaussian curvature at the corresponding object point. This causes flat planar areas to appear as impulses upon the EGI, where a weight is assigned to each point on the Gaussian sphere equal to the area of the surface having the given normal. An example of such an extended Gaussian image is shown in *Figure 5. 1*.

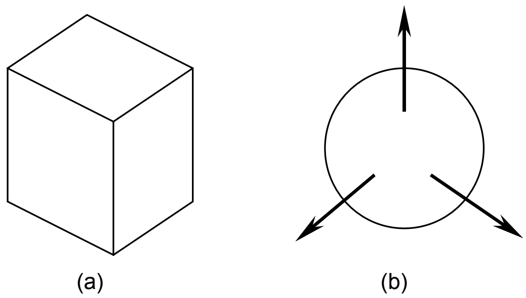


Figure 5. 1 The example of the EGI. (a) an object. (b) The EGI representation of the object in (a).

The extended Gaussian image representation is local since every surface patch on an object corresponds to a point on the sphere. Thus it has the advantage of direct computation from image data, especially when surface normal is easily available. On the other hand, the extended Gaussian image has two disadvantages:

1. The mapping to the Gaussian sphere is unique for convex objects.
2. An infinite number of non-convex objects can possess the same extended Gaussian image. For example in *Figure 5. 1*, those two objects have the same extended Gaussian image.

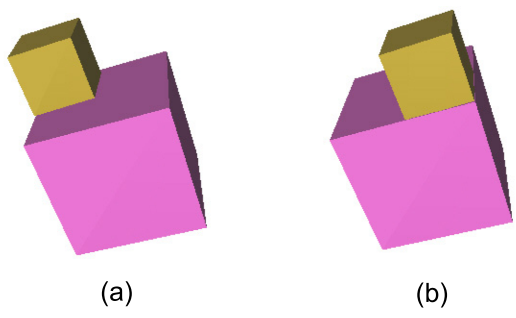


Figure 5. 2 Examples of objects with the same EGL.

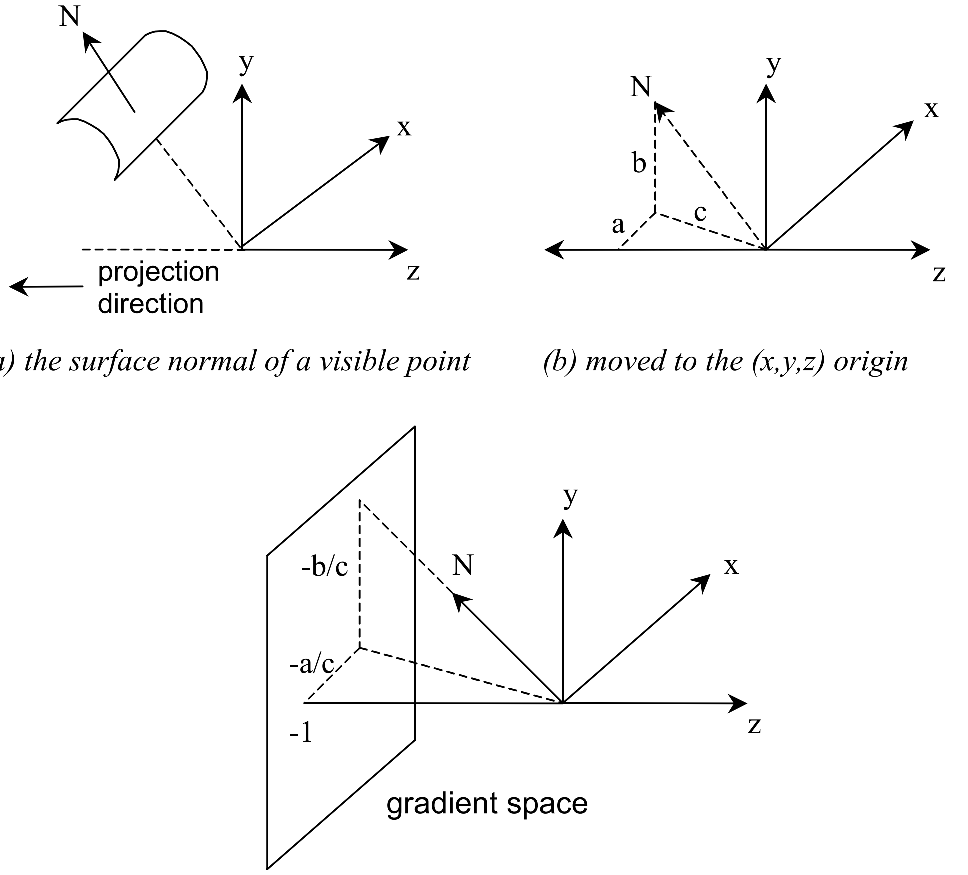
Although the mapping of EGI may be extended to non-convex objects, extended Gaussian image is itself inconvenient to use because of its three dimensional curved surface. However, given that points on the Gaussian sphere specify directions in space, and because orientations has only two degrees of freedom, an alternative representation can be obtained by projecting the visible half of the Gaussian sphere onto an infinite plane, called the *gradient space*. In the next section, we will discuss the gradient space, and in this manner a surface can therefore be represented as a distribution of points in gradient space.

5.3. From Surface Normal to Gradient Space

Gradient space is a two-space representation of the orientation of every point on a surface. It is a scatter plot between surface partial derivatives p and q. We consider that the surface that we are viewing has been projected into a two-space or image plane representation. In addition, we take the projection direction along the $-z$ axis and consider only orthographic projections. Our surface is:

$$z = f(x, y) \tag{5. 1}$$

Every visible point on the surface is projected into the image plane. Each of these points has a world space surface normal and we can represent the orientation of the surface normal of each by a single point in the two-space. This space is called the *gradient space*. *Figure 5. 3* shows the relationship between gradient space and a surface normal.



(a) the surface normal of a visible point *(b) moved to the (x,y,z) origin*

(c) extended to intersect the plane z=-1

Figure 5. 3 The mapping of surface normal data to the gradient space domain.

This shows a surface normal N at a point on a surface. All visible points on this surface will have surface normals with negative z components. To represent such normals in two-space we first extend the vector back to the (x, y, z) origin and assign coordinates (a, b, c) to it (*Figure 5. 3(b)*). If we consider a plane normal to the z axis positioned at $z=-1$ then vector intersects this plane at coordinates (p, q), where

$$\begin{aligned} p &= - a / c \\ q &= - b / c \end{aligned} \tag{5. 2}$$

Now the direction of the normal to surface $z=f(x, y)$ is $(\partial f / \partial x, \partial f / \partial y, -1)$ and we get

$$
\begin{aligned}
p &= \partial f / \partial x \\
q &= \partial f / \partial y
\end{aligned}
\tag{5. 3}
$$

Therefore the surface orientation can be presented in gradient space *G(p, q)* [Horn79] [Smith99a].

5.4. Surface Orientation in Gradient Space

5.4.1. Surface Distribution of Gradient Space

In essence, as already described, the concept of gradient space *G(p, q)* facilitates the mapping of an array of surface normals to a series of co-ordinate points, *(p, q)* within the two dimensional gradient space domain, where p and q describe the surface partial derivatives (the local surface slope). By mapping such an array of surface normals into the gradient space domain, an indication of the global surface normal distribution, and hence global surface shape, can readily be obtained [Smith99a]. Horn has described such a concept for the recognition and attitude determination [Horn79].

Figure 5. 4 shows the distribution of gradient space *G(p, q)* for four synthetic textures and *Figure 5. 5* shows their corresponding 3D frequency distribution.

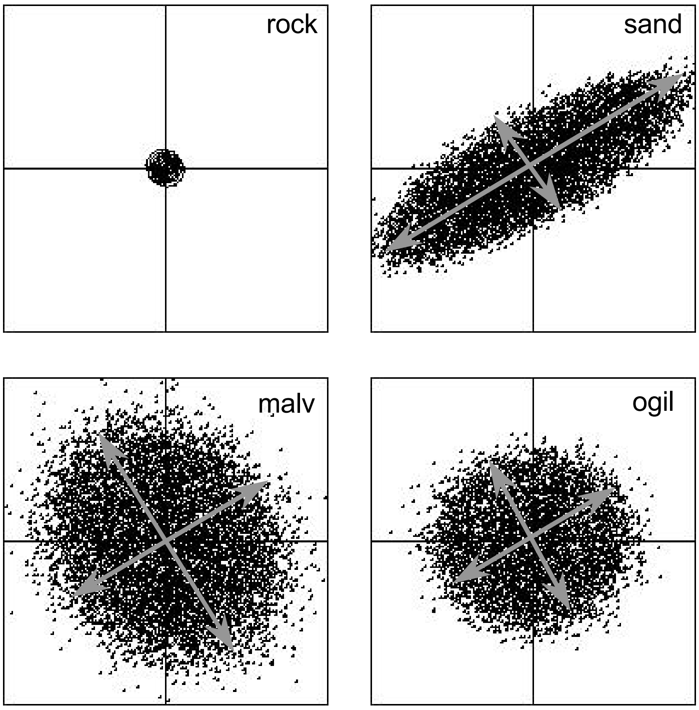


Figure 5. 4   Distribution of gradient space G(p, q) for four synthetic textures.

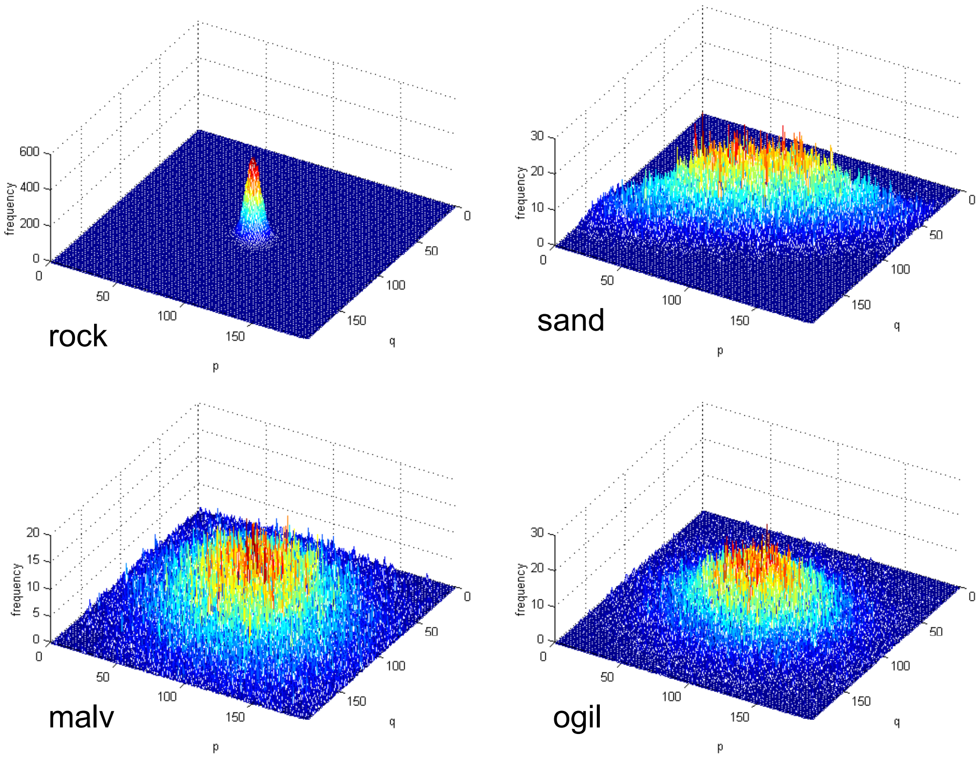


Figure 5. 5   3D Distribution of gradient space g(p, q) for four synthetic textures.

5.4.2. Presentation of Surface Orientation in Gradient Space

The mapping of surface normals data to the gradient space domain would seem to offer a useful mechanism for the representation of surface shape. It will be shown here that the distribution of a given gradient space may in fact be considered to represent an invariant description of the entire observed surface. Examples of gradient space for four synthetic textures at different surface orientations are shown in *Figure 5. 6*.

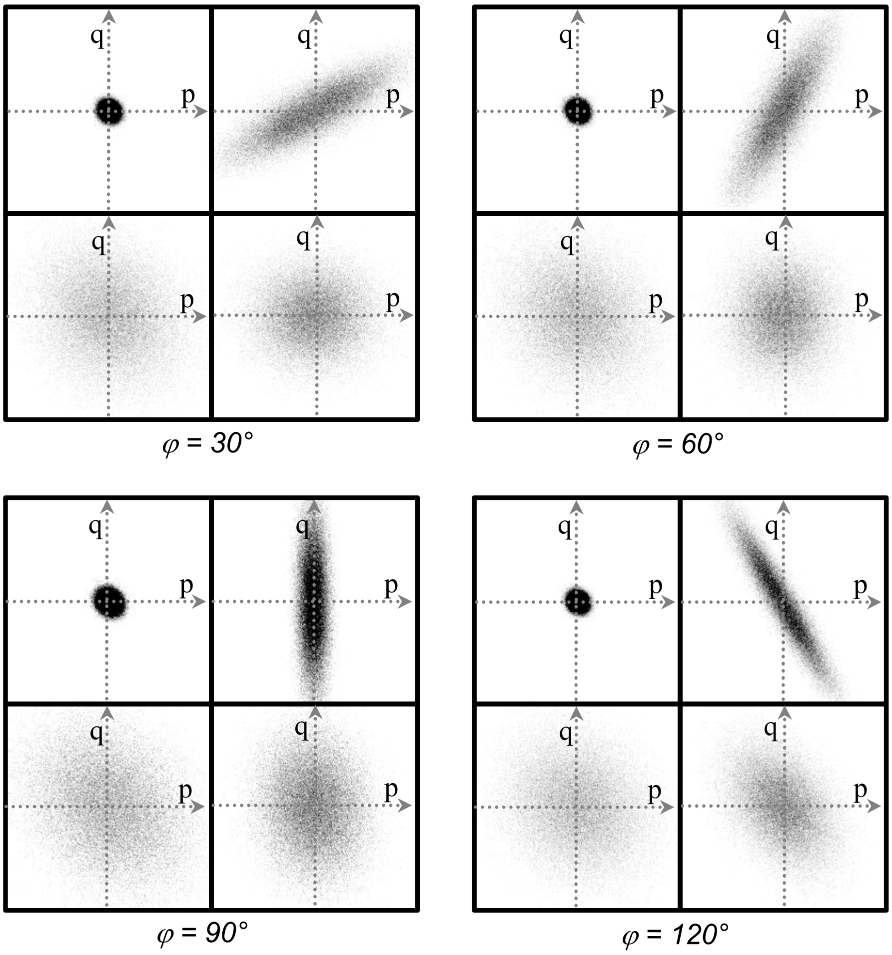


Figure 5. 6 Gradient space G(p, q) for four synthetic textures at different surface orientations φ=30°, 60°, 90° and 120°, shown in montage format. The textures are rock(left-top), sand(right-top), malv(left-bottom) and ogil(right-bottom).

As shown in *Figure 5. 6*, the presence of a texture will be manifest as a spreading of impulsive distribution within the gradient space domain. In order to describe the distribution of gradient space, it is useful to compare the rotation invariant moments among those textures. For example, consider the following two main texture feature parameters calculated by moments:

- *Eccentricity (shape factor) E_m.* Obtained from the ratio principal second order moments.

$$E_m = \frac{E_1}{E_2} \tag{5. 4}$$

Where the principal second order moments are given by:

$$E_1 = \frac{(M_{20} + M_{02})}{2} + \sqrt{\left[\frac{(M_{20} - M_{02})}{2}\right]^2 + M_{11}^2} \tag{5. 5}$$

$$E_2 = \frac{(M_{20} + M_{02})}{2} - \sqrt{\left[\frac{(M_{20} - M_{02})}{2}\right]^2 + M_{11}^2} \tag{5. 6}$$

where M_{ij}'s are second order moments of the gradient space image $G(p, q)$ given in the following:

$$M_{11} = \sum_p \sum_q (pq) G(p, q) \tag{5. 7}$$

$$M_{20} = \sum_p \sum_q (p^2) G(p, q) \tag{5. 8}$$

$$M_{02} = \sum_p \sum_q (q^2) G(p, q) \tag{5. 9}$$

- *Polar moment P_m.*

$$P_m = \sum_p \sum_q (p^2 + q^2) G\ (p, q) \tag{5. 10}$$

Table 5. 1 and *Table 5. 2* show the eccentricity E_m and polar moment P_m of the gradient space for two example synthetic texture *sand* and *malv* in various surface orientations from *0°* to *150°*. We note that both eccentricity E_m and polar moment P_m are rotation invariant moments.

Orientation	***0°***	***30°***	***60°***	***90°***	***120°***	***150°***
sand	4.18E-02	9.69E-02	9.59E-02	4.17E-02	2.84E-02	2.79E-02
malv	6.40E-01	7.15E-01	7.17E-01	6.46E-01	7.13E-01	7.07E-01

Table 5. 1 Eccentricity of gradient space (E_m) for synthetic texture sand and malv in various surface orientations.

Orientation	***0°***	***30°***	***60°***	***90°***	***120°***	***150°***
sand	1.09E+08	1.38E+08	1.30E+08	1.11E+08	1.36E+08	1.26E+08
malv	2.01E+08	1.69E+08	1.65E+08	2.02E+08	1.64E+08	1.66E+08

Table 5. 2 Polar moment of gradient space (P_m) for synthetic texture sand and malv in various surface orientations.

Classification feature space among textures (*rock, sand, malv and ogil*) with respect to various surface orientations (from *0°* to *180°*) can be seen in *Figure 5. 7*. We also note that they are not discriminative enough to enable the classifier to be robust to surface rotation for various texture surfaces. The clusters between texture class *sand* and *ogil* are too close, where the classification may be failed so that we have to seek other more feature spaces being robust to surface orientation and being more discriminative to various texture classes in the next chapter.

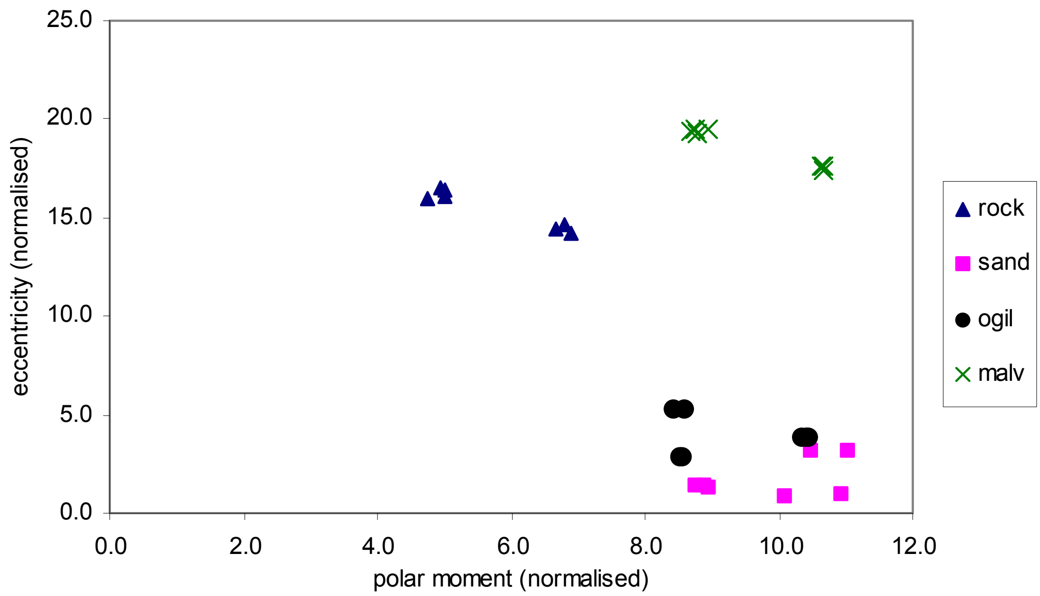


Figure 5. 7   Classification feature space among textures (rock, sand, malv and ogil) with respect to various surface orientations (from 0° to 180°).

5.4.3. Estimate Surface Orientation by Moment

From the *Figure 5. 6*, we can see that the principal axes of gradient space image (especially for texture *sand*) presents the surface orientation. Therefore we may estimate surface orientation by analysing the moment in gradient space. The publication results by Hu [Hu62], Teh and Chin [Teh86] show directly or indirectly that only moments invariants based on the moments of order two are actually almost invariant to rotation. The moments of higher order are so sensitive to digitalisation errors, minor shape deformations, etc., that the corresponding moment invariants can be hardly used for shape identification. In order to illustrate the estimation and to make the step clear, we show as a very easy and well-known example the derivation of the second order central moments invariant to rotation.

We regard the gradient space image as a function of two variables, $G(p, q)$. Therefore the principal orientation (θ) of surface is obtained from the principal axis of the gradient distribution.

$$\tan 2\theta = \frac{2M_{11}}{M_{20} - M_{02}} \qquad (5.\ 11)$$

where the origins of the coordinate have been centred at the centroid of each gradient space image.

In order to test the estimation of surface orientation by moment, we carry out an experiment by using four synthetic texture (rock, sand, ogil and malv). Their surface partial derivatives p and q at surface orientation angle of *0°* are firstly estimated by photometric stereo, and then their corresponding gradient space *G(p, q)* are built as references (training set at surface orientation $\varphi=0°$). Secondly, the test sets of gradient space at surface rotation angles of *30°, 60°, 90°, 120°, 150°* and *180°* are estimated by rotating their surface, respectively. Finally the surface orientation angles θ on both test and training sets are calculated by the moments of their gradient space through above equation.

Figure 5. 8 shows the estimated surface orientation angles *(θ)* for above four synthetic textures obtained from their corresponding gradient space *G(p, q)*. It can be readily seen that only for the directional texture *"sand"*, the estimation processing is achieved a good result, while it fails with the rest of three textures (*rock*, *malv* and *ogil*). This is because the distribution of the gradient space for these three textures (non-directional) does not relate well to the surface orientations and leads to the failures.

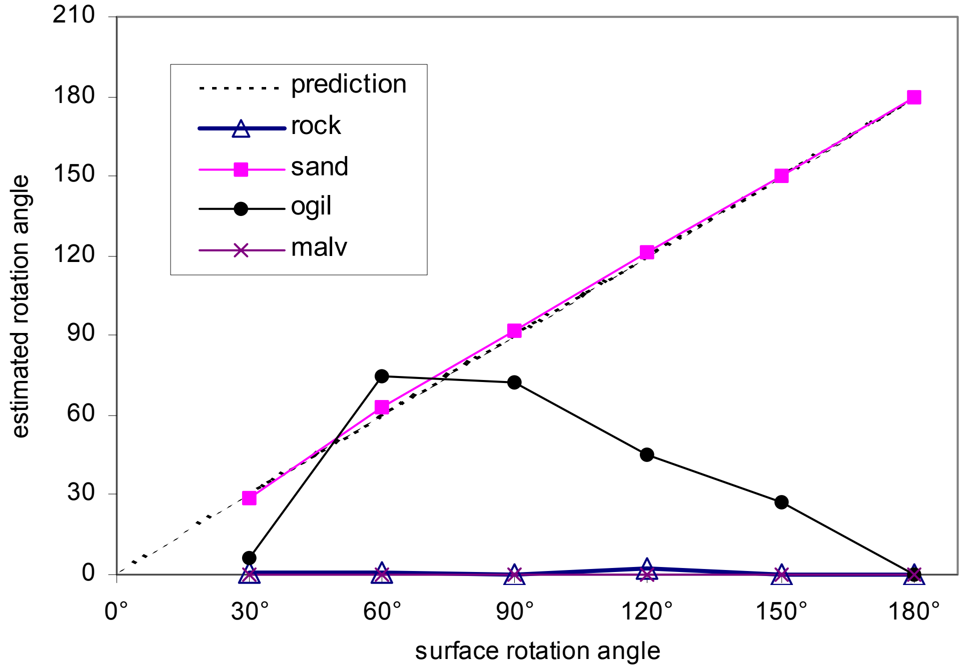


Figure 5. 8 Estimation of the surface orientation angles for four synthetic textures (rock, sand, malv and ogil) obtained from gradient space G(p, q).

We note that the angle θ obtained with equation *(5. 11)* may be with respect to either the major principal axis or the minor principal axis. The correct rotation angle will be $\theta + n\pi/2$ where n is chosen to satisfy the constraints. Problems may occur when a gradient space image is *n-fold* symmetric since there are multiple possible sets of principal axes, such as gradient spaces for texture *rock* and *malv* in *Figure 5. 6*.

We therefore present our new algorithm in next chapter, which is rotation invariant texture classification using gradient space in *frequency domain* rather than the above in *spatial domain*.

5.5. Summary

In this chapter, following by discussing the disadvantages from the Extended Gaussian Image (EGI), which is that the Gaussian sphere is unique for convex objects and an infinite number of non-convex objects can processed the same extended Gaussian image, we therefore introduce gradient space.

In general, the concept of gradient space *G(p, q)* facilitates the mapping of an array of surface normal to a series of coordinate points *(p, q)* within the two-dimensional gradient domain, where *p* and *q* describe the surface gradient in two orthogonal degrees of freedom at a given location. By mapping such an array of surface normals into gradient space, the global surface normal distribution can readily be obtained. The utilisation of gradient space mapping offers an advantage in terms of surface orientation dependence. Further, the distribution shape of gradient space about the plot centroid will remain constant, and the magnitude and directional distribution of surface gradient within the space becomes clearly visible. Hence the character of the distribution can be used to classify the texture.

We analysis the gradient space using various moments. The eccentricity moment and polar moment are considered to provide the indication of gradient space distribution. Both of them are rotation invariant features with respect to surface orientations, however, they are not robust to the various surface texture classes. In addition, we also estimate surface orientation using second order moments of principal axes in gradient space. However the estimation process on non-directional texture surfaces has been failed, because the problems may occur when a gradient space image is *n-fold* symmetric where there are multiple possible sets of principal axes.

With regard to the surface rotation invariant classification, there are some difficulties associated with the surface derivatives in gradient space *G(p, q)* as feature spaces. The first is that the surface derivatives *p* and *q* are vectors rather than scalar quantities. We will have to use these two surface derivatives together in classifier, rather than one scalar quantity. Recovery of the surface height map by integrating the surface derivatives may yield a scalar field which can be directly incorporated into an existing classifier. However the integration error may be well increase via accumulation. The other difficulty is that there is a direction-related factor, which is an artefact of the partial derivative operator. In this case, this directionality component should be removed before classification. In general, the gradient space $G(p, q)$ will not subsequently use in our classification scheme. Those matters are discussed in the next chapter.

In next chapter, we proposes a method by which the partial derivatives may be combined in the frequency domain in such a way as to remove these directional artefacts. Note that the gradient space $G(p, q)$ is a scatter plot of surface partial derivatives p and q presenting in the spatial domain, it is not the inverse Fourier transform of the measure gradient spectrum $M(\omega, \theta)$ presented in the next chapter.