Heriot-Watt University

M.Sc. in Actuarial Science

Life Insurance Mathematics I

Tutorial 9 Solutions

1. (a) Temporary initial selection refers to the effect on mortality rates (or other characteristic of interest), at any specified age, of a process of selection of lives from the general population which excludes those in ill-health or subject to other risks. The most important example is the selection of lives acceptable at normal assurance premium rates by the underwriting process of a life office. The difference between the mortality rates of the selected selected group and the general population may be expected to decrease as the time since selection increases, but even the 'ultimate' rates are lower than for the general population.
   (b) Anti selection occurs when a life office's business contains a disproportionate share of lives subject to some risk. For example, a life office which offers the same premium rates to smokers and non-smokers (when other offices charged higher life assurance premiums for smokers) would tend to find that most of its policyholders are smokers.
   (c) Class selection is the effect on mortality rates (or other characteristics of interest) of treating as one population, populations which have different mortality rates due to differences in some permanent features of the populations. An example is the difference in mortality between males and females such that two populations which have differing proportions of males and females may exhibit different mortality rates due to this difference in proportions.

2. (a) The **Crude Death Rate** is heavily influenced by mortality at older ages.
      (i) OK if population structures by age and sex are reasonably stable. Beware of large scale emigration/immigration. Easy and practical.
      (ii) Not likely to be suitable. Age and sex distributions in occupational groups are likely to vary significantly.

   The **Standardised Death Rate** is also influenced by mortality at older ages.
      (i) OK but need age specific mortality rates at each time point. Changing population structure has no effect.
      (ii) Copes well with age/sex variations provided age specific rates are available for occupational groups. Use of a fixed age structure may be unrepresentative of given occupation.

   The **Standardised Mortality Ratio** is heavily influenced by relative mortality at older ages.

(i)Fine but ensure standard rates are same each time.

(ii)Good except for possible problems gathering data on age distributions. Use of age structure maintains relevance.

(b) Crude Death Rate

$$= \frac{235}{37,000} = 0.00635.$$

Standardised Death Rate:

$$= \frac{1}{3,100,000}\left(960,000 \times \frac{52}{15000} + 1,400,000 \times \frac{74}{12000} + 740,000 \times \frac{109}{10000}\right) = 0.00646$$

Standardised Mortality Ratio:

$$235/\left(15,000 \times \frac{3,100}{960,000} + 12,000 \times \frac{7,500}{1,400,000} + 10,000 \times \frac{7,100}{740,000}\right) = \frac{235}{208.68} = 1.126$$

3. (a)

$$\text{CDR}_\text{A} = \frac{86,520}{3,800,000} = 0.0228, \quad \text{CDR}_\text{B} = \frac{78,360}{4,000,000} = 0.0196, \quad \text{CDR}_\text{W} = \frac{814,000}{40,000,000} = 0.0204.$$

(b) First calculate the table of mortality rates.

Mortality Rates

Age	Area A	Area B	Country
30-39	0.0035	0.0045	0.0040
40-49	0.0070	0.0090	0.0080
50-59	0.0125	0.0150	0.0140
60-69	0.0320	0.0370	0.0354
70-79	0.0700	0.0900	0.0800

$$\begin{aligned}\text{SDR}_\text{A} &= (0.0035 \times 10,000 + 0.0070 \times 10,000 + 0.0125 \times 9,000 + 0.0320 \times 7,000 + \\ &\quad 0.0700 \times 4,000)/40,000 \\ &= \frac{721.5}{40,000} = 0.0180\end{aligned}$$

$$\text{SDR}_\text{B} = \frac{889}{40,000} = 0.0222$$

(c)

$$\text{SMR}_\text{A} = \frac{86,520}{97,808} = 0.8846 \quad \text{and} \quad \text{SMR}_\text{B} = \frac{78,360}{71,720} = 1.0926.$$

4. (a) The crude death rate for Region 1 is 9,800/570,000 = 0.0172 and for Region 2 is 19,620/1,020,000 = 0.0192.
The standardised mortality ratio for Region 1 is

$$9,800/(0.00106 \times 220,000 + \cdots + 0.051 \times 100,000) = 1.4482$$

and the standardised mortality ratio for Region 2 is

$$19,620/(0.00106 \times 180,000 + \cdots + 0.051 \times 360,000) = 0.9015.$$

(b) The indices give conflicting indications. The CDR suggests mortality is heavier in Region 2, whereas the SMR suggests it is heavier in Region 1.

(c) The SMR gives us a comparison against a common standard. It is therefore a more reliable measure for comparisons than the CDR. Hence we would say that Region 1 experiences the heavier mortality. Note that the exposure in Region 2 is more heavily weighted towards the older ages, while the opposite is true for younger ages.

5. I would disagree with this statement. Social class impacts on income and education, and consequently on living standards. Each of these impacts on mortality.

   Income: Those on high incomes can afford to eat properly and live in decent conditions. Those on low incomes are constrained by their budget. Diet affects mortality, as does quality of living - clean, warm, dry, etc. Population density may increase exposure to infectious diseases.

   Education: Those who are better educated tend to have professional or white collar employment, compared with blue collar and manual employment. Mortality rates are lower for the former. Education also affects personal behaviour. In general, those with a higher level of education heed health warnings on issues like smoking, drinking and safe sex.

6. The order is not surprising. There are links between mortality, occupation, education and income.

   Those in the first three groups will be tertiary educated, well paid (teachers - maybe not as much compared to the others), working in a safe environment, likely to live in good housing with the means to afford a balanced diet.

   Ministers of religion are also educated people, but not well paid, and subject to stress. They have, however, a reasonably good working environment.

   The last three groups will all be subject to some hazard caused by their occupation, e.g. dangerous chemicals, working at heights, operating machinery. They will be on low incomes, affecting housing and diet. On the positive side for these groups, manual work may lead to an improved level of fitness.

   Foremen are in supervisory role, so would not be exposed to the same risks as those in the last three groups and hence we would expect them to have lighter mortality. Also, their incomes will generally be higher than for those in the last three groups.