Chapter 2

Image models of topological texture

This chapter surveys possible sources of mathematical models or empirical studies that would enable predictions to be made about the effect of illuminant vector variation on images of topological texture. It reviews models of image texture in general, and places particular emphasis on models that define image characteristics in terms of topological texture and illumination parameters. The latter type will be referred to as *image models of topological texture.* This terminology is necessary in order to differentiate such models from the purely two-dimensional *image texture models,* used extensively by the texture classification community, and the *"three-dimensional texture models"* — models of albedo texture on three-dimensional surfaces used by computer graphics and shape from texture researchers [Cohen91c] [Patel91].

Thus the term *image model of topological texture* is exclusively used to refer to a model that, given certain characteristics of the physical surface together with a description of the illumination and the viewer's position, can be used to predict characteristics of texture in the image.

Four areas of research would seem to be likely candidates for the development of such models :

(i) texture synthesis (mainly used in computer graphics to add realism to images),

(ii) texture segmentation and classification,

(iii) shape from texture, and

(iv) scattering theory.

Each of these areas will now be reviewed in turn with the objective of identifying sources of suitable theory or empirical studies, that will enable predictions to be made about the behaviour of texture under varying illumination conditions. These reviews are brief — as there is surprisingly little in the way of published literature on the effects of lighting on image texture in the first three areas, while the last is not directly applicable. These review sections are followed by an examination of one model in detail, and the chapter concludes by considering the implications that this model has for texture analysis.

2.1. Review

2.1.1. Texture synthesis

One of the most frequent criticisms of early computer graphics was the lack of realism due to the apparent smoothness of the three dimensional surfaces portrayed. It is not surprising therefore that one of the main uses of texture synthesis has been to improve the realism of such graphics. Heckbert [Heckbert86] surveyed texture mapping techniques which are concerned with mapping *two-dimensional* arrays or functions of texture onto screen space according to the three-dimensional surfaces contained in object space. Texture mapping is most commonly used to modulate surface colour [Blinn90] and for "bump mapping" i.e. surface normal perturbation [Blinn78] [Haruyama84] [Baston75]. The former treats texture purely as a set of surface markings, while the latter provides a simplified way of imitating the effects of topological texture (occlusion and shadowing are ignored). Blinn [Blinn90] states that surface marking based schemes produce images that look like smooth surfaces with photographs of wrinkles glued on — as the *light source directions* are rarely the same in the original texture map and graphics model. He notes that the effect of wrinkles on intensity is primarily due to variation of the surface normal, and therefore goes on to develop a texture scheme based upon *small perturbations* of surface normals. His results are extremely realistic, and justify his assumption that the major effects of topological texture (consisting of small perturbations) can be modelled solely as variations of the surface normal. This assumption is also made in a later section of this chapter, which presents an image model of topological texture due to Kube and Pentland [Kube88].

An alternative to texture mapping was first developed by Gagalowicz and Ma [Gagalowicz86]. Their model-based approach essentially parameterises a planar texture model (based on second order statistics) with three-dimensional spatial parameters.

Synthesis is thus performed directly on the surface and avoids the mapping process above. Cohen and Patel developed a similar model-based approach but they used the more parsimonious Markov random field (MRF) model [Cohen91c] [Patel91] [Patel93]. Their "three-dimensional texture model" is a two-dimensional MRF model, with additional surface shape parameters, that enables the foreshortening effects due to surface orientation and perspective projection to be taken into account. Neither of these approaches are however of direct interest to this survey, as they do not consider topological texture or illuminant effects.

A very popular area of computer graphics that does use models of topological texture, and does take illumination into account, is that of fractals [Mandelbrot85]. Spectacular "natural" images have been generated by Voss [Voss88], Saupe [Saupe88], Bouville [Bouville85] and others, using random fractals. These researchers are primarily concerned with the appearance of the final image, and while they do use stochastic topological texture models, and do explicitly take into account illumination, they have not in general developed corresponding image models. Pentland [Pentland84] [Pentland86] in his shape from shading work did however investigate such a model, and this is discussed in the *Shape from texture* section of this review.

Boulanger, Gagalowicz, and Rioux [Boulanger89] also used topological texture models – but for data compression purposes. They recreated the appearance of surface texture on museum artefacts using an autoregressive model and Lambertian shading model, but, as for the majority of the fractal work, they were primarily concerned with the appearance of the resulting image and its image model was therefore not investigated.

To summarise : texture synthesis researchers have explicitly considered and used models of topological texture (e.g. fractals) and have taken into account lighting conditions. However, as their primary concern is the appearance of the final image, they have no requirement or motivation to develop mathematical models of the resulting image texture. Cohen et al, and Gagalowicz et al, developed texture models that incorporate surface orientation and camera projection parameters. However, they did not take lighting effects into account and their texture models are two-dimensional.

2.1.2. Texture analysis - segmentation and classification

Random field models have been used for the synthesis of perfect test textures with known and consistent characteristics for the testing of texture segmentation and classification algorithms. The models themselves have also been used as the basis of texture segmentation and classification methods. If a model is capable of representing and synthesising a range of textures, then estimates of its parameters may provide a useful feature set. For such a model-based approach to be successful there must exist a reasonably efficient and appropriate parameter estimation scheme and the model itself should be parsimonious, i.e. use the minimum number of parameters. Popular random field models used for texture analysis and testing include fractals [Pentland84] [Peleg84] [Medioni84], autoregressive models [Kashyap80] [Khontanzad87], fractional differencing models [Kashyap84] [Choe91a], and Markov random fields [Chellappa85a] [Cohen91b]. To the best of the author's knowledge (see chapter 4 for a detailed review) all of these models are used purely as image texture models; that is they are used to represent and synthesise two-dimensional intensity textures directly in the image plane. Only very rarely is consideration given to topological texture and lighting. Indeed, even on the wider subject of machine vision, few papers or books give details of lighting schemes used for image acquisition, and fewer still give any background theory for such schemes [Davies90].

Davis [Davis81a] describes two approaches to modelling image texture. A *"physically based"* model takes into account surface relief, albedo, illumination, and the position and frequency response of the viewer. *"Image-based"*, models on the other hand, model textures directly in the image plane without regard to their physical origin. He states that physically based models are ordinarily very difficult to construct, and this in part explains their scarcity. Davis however, does not consider the effects of illumination in his experiments. Nor do many of the other papers on texture segmentation and classification (see chapter 4). This is particularly surprising for papers on *"rotation invariant"* schemes, where one might reasonably expect researchers to rotate the physical textures on their own without rotating the associated lighting.

Pentland is one of the few researchers to develop a segmentation scheme who does consider topological texture and lighting [Pentland84]. He states that *"the lack of a 3-D model for such naturally occurring surfaces has generally restricted image-understanding efforts to a world populated exclusively by smooth objects, a sort of 'Play-Doh' world".* He uses fractal dimension as a feature measure, and shows that it is theoretically independent of illuminant direction. That is he proves that the fractal dimension of texture in the image plane, is the same as the fractal dimension of the components of the normals of the physical surface being imaged (assuming a Lambertian reflectance function, constant illumination and constant albedo).

Pentland also used fractal models in his shape from texture algorithms, and these are described in the next section.

2.1.3. Texture analysis - shape from texture

This section gives a brief overview of the two main *shape from texture* techniques — texture gradient based approaches and isotropy based approaches. This is followed by a more detailed review of the associated literature that has considered topological texture and illumination issues. *Shape from shading* techniques - e.g. [Ikeuchi81], [Horn89]- have not been reviewed here, as they normally assume that the surfaces under consideration are smooth [Pentland86], and they do not employ models of topological texture.

a) Texture gradient and isotropy approaches

There are two ways that surface shape affects images of texture. Firstly, perspective projection effects a uniform compression which is dependent on the distance of the surface from the viewer, the greater the distance the greater the compression. Secondly, projection of surfaces that are not perpendicular to the viewing direction will result in a foreshortening effect. The degree of foreshortening is proportional to the cosine of the surface inclination angle (surface slant angle σ_S), and the direction of maximum foreshortening is the direction of the steepest descent (surface tilt angle τ_S).

Correspondingly, two approaches have been employed to estimate the tilt and slant of surfaces. The first exploits the concept of "texture gradients" and is due to Gibson

[Gibson50]. It assumes that physical texture is homogeneous and exploits the gradient of texture densities caused by perspective projection. The second uses a statistical approach first proposed by Witkin [Witkin81]. The distribution of orientations in a texture image is biased towards a direction perpendicular to the tilt angle and the degree of biasing is a function of the slant. Both surface slant and tilt can therefore be estimated from the distribution of orientations (assuming that the original texture is isotropic). These two approaches have been extensively researched. Bajcsy [Bajcsy76] uses a texture gradient based on "preferred" frequencies derived from Fourier transforms of 128x128 windows. Blostein [Blostein89] expicitly identifies texture elements (*textels*) in textures. She defines a *texel* as "*the repetitive unit of which the texture is composed*", and uses the *texel* area gradient to extract depth information. Rosenfeld [Rosenfeld75] suggested the use of an edge operator as a simple method of measuring texture gradient. Researchers who have built upon Witkin's ideas include Davis [Davis83], Kanatani [Kanatani84] and Blake [Blake90], who have all suggested ways of estimating surface orientation from the distribution of orientations of the texture.

b) Topological texture and illumination

The subject of *shape from texture* is not an easy one and it is therefore not surprising that both of the above schools (i.e. both texture gradient and isotropy researchers) have implicitly assumed that the effects of occlusion in topological textures and the effects of illuminant vector variations do not significantly affect image textures. That is they have effectively assumed that image texture results only from surface markings. Exceptions include Kender, Chen & Keller, Choe & Kashyap, Pentland, and Kube & Pentland.

Kender [Kender80] considered surfaces in which texture primitives were either "painted" (parallel to the surface plane) or "pointed" (perpendicular to the surface plane). He did not consider both simultaneously and commented that "textures formed by arbitrary angles to a surface are almost intractable". He did not consider illumination effects on texture.

Chen & Keller [Chen90] state that most shape from texture techniques are based on the assumption that the surfaces are smooth and uniformly covered with flat textured markings or texels. Although Chen & Keller do discuss the use of fractional Brownian motion (FBM) to model the topological texture, and do use a topological model to *generate* test images, they do not use such a model in their shape from texture algorithm directly. Instead they use it to model the intensity texture which could result from either height or albedo variation. They make use of the "*average Holder constant*", which is a fractal-related parameter that changes with scale [Keller87]. This parameter is used to calculate the distance ratio between points on a "planar" surface in order to determine the surface's orientation (i.e. it is a gradient measure). As an intensity model of texture is used it cannot be shown that the average Holder constant is invariant to illumination. This lack of consideration of illumination effects is reinforced by the use of test textures consisting of *computer scaled and rotated* Brodatz images [Brodatz66]. Such rotation ignores illumination effects or at best implicitly assumes that the illumination has been similarly rotated.

Choe and Kashyap [Choe91a] [Choe91b] presented a hybrid *shape from shading*/*shape from texture* technique. They assume that the image is made up of a random texture component and a component due to a smoothed version of the surface (i.e. a surface without topological texture). The smoothed surface is assumed to be Lambertian. An explicit model of the surface's topological texture is not used. Rather, the intensity texture in the surface normal plane is modelled directly as a "fractional differencing model" which has the ability to model anisotropic textures and has a separate variance parameter (see chapter 4).

The key point however, as concerns this thesis, is that Choe & Kashyap effectively assume that a two-dimensional or albedo texture pattern is mapped onto a three dimensional surface and no account is taken of lighting effects. Furthermore, as with Chen & Keller, these assumptions are implicit in the selection of the image test set : images from Brodatz's standard texture album [Brodatz66] were digitised, and *then* subjected to a projection/rotation process. The effects of lighting on three-dimensional or topological textures were therefore not investigated.

Kashyap and his colleagues have also used the fractional differencing model for rotation invariant texture classification [Choe91a] and this will be discussed later in this thesis.

Pentland was the first to report the use of a realistic model of natural topological texture for the purposes of determining shape from texture and texture segmentation. He uses a "spatially isotropic fractal Brownian surface" [Pentland84]. He proposes a proof that the fractal dimension of an imaged texture is identical to that of the components of the surface normals of a spatially isotropic fractal Brownian surface, and goes on to use the fractal dimension as a feature measure for texture segmentation. The two main conclusions of [Pentland84], for shape from shading, are (i) that as real fractal surfaces are fractal over a finite range of scales the perspective gradient of these limits can provide orientation information, and (ii) that fractal dimension can be used as a test for non-isotropy.

In [Pentland86] the use of fractal models for shape from shading is further developed. An image texture measure is presented which is a function of the expectation of the 2nd derivative of the surface normal. This measure is independent of illuminant direction i.e. it is intrinsic to the surface (however it is not clear as to how the illuminant vector is eliminated). As it is affected by foreshortening it can be used to estimate surface tilt and slant. Thus the main conclusion that can be drawn from Pentland's work, for the purposes of this research, is that the fractal dimension of the image of a *spatially isotropic fractal Brownian surface* is identical to that of the components of the surface normals.

Kube and Pentland [Kube88] further investigated the effects of illumination on images of topological texture. They developed a frequency domain model which, given the illuminant vector and the power roll-off factor of a fractal model of the physical texture, allows the two-dimensional power spectrum of the image texture to be predicted. They concluded that the resulting image texture would also be fractal, having a power roll-off factor two less than that of the surface. Their model may be used to predict the directional characteristics of image texture and these predictions have important implications for the majority of texture segmentation and classification schemes (this is discussed further in the last part of this chapter) .

Thus of the shape from texture work Kube and Pentland's fractal-based model would seem to offer the most promising theory. However, before this is described in greater detail the last category of this short review will be presented — i.e. that of scattering theory.

2.1.4. Scattering theory

The effect of rough surfaces on wave scattering has been the subject of many papers and books over the last thirty years. Both electromagnetic and acoustic waves have been investigated and application areas include ultrasonics, sonar, radar imaging, and optics [Ogilvy91]. A vast wealth of literature has been published on this subject. Ogilvy gives an excellent in-depth introduction to this area [Ogilvy91] [Ogilvy87]. Bennett provides a layman's guide to measuring surface roughness of optical and machined components [Bennett89], while Beckmann & Spichino's book [Beckmann63] still provides an often cited reference on the scattering of electromagnetic waves. With many of the titles and abstracts including terms such as "*random rough surfaces*" the area would seem to be extremely relevant to this thesis. However, as the work is concerned with the *scattering* of acoustic or electromagnetic waves, the term "*rough surface*" is defined with respect to the wavelength of the incident irradiation. Thus the typical root mean square (rms) roughness taken into consideration is of the order of 0.2µm or less [Vorburger93], whereas the rms roughnesses of typical test textures used in classification are of the order of millimetres (see [Brodatz66]). The research into scattering is thus concerned with the intimate details of reflection characteristics, whereas for the work described here it is sufficient to assume a reflection characteristic, and use this to investigate the effect of changes in illumination direction on images of comparatively gross surface relief.

Note that some work has been done on composite roughness models [Jackson86] [McDaniel83] in which the surface is modelled as a small-scale roughness superimposed on a higher amplitude, lower frequency, large-scale roughness [Ogilvy91]. The large-scale roughness is normally used to modify the surface normals of the small-scale roughness in a similar manner to Kube and Pentland [Kube88]. Kube and Pentland's theory is however much simpler, as it assumes a Lambertian reflection model — whereas the composite roughness work uses modified normals in the standard Kirchoff or small perturbation theory [Ogilvy91]. The resulting theory is therefore very complex, but it allows the characteristics of the small-scale roughness to be taken into account. Here however, it is mainly the effects that variation in the direction of illuminant incident upon "large-scale" roughness that are of concern. Hence the simpler theory due to Kube and Pentland will be used in this thesis.

2.1.5. Summary

The preceding sections have briefly reviewed four potential areas of image models of topological texture : texture synthesis; texture segmentation and classification; shape from texture; and scattering theory.

Texture synthesis researchers have extensively used three-dimensional models of texture — both for "bump mapping" and generation of fractal landscapes. They have not however generated corresponding models of image texture, which is not surprising given that they are primarily concerned with the *appearance* of their images. On the other hand the texture segmentation and classification researchers might have been more reasonably expected to have developed such models — as "rotation invariant" classification schemes have been reported. However, the majority of this research has not considered problems associated with illuminant variation and surface relief (see chapter 4 for a more detailed review). The third category, shape from texture, yielded Kube and Pentland's frequency domain model which allows the effect of illuminant variation on images of topological texture to be predicted. They assume perfectly diffuse reflection, whereas the last category, scattering theory, is intimately concerned with the details of reflection characteristics. "Surface roughness" in this case refers to variations of the same order as the illuminant wavelength (i.e. hundreds of nanometres). In this thesis however, rms roughness of the test textures is several order of magnitudes higher. In addition the theory is extremely complex. For these reasons it was decided to investigate Kube and Pentland's model in more detail.

2.2. An image model of topological texture

In this section a model of the image of an illuminated fractal surface due to Kube and Pentland [Kube88] is presented. More specifically, an expression for the spectrum of the image that results when such a surface is illuminated by a distant point light source is developed. The theory here differs from [Kube88] in that a simplifying axis-rotation is introduced — this both reduces the complexity of the derivation, and results in an expression for the model, in which the directional effects of lighting are more easily understood. The model is generalised to non-fractal surfaces and this is followed by an examination of the implications that the theory has for texture analysis.

2.2.1. A fractal based image model

A prerequisite for the development of an image model of topological texture is the choice of representation of surface relief. Kube and Pentland chose fractal Brownian motion [Mandelbrot83] to model natural surfaces, as it is widely used in computer graphics [Voss88] [Saupe88]. Their paper essentially applies a simplified version of the Lambertian surface reflectance model to an expression for the power spectral density of the fractal height-map. The theory is split into two parts. *Case 1* considers the situation where the illuminant vector is not perpendicular to the reference plane of the surface texture — allowing the Lambertian reflectance model to be linearised and used in the frequency domain. *Case 2* considers the situation in which the direction of illumination is perpendicular or close to the perpendicular. Here the quadratic term becomes significant and cannot be ignored. The theory becomes complex, involves additional assumptions, and does not yield an expression as a function of either illuminant slant or tilt. Hence only *case 1* will be considered here.

The following theory assumes :

(i) a Lambertian surface (i.e. perfectly diffuse reflection),

(ii) that the fractal Brownian surface $V_H(x,y)$ is band limited such that it is differentiable,

(iii) an orthogonal camera model,

(iv) a constant illuminant vector over the scene, and

(v) a viewer-centred co-ordinate system, in which the reference plane of the surface is perpendicular to the viewing direction.

The following theory first develops a linear model of the intensity image; second, it shows that the two-dimensional partial derivative is a linear operator; third, it introduces a fractal model of the surface; and fourth, it combines the three preceding elements together to provide the frequency domain model.

a) A linear image model of topological texture

The normalised image intensity $I(x,y)$ of the surface is

$$I(x,y) = \mathbf{n} \cdot \mathbf{L}$$

$$= \frac{-p\cos\tau\sin\sigma - q\sin\tau\sin\sigma + \cos\sigma}{\sqrt{p^2 + q^2 + 1}} \qquad (2.1)$$

where

$\mathbf{n}$ = the unit vector normal to the surface at the point (x,y)

$$= \left(\frac{-p}{\sqrt{p^2+q^2+1}}, \frac{-q}{\sqrt{p^2+q^2+1}}, \frac{1}{\sqrt{p^2+q^2+1}} \right)$$

$$p = \frac{\partial V_H}{\partial x} \qquad q = \frac{\partial V_H}{\partial y}$$

$\mathbf{L}$ = $(\cos\tau.\sin\sigma, \sin\tau.\sin\sigma, \cos\sigma)$ is the unit vector towards the light source

τ and σ are the illuminant vector's *tilt* and *slant* angles as defined in figure 2.1.

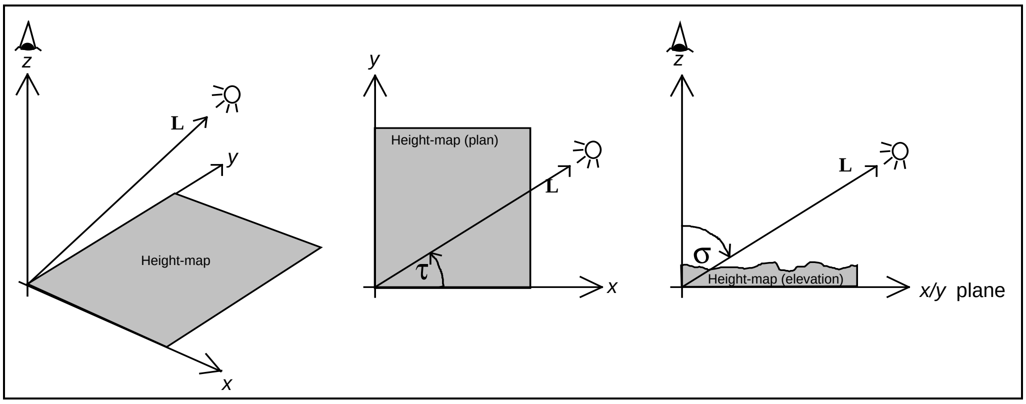

Figure 2.1 - Definition of axis and illumination angles

Now in a departure from [Kube88] and without loss of generality, choose a new axis (*x'*,*y'*,*z*) which is rotated τ about the *z* axis such that the projection of **L** onto the *x*-*y* plane will be parallel to the *x'* axis, as shown in figure 2.2.

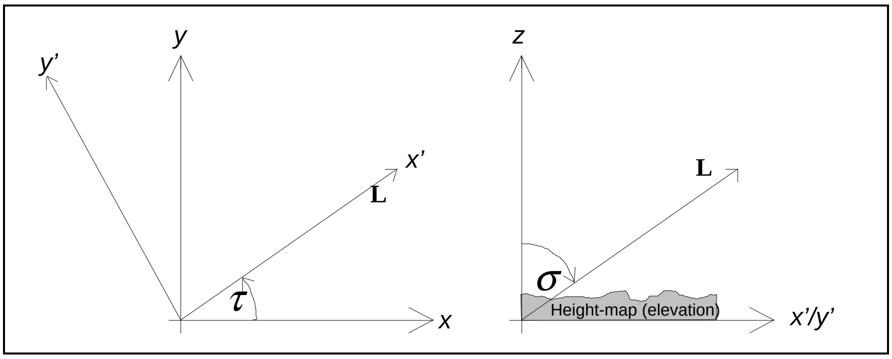

Figure 2.2 - (x,y,z) and (x',y',z) axes.

In this new axis system the expression for intensity simplifies to

$$I(x,y) \;=\; \mathbf{n} \cdot \mathbf{L} \;=\; \frac{-r\sin\sigma + \cos\sigma}{\sqrt{r^2+t^2+1}} \tag{2.2}$$

where

$$r = \frac{\partial V_H}{\partial x'}, \text{ and } t = \frac{\partial V_H}{\partial y'}$$

Taking the MacLaurin expansion of $1\big/\sqrt{r^2+t^2+1}$ yields

$$I(x,y) = (-r\sin\sigma + \cos\sigma)\left[1 - \frac{(r^2+t^2)}{2!} + \frac{9(r^2+t^2)^2}{4!}\ldots\ldots\right] \tag{2.3}$$

A proof of this expansion is provided in appendix A.

Now if the surface slope angles are less than 15°, then $r^2, t^2 << 1$; and the quadratic and higher order terms may be neglected. Note that the error introduced by this approximation, for a slope angle of 15°, is 3.5% (see figure 2.3). With this approximation (2.3) becomes

$$I(x, y) = (-r \sin\sigma + \cos\sigma) \qquad (2.4)$$

which is simply the mean, plus a linear contribution of the surface gradient measured in the direction of the illuminant's tilt angle. Thus equation (2.4) is a *linear* model of image intensity, while (2.3) which retains the quadratic and higher order terms is a *non-linear* model of image intensity. It is the former which is of interest here, but both will be referred to in later chapters.

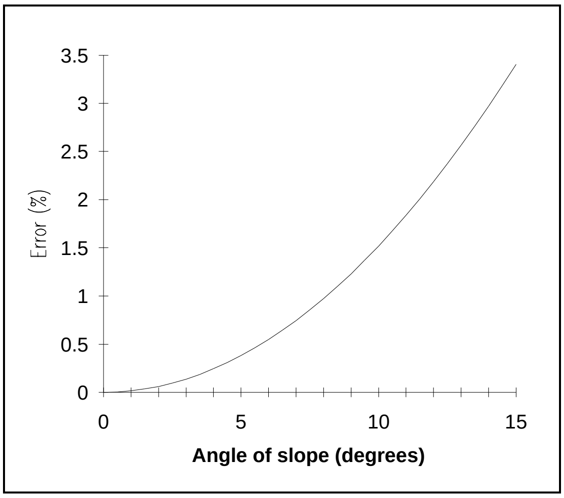

Figure 2.3 - Error due to linear approximation.

Note that if the slant angle is small then sin σ ≈ 0 and the quadratic terms in (2.3) will become important (this is Kube's case 2). For case 1, Kube therefore further assumes sin σ > 0.1, i.e. the illuminant vector **L** is not within 6° of the *z*-axis.

b) The partial derivative operator $\frac{\partial}{\partial x'}$

Consider a single sinusoid surface $V_1(x, y)$ of spatial angular frequency ω_1, angle θ_1 (w.r.t. the *x*-axis), and phase ϕ_1 :

$$V_1(x, y) = \sin[\omega_1(x\cos\theta_1 + y\sin\theta_1) + \phi_1] \qquad (2.5)$$

Transforming to the (*x',y',z*) co-ordinate system gives :

$$V_1'(x', y') = \sin\left[\omega_1\left(x'\cos(\theta_1 - \tau) + y'\sin(\theta_1 - \tau)\right) + \phi_1\right] \tag{2.6}$$

and

$$\frac{\partial V_1'}{\partial x'} = i\omega_1 \cos(\theta_1 - \tau) V_1'(x', y') \tag{2.7}$$

where *i* represents a 90° phase shift.

Taking the Fourier transform yields

$$\mathrm{F}\left[\frac{\partial V_1'}{\partial x'}\right] = i\omega_1 \cos(\theta_1 - \tau) F_1(\omega, \theta) \tag{2.8}$$

where

ω is the angular frequency of the Fourier component

θ is its direction w.r.t. the *x*-axis

$\mathcal{F}[g(x,y)]$ is the two-dimensional Fourier transform of *g(x,y)*, and

$F_1(\omega,\theta) = \mathcal{F}[V_1(x,y)] = \mathcal{F}\left[V_1'(x',y')\right]$

Thus the partial derivative operator $\frac{\partial}{\partial x'}$ is a linear operator, as it does not change either the angular frequency (ω) or the direction (θ) of a two-dimensional sine wave.

c) A fractal model of the surface

From [Kube88] a fractal surface is represented by

$$\mathrm{F}[V_H(x,y)] = F_H(f,\theta) = f^{-\beta_H/2} e^{i\phi} \tag{2.9}$$

where

f is the spatial rotational frequency = $\omega/2\pi$,

ϕ is a random phase element,

β_H is the power roll-off factor[3].

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[3]Note that for a surface the power roll-off factor β is related to the fractal dimension *D* by : *D* = (7 - 2β)/2 [Voss88]. The power roll-off factor will be used in preference to fractal dimension, as

d) Combining the linear intensity model, the partial derivative operator, and the fractal surface model

By superposition, (2.8) and (2.9)

$$\mathrm{F}\left[\frac{\partial V_H}{\partial x'}\right] = i\omega\cos(\theta - \tau)\left(\frac{\omega}{2\pi}\right)^{-\beta_H/2} e^{i\phi} \tag{2.10}$$

Now from (2.4)

$$I(x,y) = -\frac{\partial V_H}{\partial x'}\sin\sigma + \cos\sigma \tag{2.11}$$

Hence if the mean is ignored the Fourier transform of the intensity image is :

$$\begin{aligned} F_I(\omega,\theta) &= \mathrm{F}[I(x,y)] \\ &= \mathrm{F}\left[-\frac{\partial V_H}{\partial x'}\sin\sigma\right] \\ &= -\sin\sigma.\mathrm{F}\left[\frac{\partial V_H}{\partial x'}\right] \\ &= -\sin\sigma.\left[i\omega\cos(\theta-\tau)\left(\frac{\omega}{2\pi}\right)^{-\beta_H/2} e^{i\phi}\right] \\ &= -i\cos(\theta - \tau).\sin\sigma.(2\pi)^{\beta_H/2}.\omega^{-(\beta_H-2)/2}e^{i\phi} \end{aligned} \tag{2.12}$$

The above is mathematically equivalent to Kube and Pentland's case 1, but it contains a simpler expression in terms of θ and τ, which allows the directional effects of illumination to be more easily understood.

Note that the image is predicted to be fractal with a magnitude roll-off of $-^{(\beta_H-2)}/_2$, but that its directional properties have been altered compared with the original surface, i.e. the magnitude of the frequency components is now a function of their angle (θ) in relation to the tilt angle (τ) of the lighting.

e) Generalisation

Although Kube and Pentland used a fractal model of topological texture it is straight-forward to generalise their theory to non-fractal surfaces. If the requirement for the

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the latter depends upon the measurement method and choice of measurement scale [Voss88] [Mandelbrot85].

surface to have fractal PSD characteristics is relaxed, then as the partial derivative is a linear operator, the Fourier transform of the partial derivative $\frac{\partial}{\partial x'}$ of the surface $V_H\ (x,y)$ is

$$\mathrm{F}\left[\frac{\partial V_H}{\partial x'}\right] = i\omega\cos(\theta-\tau)F_H(\omega,\theta) \tag{2.13}$$

where

> $F_H(\omega,\theta)$ is the Fourier transform of the surface $V_H(x,y)$ which now need not have fractal characteristics.

Hence taking the Fourier transform of (2.11), ignoring the mean, and substituting (2.13) gives :

$$\begin{aligned} F_I(\omega,\theta) &= \mathrm{F}\left[-\frac{\partial V_H}{\partial x'}\sin\sigma\right] \\ &= [-i\omega F_H(\omega,\theta)][\cos(\theta-\tau)][\sin\sigma] \end{aligned} \tag{2.14}$$

This model (2.14) is now divided into three parts, both to aid understanding, and to facilitate future discussion. The three parts of the model are :

(i) The *surface response component*

$$F_s(\omega,\theta) = -i\omega F_H(\omega,\theta) \tag{2.15}$$

(ii) The *tilt response component*

$$F_\tau(\omega,\theta) = \cos(\theta-\tau) \tag{2.16}$$

(iii) The *slant response component*

$$F_\sigma(\omega,\theta) = \sin\sigma \tag{2.17}$$

Thus Kube and Pentland’s model provides theory which allows the influence of illuminant tilt (τ), illuminant slant (σ), and surface characteristics, to be clearly identified.

2.2.2. Implications for texture analysis

In order to aid the design of texture analysis schemes that are robust under lighting variations, it is useful to know which texture characteristics are *intrinsic* to the physical texture, i.e. independent of illuminant, and which are *extrinsic,* i.e. dependent upon the illuminant. In the case of the latter, a knowledge of the behaviour of the texture property under varying illumination conditions, would aid the design of suitable compensation schemes and/or systems that could determine the illuminant's directional characteristics.

The main conclusion of [Kube88] is that a fractal surface with a power spectrum proportional to $f^{-\beta_H}$ produces an image with a power spectrum proportional to $f^{2-\beta_H}$. That is the power roll-off factor (β_I) of an image is predicted to be an *intrinsic* property of a fractal texture — as it is predicted to be independent of the illuminant vector. As far as the directionality of the image is concerned, they merely noted that *"the spectrum depends, as expected, upon the illuminant direction"* and that one of the directional effects could be used for determining the direction of the illuminant. It is however, the directional effects of lighting that are most likely to significantly affect the performance of existing texture analysis schemes — as the majority of texture features surveyed in chapter 4 exploit image texture directionality.

In the following sections the more general model (2.14) is examined with the objective of identifying potentially *intrinsic* or *extrinsic* characteristics of image texture.

(i) The *radial shape* of an image's magnitude spectrum is predicted to be directly related to the radial shape of its surface's spectrum. The term "radial shape" is used here to refer to the shape of a section or slice passing through the coefficient representing the mean. It is purely a function of the surface response F_s, and is therefore an *intrinsic characteristic*. That is, for any value of θ, θ_1 the spectrum in direction θ_1 is

$$F_I(\omega,\theta_1) = k_{\tau\sigma}\left(-i\omega F_H(\omega,\theta_1)\right) \tag{2.18}$$

where

$k_{\tau\sigma}$ is the constant $\cos(\theta_1 - \tau).\sin\sigma$

Thus the radial shape of the log-magnitude/log-frequency graph in any direction θ is predicted to be constant under changes in illumination except for an additive term. This is summarised graphically in figure 2.4.

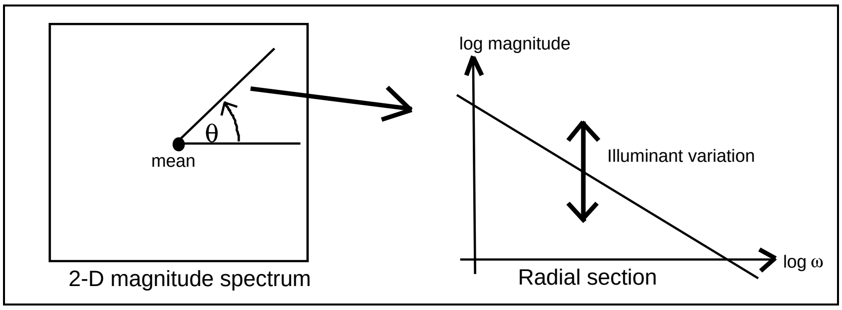

Figure 2.4 -The predicted effect of variation in illuminant direction on the radial shape of the magnitude spectrum.

(ii) The magnitude of any point in the spectrum is a function of illuminant slant (and hence the variance is a function of illuminant slant). So if surface relief and illuminant tilt are held constant the magnitude of a component at any point (ω_1, θ_1) is

$$F_I(\omega_1, \theta_1) = k_{s\tau} \sin\sigma \qquad (2.19)$$

where

$k_{s\tau}$ is the constant $-i\omega_1 F_H(\omega_1, \theta_1).\cos(\theta_1 - \tau)$

Thus the absolute values of the magnitude spectrum are a function of σ and any feature based upon these absolute values is an *extrinsic* measure of texture.

(iii) The *angular distribution* of frequency components for an isotropic surface is related to the illuminant tilt angle τ by a cosine function. That is for a "ring" of magnitude spectrum components with radius $\omega = \omega_1$

$$F_I(\omega_1, \theta) = k_{s\sigma} \cos(\theta - \tau) \qquad (2.20)$$

where

$k_{s\sigma}$ is the constant $-\omega_1 F_H(\omega_1).\sin\sigma$

In addition the angular distribution of energy of images of anisotropic surfaces (except those surfaces that are perfectly unidirectional) will be a combination of the tilt response and surface directionality[4]. Thus, except for the purely unidirectional case, the directional nature of image texture is predicted to be an *extrinsic*

[4]The term *surface directionality* is used to refer to the angular distribution of a *surface's* variance.

characteristic. This has important implications for texture classification schemes as many make use of directional features.

Normalisation

Some texture classification and segmentation schemes employ normalisation to account for variation in lighting conditions [Greenhill93] [duBuf90] [Laws79] [Weska76] [Haralick73]. Thus they remove any dependence upon absolute magnitude, and so variation in illuminant slant will, according to point (ii) above, be compensated for automatically. However, tilt angle variation cannot be compensated for in the same manner (except if all textures are perfectly unidirectional and all have the same direction). Thus many of texture feature sets that do exploit directionality will *not be* invariant to variation in tilt, unless the test data consists of individually normalised directional textures. This point seems obvious but has not, to the author's knowledge, been considered explicitly in the texture analysis literature (see chapter 4).

2.3. Conclusions

This chapter has briefly reviewed four possible sources of image models of topological texture :

(i) texture synthesis,

(ii) texture segmentation and classification,

(iii) shape from texture, and

(iv) scattering theory.

From these areas a simple frequency domain image model, due to Kube and Pentland, has been selected and presented. This model predicts that image variance and directionality are dependent upon illuminant direction, and only radial shape of magnitude spectra may be intrinsic to the underlying physical texture.

The most important implication that this model has for texture classification and segmentation is that it predicts that many schemes are not invariant to changes in illuminant tilt, and that, unlike slant variation, these effects may not normally be compensated for through the use of normalisation.

However, a number of significant assumptions were made in the derivation of the preceding theory, and validity of the model is therefore the subject of the next chapter.