Heriot-Watt University

M.Sc. in Actuarial Science

Life Insurance Mathematics I

Tutorial 3 Solutions

1. We have:

$$\begin{aligned} {}_hq_x &\approx {}_0q_x + h\,\frac{d}{dt}{}_tq_x\bigg|_{t=0} \\ &= {}_0q_x + h\,[(1-{}_0q_x)\,\mu_{x+0}] \\ &= 0 + h\,\mu_x \end{aligned}$$

and:

$$\begin{aligned} {}_{2h}q_x &\approx {}_hq_x + h\,\frac{d}{dt}{}_tq_x\bigg|_{t=h} \\ &= {}_hq_x + h\,[(1-{}_hq_x)\,\mu_{x+h}] \\ &\approx h\,\mu_x + h\,(1-h\,\mu_x)\mu_{x+h}. \end{aligned}$$

2. (a) We have:

$$\begin{aligned} V(n-h) &\approx V(n) - h\,\frac{d}{dt}V(t)\bigg|_{t=n} \\ &= V(n) - h[V(n)\,\delta - 1 + \mu_{x+n}\,V(n)] \\ &= h \qquad (1) \end{aligned}$$

and:

$$\begin{aligned} V(n-2h) &\approx V(n-h) - h\,\frac{d}{dt}V(t)\bigg|_{t=n-h} \\ &= V(n-h) - h[V(n-h)\,\delta - 1 + \mu_{x+n-h}\,V(n-h)] \\ &\approx h - h[h\,\delta - 1 + h\,\mu_{x+n-h}]. \qquad (2) \end{aligned}$$

(b) We have:

$$\begin{aligned} V(n-h) &\approx V(n) - h\,\frac{d}{dt}V(t)\bigg|_{t=n} \\ &= V(n) - h[V(n)\,\delta + \mu_{x+n}\,V(n)] \\ &= V(n)\,(1 - h(\delta + \mu_{x+n})) \\ &= 1 - h(\delta + \mu_{x+n}) \end{aligned} \tag{3}$$

and:

$$\begin{aligned} V(n-2h) &\approx V(n-h) - h\,\frac{d}{dt}V(t)\bigg|_{t=n-h} \\ &= V(n-h) - h[V(n-h)\,\delta + \mu_{x+n-h}\,V(n-h)] \\ &= V(n-h)\,(1 - h(\delta + \mu_{x+n-h})) \\ &\approx (1 - h(\delta + \mu_{x+n}))\,(1 - h(\delta + \mu_{x+n-h})). \end{aligned} \tag{4}$$

3. An example of an Excel worksheet (`tut3_q3.xls`) for solving this problem can be downloaded from the course web page at:

   `www/ma.hw.ac.uk/~andrea/f79af`.

   (a) In `tut3_q3.xls`, the result of the Euler scheme is ${}_{10}p_{20} \approx 0.965070$. This compares with the correct value of:

$$\begin{aligned} {}_{10}p_{20} &= \exp\left(-\int_{20}^{30} 0.0004\,\exp(\ln(1.09)\,x)\,dx\right) \\ &= \exp\left(\frac{-0.0004}{\ln(1.09)}\left(1.09^{30} - 1.09^{20}\right)\right) \\ &= 0.965056. \end{aligned}$$

   (b) See `tut3_q3.xls`.

   (c) We have $V(0) = 0$ provided the bases used to calculate premiums and policy values are the same. Evidently, the basis given in the question is *not* the same as the basis which gave the rate of premium as 0.003 per annum.

   (d) By trial and error, a rate of premium of 0.003425 per annum gives $V(0) = 0$.

4. An example of an Excel worksheet (`tut3_q4.xls`) for solving this problem can be downloaded from the course web page at:

   `www/ma.hw.ac.uk/~andrea/f79af`.

   (a) We have:

	${}_5p_{30}$	${}_{10}p_{30}$	${}_{15}p_{30}$
Yellow Tables	0.996899	0.993056	0.987517
Approximate, $h = 1$	0.996941	0.993206	0.987902
Approximate, $h = 0.1$	0.996902	0.993066	0.987547
Approximate, $h = 0.01$	0.996898	0.993052	0.987511

   In practice, much better procedures than the Euler scheme would be used, e.g. a *4th order Runge-Kutta scheme*.

   (b) We have:

Step Size h	Rate of Premium (£p.a.)
1 year	481.77
0.1 year	493.29
0.01 year	494.44

   Clearly, a better procedure than Euler's scheme would be needed in practice.