HERIOT-WATT UNIVERSITY

M.SC. IN ACTUARIAL SCIENCE

Life Insurance Mathematics I

Tutorial 1

1. A whole life policy with sum assured £100,000 is issued to a life aged x. The sum assured is paid at the end of year of death while premiums are paid annually in advance. Let K_x denote the curtate future lifetime of x and L_0 denote the present value of the future loss on the contract, using the premium basis. Derive an expression for the standard deviation of L_0.

2. (a) Let $V(t)$ be the net premium policy value at duration t, of a whole-life insurance sum assured £1 payable immediately on death, sold to a life age x, with level premiums payable continuously for life. Show that:

$$V(t) = 1 - \frac{\bar{a}_{x+t}}{\bar{a}_x}.$$

   (b) Show that:

$$\frac{d}{dt}\bar{a}_{x+t} = \mu_{x+t}\,\bar{a}_{x+t} - \bar{A}_{x+t}.$$

   [Note: These results will be needed in the proof of Thiele's Differential Equation in the lectures.]

3. (a) Explain briefly why a life office holds reserves in respect of its policies.
   (b) A life office sells non-profit endowment assurances with term 25 years to lives age 30. It calculates net premium policy values assuming the mortality of the AM92 ultimate table. Calculate $V(5)$, $V(10)$, $V(15)$, and $V(20)$, at 4% p.a. interest.
   (c) A life office sells term assurances with term 25 years to lives age 30. It calculates net premium policy values assuming the mortality of the AM92 ultimate table. Calculate $V(5)$, $V(10)$, $V(15)$, and $V(20)$, at 4% p.a. interest.

4. A certain life office sells without profits term assurances to lives age 30, with term 25 years, sum assured £100,000 payable at the end of year of death and a level annual premium of £200 p.a.. It calculates policy values using the gross premium method, on the following basis:

   mortality: AM92 ultimate

   interest: 4% per annum

future expenses: 10% of office premiums.

Calculate the policy value for this contract, immediately before payment of the 16th premium.

5. An endowment assurance with term 25 years was sold to a life aged 40, 3 years ago. The sum assured is £60,000 payable at the end of year of death and premiums are payable annually in advance. Bonuses are supercompound (two-tier) as follows: (a) each year there is a *simple* bonus of of 3% on the sum assured; and (b) an additional bonus of 4.5% on *all existing bonuses*. What is the policy value (just before payment of the 4th premium)?

   Basis:

   Method: net premium policy valuation.

   Mortality: AM92 select.

   Interest: 5% per annum.

6. 15 years ago a life office sold a with-profits whole life policy to a person then aged 40. The sum assured was £20,000 payable at the end of year of death and a premium of £350 was payable annually in advance. Compound reversionary bonus accrues at the start of each year. The bonus added to date is £13,000. The office values the policy on the following gross premium basis.

   Mortality: A1967–70 select.

   Interest: 6% per annum.

   Future bonuses: 2.913% p.a. (compound).

   Expenses: Nil.

   Calculate the policy value just before the 16th premium is paid.

7. A life office issues a large number of 3-year non-profit endowment assurances policies to males aged exactly 62. The sum assured under each policy is £2,000 payable at the date of maturity or at the end of year of earlier death. Under each policy, level annual premiums are payable in advance throughout the term of the policy.

   The premium and reserving bases are as follows.

   Premium basis:

   Mortality: A1967–70 select.

   Interest: 6% per annum.

   Expenses: Initial £150

   Renewal £20 at the start of the second and third policy years.

   Reserving basis: Net premium method

Mortality: A1967–70 ultimate

Interest: 3% per annum.

(a) Show that the annual premium is £665.07.
(b) Calculate the policy values per policy at durations 0, 1, and 2 years respectively.

8. Tables of $q_{[x]}$, $q_{[x]+1}$ and q_x can be downloaded from the course web-page:

   `www.ma.hw.ac.uk/~andrea/f79af`

   in the form of Excel files, for the following life tables:

A1967–70	(assured lives, males, based on 1967–70 data)
AM92	(assured lives, males, based on 1991–94 data)
AF92	(assured lives, females, based on 1991–94 data).

(a) For these life tables, set up the following commutation functions at 4% per annum effective interest: $D_{[x]}, D_{[x]+1}, D_x$; $N_{[x]}, N_{[x]+1}, N_x$; $C_{[x]}, C_{[x]+1}, C_x$ and $M_{[x]}, M_{[x]+1}, M_x$, as follows:

Table	Age Range	Starting Point
A1967–70	ult 0–109	$l_0 = 34,489.00$
	sel 0–80	
AM92	ult 17–120	$l_{17} = 10,000.0000$
	sel 17–90	
AF92	ult 17–120	$l_{17} = 10,000.0000$
	sel 17–90.	

You can check your results for A1967–70 against the Green Tables, and for AM92 against the Yellow Tables.

[Note: Keep these worksheets of commutation columns handy for future work.]

(b) Return to Q.3, (b) and (c), but now using your spreadsheets calculate the policy values at all integer durations 0, 1, ... 25. Comment on your results.