HERIOT-WATT UNIVERSITY

M.SC. IN ACTUARIAL SCIENCE

Life Insurance Mathematics I

Tutorial 2

1. State and prove the recursive relationship between policy values of (i) a pure endowment with level annual premiums; (ii) a level annuity payable annually in advance for life.

2. (Excel exercise: compare with Tutorial 1, Q.8 (b).)
   (a) A life office sells non-profit endowment assurances with term 25 years to lives age 30. It calculates net premium policy values assuming the mortality of the A1967-70 ultimate table and 4% p.a. interest. Calculate the policy values at all integer durations using the recursive relationship starting with $V(25) = 1$.
   (b) A life office sells term assurances with term 25 years to lives age 30. It calculates net premium policy values assuming the mortality of the A1967-70 ultimate table 4% p.a. interest. Calculate the policy values at all integer durations using the recursive relationship starting with $V(25) = 0$.

3. Consider a decreasing term assurance, issued to a life age x, with term n years, initial sum assured n decreasing by 1 per annum on each policy anniversary, and level premiums payable annually in advance.
   (a) State and prove the relation between policy values at integer durations.
   (b) If $x = 30$ and $n = 30$, find the policy value just before payment of the 29th premium, on the basis of A1967–70 Ultimate mortality and 4% interest.
   (c) What problems do you think this policy value could cause in practice? Can you think of any changes to the terms of the policy that would help?

4. Calculate the following on the basis of AM92 Ultimate mortality and 4% interest, stating any approximations you use:
   (a) For a 25-year endowment sold to a man age 35, with sum assured £1 paid at the end of the year of death or on maturity, and with level annual premiums: $V(6.5)$.
   (b) For a 25-year pure endowment sold to a man age 35, with sum assured £1 paid on survival to maturity, and with level annual premiums: $V(16.25)$.
   (c) For a 25-year endowment sold to a man age 35, with sum assured £1 paid at the end of the year of death or on maturity, and with level quarterly premiums: $V(16.75)$ and $V(16.5)$.

5. A life insurer issued a special endowment insurance policy to a life age 40, with term 5 years. The death benefit, payable at the end of the year of death, is equal to £1,000 *plus* the reserve that would have been held at the end of the year had the policyholder been alive. The maturity benefit is £1,000. The premium is £179.30, payable annually in advance. By using the recursive relation between reserves, or otherwise, find the reserve just before payment of the 3rd premium. The basis for all calculations is A1967–70 Ultimate mortality and 4% interest.

6. State and prove Thiele's differential equation for the following contracts, and in each case give the appropriate boundary condition at maturity/expiry.
   (a) A temporary annuity with term n years, payable continuously at rate £1 per annum to a life now age x as long as they are alive during the term.
   (b) A pure endowment with term n years, and sum assured £1 payable to a life now age x if they survive to the end of the term. There is a single premium paid at outset.
   (c) A deferred annuity sold to a life now age x, paid for by a single premium. Payment is continuous at rate £1 per annum starting in n years, if the person is then alive, and then continuing for life.

7. A man age x buys a special endowment assurance with term n years, with the following benefits:

   - On maturity, a sum assured of £10,000.
   - On death before maturity, a sum assured of £10,000 *plus* the policy value at the time of death.

   Premiums are payable continuously at rate P per annum. Write down Thiele's differential equation for this policy, and state what boundary condition you would use in its solution.