

MSc IN ACTUARIAL SCIENCE 2007/08 - TERM III
F79BF3 - LIFE INSURANCE MATHEMATICS II
TUTORIAL THREE (PROFIT TESTING II)

1. (Excel) Consider the endowment assurance from Tutorial 2 Question 6 (20 year term, £25,000 sum assured). In this exercise we will work our way through a full profit test of this contract.

a) We start by opening up the spreadsheet from tutorial 2. Let this represent a contract sold a few months ago with a premium of £910.

Over time, economic and demographic conditions change. The insurer will want to check the revised profit under these new situations. The actuary has just received a new set of mortality tables which show that people are living longer. Inflation has also fallen. We incorporate these changes into the following **experience basis**:

Interest	4% <i>p.a.</i>
Mortality Rate	0.0014 in the first year, increasing by 0.0001 each year i.e. the mortality rate in year 20 will be 0.0033
Initial Expenses	£200 plus 80% of premium
Renewal Expenses	In each year except the first £9 plus 2.5% of premium

Update your spreadsheet for the new experience basis. (Be careful to keep the reserves and premiums fixed. However the 'Interest on Reserve' column will change. The change in mortality will affect the benefits paid, the increase in reserve and the profit signature.)

We have now performed two scenario tests. The high interest scenario (tutorial 2) gave an expected present value of profit of £62.56. The low interest scenario gives an expected present value of profit of -£481.05.

We see that the low interest scenario causes a loss, because we earn less interest on the premiums. However there is some good news from the lower expenses and lower mortality. (This is why we test the 3 changes together)

Save the spreadsheet from part a) as we will need it later.

b) The scenario test in part a) was very useful because it tells us that we expect to make losses in the future. However we do not know how much of this loss is due to the change in the interest rate, and how much is profit from the change in mortality. Hence a **sensitivity analysis** is used to change the interest only, while keeping everything else constant.

Take a copy of the spreadsheet from part a) and then change interest from 4 % to 5 %. We therefore have a new **experience basis** of:

Interest	5%p.a.
Mortality Rate	0.0014 in the first year, increasing by 0.0001 each year i.e. the mortality rate in year 20 will be 0.0033
Initial Expenses	£200 plus 80% of premium
Renewal Expenses	In each year except the first £9 plus 2.5% of premium

The expected present value of profit is now £141.33.

Just by changing the interest rate by 1 % we have increased the profit by £622.38 [= 141.33 - (- 481.05)] so the profitability is very sensitive to changes in the interest rate.

However we can see that the profitability is not very sensitive to even quite large changes in mortality. (Compare £62.56 and £141.33 - remember the only difference between these two profit tests is mortality, and a small change in expenses. Both tests have the same 5% interest rate.)

You can now experiment with different interest rates, mortality and expenses, and compare the profits they give.

c) There is further bad news for the insurer. The government has decided to strengthen the reserving basis, in light of the lower inflation. The new net premium valuation uses a **valuation basis**:

Interest	3.5%p.a.
Mortality Rate	0.0014 in the first year, increasing by 0.0001 each year i.e. the mortality rate in year 20 will be 0.0033

Take a copy of the spreadsheet from part a) and update it for the new reserves. You are given that the new net premium is £882.15 (calculate the reserves recursively in the same way as tutorial 2).

The new valuation basis has a weaker assumption for mortality, but a much stronger interest assumption. Therefore the new reserves are larger and the profit reduces to - £559.46.

d) So from part c) we see that the policy is unprofitable. There is nothing we can do about the policies we have already sold - this is the risk we run by selling the policies, and is why we want a high **expected** rate of return. However we can increase the premiums on the policies we will sell in the future.

Take a copy of the spreadsheet from part c). This spreadsheet is already set up to use the experience basis from part a). We choose to use this experience basis as the **premium basis** also, so we need make no further changes to the spreadsheet. Experiment with different premiums until you are happy with the profit.

For example a premium of £990 gives us an expected profit of £64.13 and a discounted payback period of 18 years.

2. The profit signature of a 3 year assurance contract issued to a life aged 60 is $(-250, 150, 170)$.

The premium payable at the start of each year is £200. Mortality is assumed to follow AM92 ultimate mortality and there are no lapses.

Assuming that the office uses a risk discount rate of 15% per annum, what is the profit margin of the contract?

(This question is taken from the Faculty / Institute paper A2 April 1997, question 8, and was worth 3 marks.)

3. A life office issues a 3 year without profits endowment assurance policy to a life aged 62. The sum assured of £30,000 is payable on maturity or at the end of the year of death, if within 3 years, and there are level annual premiums of £9,350 payable in advance.

The office uses the following ‘experience’ basis:

Interest	6% <i>p.a.</i>
Mortality	AM92 Ultimate
Initial Expenses	£400
Renewal Expenses	£100 at the beginning of the second and the third policy years

The reserves to be held at the end of the first and second policy years are £9,532 and £19,514 respectively.

a) Determine the profit signature of this policy.

b) Find the net present value of the profit signature at a risk discount rate of 9 % per annum.

(This is taken from an old exam paper and was worth 10 marks.)

4. A five year temporary annuity pays £1,000 annually in arrear. The single premium has been set at £3,300. The reserves are simply the sum of the remaining annuity payments ignoring mortality and interest (e.g. ${}_2V = 3 * 1000$). The pricing actuary wishes to profit test on two possible experience bases; a high inflation scenario and a low inflation scenario. For both scenarios calculate the present value of future profits at a risk discount rate of 10% p.a.. Also calculate the profit margin and the discounted payback period. Comment on your results.

High Inflation Scenario:

Interest - 8% p.a.

Mortality - probability of death is 15% p.a.

Expenses - £50 at the start of each year increasing at 10% p.a.

Low Inflation Scenario:

Interest - 5% p.a.

Mortality - probability of death is 20% p.a.

Expenses - £50 at the start of each year fixed

5. You are an actuary working for the life office in question 3. You believe that the government will cut interest rates over the next few years. You assume that the interest rate will be 6%, 5% and 5% in the first, second and third years respectively.

a) Find the net present value of the profit signature at a risk discount rate of 9% p.a. of the contract if the interest rate changes as above but all other details are as question 3.

b) Show that the internal rate of return lies between 7% and 8%.