NuSPADE: An Integrated Approach to Program Reasoning

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Context

- Investigate the role of proof planning within the SPARK Approach to high integrity software
- Bill Ellis (Research Associate)
- Funded by the EPSRC Critical Systems Programme (GR/R24081) in collaboration with Praxis

`www.macs.hw.ac.uk/nuspade`

Overview

- Focus on the problems that arise when proving exception freedom verification conditions (VCs)
- Present an integrated approach to these problems
- Results and future directions

Program Reasoning

Long history: Goldstine & von Neumann 1947, Turing 1949, Floyd 1967, Hoare 1969, ...

Early systems: "A Program Verifier" (King 1969), Stanford Pascal Verifier (Luckham et al 1973), Gypsy (Good et al 1974), VISTA (German & Wegbreit, 1975), Hierarchical Development Methodology (Levitt et al 1977), ...

Renewed interest: ESC/Java (DEC/Compaq/HP), SPARK (Praxis), Caveat (Airbus), Auto-Coding/Auto-Proving (QinetiQ), Perfect Developer (Escher Tech), Pathfinder (NASA), Bandera (Kansas), SLAM (Microsoft), Proof Carrying Code (Necula & Lee), Java bytecode verification (Moore), Program logics for dynamic structures (O'Hearn & Reynolds), ...

The SPARK Approach

- SPARK is a subset of Ada that includes annotations
- Expressive enough for industrial applications, but restrictive enough to support rigorous analysis
- SPARK toolset supports data & information flow analysis and formal verification
- SPARK proof tools support partial correctness and exception freedom proof, *i.e.* proving that a system is free from run-time errors, *e.g.* buffer overflows, division by zero, range violations, *etc*

SPARK Applications

- Railway signalling, smartcard security and avionics systems such as the Lockheed C130J, EuroFighter and Hawk projects
- Ship Helicopter Operating Limits Information System (SHOLIS): First system developed to meet the UK MOD Defence Standards 00-55 and 00-56
- Recognized by US National Cyber Security Partnership (NSA) as one of 3 software development processes that can deliver sufficient assurance for security critical systems

An Example

```
subtype AR_T is Integer range 0..9;
type A_T is array (AR_T) of Integer;
...
procedure Filter(A: in A_T; R: out Integer)
is
begin
   R:=0;
   for I in AR_T loop
       if A(I)>=0 and A(I)<=100 then
           R:=R+A(I);
       end if;
   end loop;
end Filter;
```


Exception Freedom VC

```
H1: for_all (i___1: integer, ((i___1 >= ar_t__first) and
    (i___1 <= ar_t__last)) -> ((element(a, [i___1]) >=
      integer__first) and (element(a, [i___1]) <=
      integer__last))) .
H2: loop__1__i >= ar_t__first .
H3: loop__1__i <= ar_t__last .
H4: element(a, [loop__1__i]) >= 0 .
H5: element(a, [loop__1__i]) <= 100 .
H6: r >= integer__first .
H7: r <= integer__last .
    ->
C1: r + element(a, [loop__1__i]) >= integer__first .
C2: r + element(a, [loop__1__i]) <= integer__last .
C3: loop__1__i >= ar_t__first .
C4: loop__1__i <= ar_t__last .
```

A Closer Look

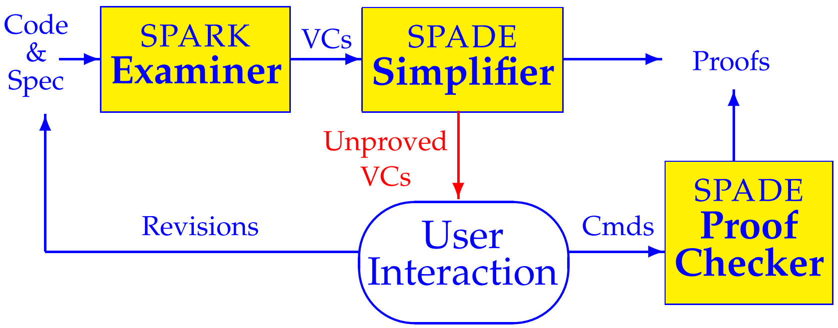
```
...
H4: element(a, [loop__1__i]) >= 0 .
H5: element(a, [loop__1__i]) <= 100 .
H6: r >= integer__first .
H7: r <= integer__last .
   ->
...
C2:r+element(a,[loop__1__i])<=integer__last
...
```


A Closer Look

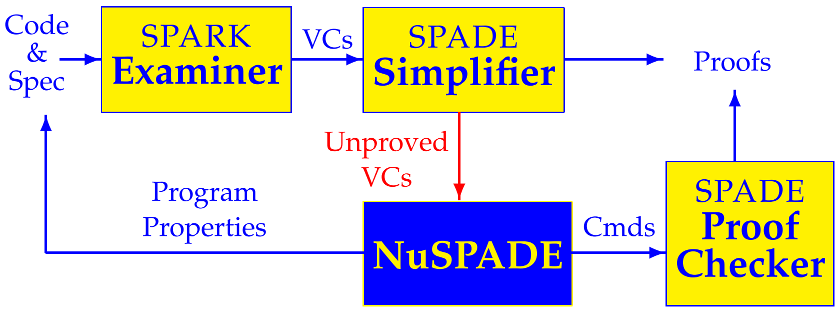
```
...
H4: element(a, [loop__1__i]) >= 0 .
H5: element(a, [loop__1__i]) <= 100 .
H6: r >= integer__first ← −32768 .
H7: r <= integer__last ← 32767 .
   ->
...
C2:r+element(a,[loop__1__i])<=integer__last
...          ↑
```

VC is unprovable, *i.e.*
if (32668 $\leq r \leq$ 32767) then possible overflow error

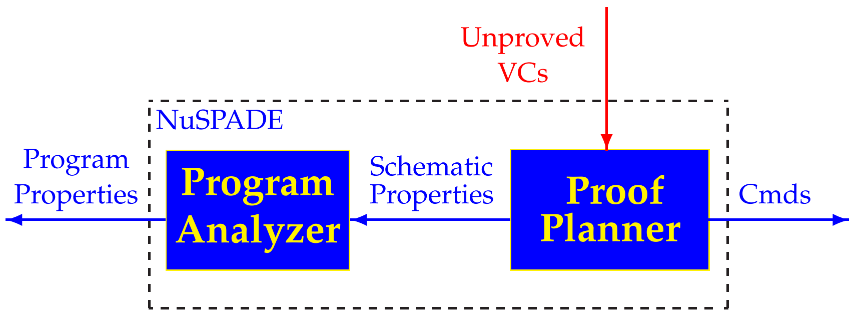
The SPARK Approach



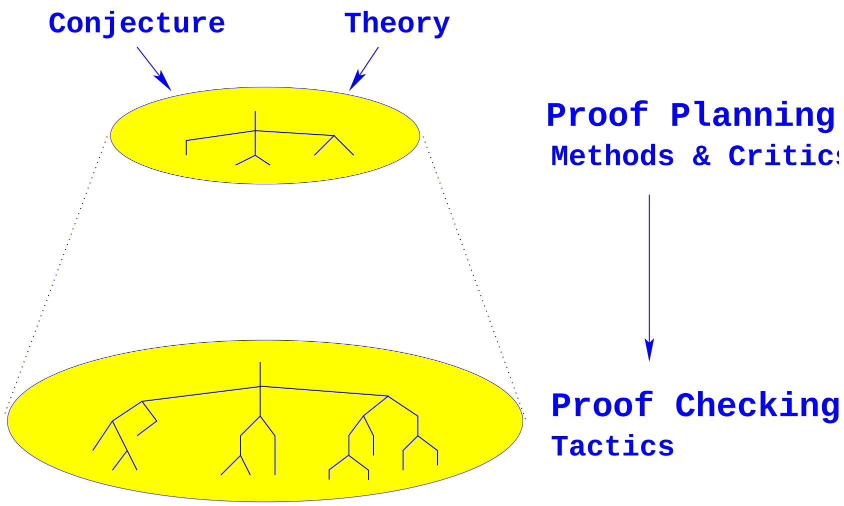
Extending the SPARK Approach



NuSPADE



Proof Planning



Proof Planning

- Proof planning supports:
  - a flexible style of proof search, *e.g.* use of meta-variables to delay choice during proof search
  - automatic proof patching via proof-failure analysis, *e.g.* conjecture generalization, lemma discovery, invariant discovery
  - diversity of proof, *i.e.* tactics can be ported to different proof checkers
- Applications include inductive and non-inductive proof

Proof Methods

Exception Freedom	Loop Invariant
`elementary`	`annotate`
`simplify`	`wave`
`transitivity`	`induction`
`decomposition`	`generalize`
`fertilize`	

Program Analysis

- Focus on the generation of program properties that support exception freedom proof
- Program analysis methods:
  - `subtype`
  - `non-looping code`
  - `looping code`
  - `loop guards`

Integration via Proof-Failure Analysis

- A proof method is applicable if all its preconditions are true
- A proof critic is applicable if its associated proof method fails to apply
- Proof-failure analysis is used to guide the selection of program properties that will progress the proof process

Proof-Failure Analysis

Preconditions for elementary critic:

- All preconditions for the elementary method fail.
- There exists a top-level goal of the form $E\ Rel\ C$, *e.g.*

  $r + element(a, i) \leq integer\_last$
- There exists a variable V_i that occurs within E such that there exists a hypothesis of the form $V_i\ Rel\ E_i$, *e.g.*

  $r \leq integer\_last$
- A counter-example can be found that shows that the bound E_i is insufficient to prove exception freedom, *e.g.*

  $32668 \leq r \leq 32767$

Patch: generate schematic properties, *e.g.* $(r \geq X) \wedge (r \leq Y)$

A Program Analysis Technique

- Recurrence relations are recursive definitions of mathematical functions or sequences, *e.g.*

$$\begin{cases} g(0) & = & 0 \\ g(n) & = & g(n-1) + (2 * n) - 1 \end{cases}$$

- Solving a recurrence relation corresponds to finding a "closed form" of the function, *e.g.* $g(n) = n^2$
- Recurrence relations can be used to express the value of variables within loops, where the solutions provide loop invariants
- There are many off-the-shelf recurrence relation solvers, *e.g.* PURRS (University of Parma)

Recurrence Relation for Variable `I`

Recurrence Relation

$$\begin{cases} I_0 &= 0 \\ I_n &= I_{n-1} + 1 \end{cases}$$

Solution

$$\begin{aligned} I_n &= \underline{I_0 + n} \\ & \qquad I_0 \Rightarrow 0 \\ I_n &= \underline{0 + n} \\ & \qquad 0 + Y \Rightarrow Y \\ I_n &= n \end{aligned}$$

Recurrence Relation for Variable `R`

Recurrence Relation

$$\begin{cases} R_0 = 0 \\ R_n = R_{n-1} \\ R_n = R_{n-1} + \underbrace{ele(A, I)}_{\text{problem term}} \end{cases}$$

Extreme Recurrence Relation

$$\begin{cases} R_0 = 0 \\ R_n = R_{n-1} \\ R_n = R_{n-1} + 0 \\ R_n = R_{n-1} + 100 \end{cases}$$

Note problem term is eliminated by generalizing the recurrence relation, *i.e.* by considering the bounds of $ele(A, I)$.

Solving for Variable `R`

- **true**-branch:

lower – bound	**upper – bound**
$R_n = R_{n-1} + 0$	$R_n = R_{n-1} + 100$
$R_n = R_0 + n * 0$	$R_n = R_0 + n * 100$
$R_n = 0$	$R_n = n * 100$

- **false**-branch:

$$R_n = R_{n-1}$$
$$R_n = R_0$$
$$R_n = 0$$

Combining Solutions

$$I_n = n \ \wedge\ \underline{(R_n = 0 \ \vee\ ((R_n \geq 0) \ \wedge\ (R_n \leq n * 100)))}$$
$$\Downarrow$$
$$\underline{I_n = n \ \wedge\ (R_n \geq 0) \ \wedge\ (R_n \leq n * 100)}$$
$$\Downarrow$$
$$(R_n \geq 0) \ \wedge\ (R_n \leq I_n * 100)$$
$$\Downarrow$$

```
--# assert R >= 0 and R <= I*100;
```

Revised Filter Code

```
procedure Filter(A: in A_T; R: out Integer)
is
begin
   R:=0;
   for I in AR_T loop
   --# assert R >= 0 and R <= I*100;
      if A(I)>=0 and A(I)<=100 then
           R:=R+A(I);
      end if;
   end loop;
end Filter;
```


Revised Exception Freedom VC

```
H1: r >= 0 .
H2: r <= loop__1__i * 100 .
...
H6: element(a, [loop__1__i]) >= 0 .
H7: element(a, [loop__1__i]) <= 100 .
...
   ->
...
C2:r+element(a,[loop__1__i])<=integer__last
...
```


Proof Planning: Exception Freedom

Given: $r \leq i * 100 \ \wedge \ ele(a, i) \leq 100$

Proof:

$$r + ele(a, i) \leq integer\_last$$

`transitivity`

$$r + ele(a, i) \leq X_0 \wedge X_0 \leq integer\_last$$

`decomposition`

$$r \leq X_1 \wedge ele(a, i) \leq X_2 \wedge X_1 + X_2 \leq integer\_last$$

`fertilize`

$$((i * 100) + 100) \leq integer\_last$$

`simplify`

$$((i * 100) + 100) \leq 32767$$

`elementary`

Note: `fertilize` produces $\{i * 100/X_1,\ 100/X_2\}$

Loop Invariant VC

```
H1: r >= 0 .
H2: r <= loop__1__i * 100 .
...
H6: element(a, [loop__1__i]) >= 0 .
H7: element(a, [loop__1__i]) <= 100 .
...
   ->
C1:r+element(a,[loop__1__i]) >= 0 .
C2:r+element(a,[loop__1__i]) <=
                     (loop__1__i + 1) * 100 .
...
```

Proof Planning: Loop Invariant

Given: $r \le i * 100 \ \wedge\ ele(a,i) \le 100$

Proof: $r + ele(a,i) \le (i+1) * 100$

`annotate`

$$\boxed{r} + ele(a,i)^{\uparrow} \le (\boxed{i} + 1)^{\uparrow} * 100$$

`wave`

$$\boxed{r} + ele(a,i)^{\uparrow} \le (\boxed{i * 100}) + (1 * 100)^{\uparrow}$$

`wave`

$$r \le i * 100 \wedge ele(a,i) \le 1 * 100^{\uparrow}$$

`fertilize`

$$true \wedge ele(a,i) \le 1 * 100$$

`simplify`

$$ele(a,i) \le 100$$

`elementary`

Loop Invariant Generated Tactic

```
plan(vc3_2,c2,
annotate(c2,h2) then
  wave(distribute(1),[2],equ) then
    wave(inequals(80),[],imp) then
      fertilize(h2) then
        simplify(logical_and(2),[],equ) then
          simplify(filter_rules(7),[2],equ) then
            simplify(filter_rules(8),[2],equ) then
              simplify(arith(2),[2],equ) then
                elementary)
```


Tactic Translation

```
plan(vc3_2,c2,
annotate(c2,h2) then
 wave(distribute(1),[2],equ) then
   wave(inequals(80),[],imp)  then
     fertilize(h2) then
       simplify(logical_and(2),[],equ) then
         simplify(filter_rules(7),[2],equ) then
           simplify(filter_rules(8),[2],equ) then
             simplify(arith(2),[2],equ) then
               elementary)
```

→

```
...
dosubgoalproperty r<=loop__1__i*100 and
element(a,[loop__1__i])<=1*100.
...
done c#1.
alldone.
dosubgoalproperty r+element(a,[loop__1__i])<=
                         loop__1__i*100+1*100.
   dosubgoalproperty r<=loop__1__i*100 and
              element(a,[loop__1__i])<=1*100->
              r+element(a,[loop__1__i])<=
              loop__1__i*100+1*100.
      prove c#1 by implication.
      dosimplifyhypsconj.
      infer r+element(a,[loop__1__i])<=
            loop__1__i*100+1*100
      using inequals(80).
      done c#1.
      alldone.
   done c#1.
   alldone.
forwardchain h#21.
done c#1.
alldone.
...
```


Results

- Our evaluation was based upon examples drawn from the literature and industrial data provided by Praxis, *e.g.* SHOLIS
- SPADE Simplifier is very effective on exception freedom VCs, *i.e.* typical hit-rate of 90%
- NuSPADE targeted the VCs which the SPADE Simplifier failed to prove *i.e.* typically loop-based code
- While critical software is engineered to minimize the number and complexity of loops, we found that 80% of the loops we encountered were provable using our techniques

Phase	Goal	Plan		Critic				PropGen		
		a1	**a2**	**b1**	**b2**	**b3**	**b4**	**c1**	**c2**	**c3**
P1	P1_1	○		•				•		
	P1_2	○		•				•		
	P1_3		○		•				•	
	P1_4		○				•			•
	P1_5		○				•			•
P2	P2_1	•								
	P2_2	•								
	P2_3		•							
	P2_4		•							

Note: the key to the results table appears at the end of these notes

Limitations & Future Work

- Constraint solving fails when reasoning with "big numbers", *i.e.* integers out with $-(2^{25})\ldots 2^{25}-1$
- Precondition strengthening would improve our hit-rate, constraint solving may have a role to play
- We could make greater use of constraint solving for debugging
- Integrating decision procedures within NuSPADE would also improve our hit-rate
- A follow-on "knowledge transfer" project, funded by the EPSRC RAIS Scheme starts early 2005
- "Critical Software Components in SPARK", in collaboration with Kung-Kiu Lau (University of Manchester) and Praxis

Conclusion

- NuSPADE = Proof Planning + Program Analysis
- Proof planning guides proof search
- Proof-failure analysis coupled with program analysis selectively strengthens program specifications
- NuSPADE increases automation for exception freedom proof

Results Table Key

Plan	**a1**	**loop invariant**
	a2	**exception freedom**
Critic	**b1**	**fertilize**
	b2	**elementary**
	b3	**transitivity**
	b4	**decomposition**
PropGen	**c1**	**entry**
	c2	**for-loop range**
	c3	**range constraint**

Note that • denotes the successful application of a proof plan while ○ denotes partial success