Heriot-Watt University

M.Sc. in Actuarial Science

Life Insurance Mathematics I

Tutorial 4 Solutions

1.

$$\begin{aligned} L &= 100,000v^{K_x+1} - P_x\ddot{a}_{\overline{K_x+1}|} = 100,000v^{K_x+1} - P_x\left[\frac{1-v^{K_x+1}}{d}\right] \\ &= v^{K_x+1}\left[100,000 + \frac{P_x}{d}\right] - \frac{P_x}{d}. \end{aligned}$$

We have, for ${}^2A_x$ denoting the value of A_x calculated at rate of interest $i^2 + 2i$,

$$S.D.[L] = \sqrt{\text{Var}[L]} = \left[100,000 + \frac{P_x}{d}\right]\sqrt{\text{Var}[v^{K_x+1}]} = \left[100,000 + \frac{P_x}{d}\right]\sqrt{({}^2A_x - A_x^2)}$$

2. (a) Reserves are required to cover the difference between the present value of future liabilities and the present value of future premium income. Reserves are needed for various purposes including to provide surrender values, to demonstrate solvency, to decide bonus rates and to determine the shareholders profits.

   (b) The policy value is given by

$$_{15}V^1_{30:\overline{25}|} = 100,000\left[A^1_{45:\overline{10}|} - P^1_{30:\overline{25}|}\cdot\ddot{a}_{45:\overline{10}|}\right].$$

   Substituting $A^1_{45:\overline{10}|} = 0.01946$, $P^1_{30:\overline{25}|} = 0.00121$ and $\ddot{a}_{45:\overline{10}|} = 8.3658$ we get $_{15}V^1_{30:\overline{25}|} = 934$

   (c) Since we are given the premium, we can write

$$\begin{aligned} _{15}V^1_{30:\overline{25}|} &= 100,000A^1_{45:\overline{10}|} - 0.9(200)\cdot\ddot{a}_{45:\overline{10}|} \\ &= 100,000(0.01946) - 180(8.3658) \\ &= 440.20 \end{aligned}$$

3. Using the formula $_tV_{30:\overline{25}|} = A_{30+t:\overline{25-t}|} - P_{30:\overline{25}|}\cdot\ddot{a}_{30+t:\overline{25-t}|}$, we get

$$_5V_{30:\overline{25}|} = 0.1301 \quad {}_{10}V_{30:\overline{25}|} = 0.28829 \quad {}_{15}V_{30:\overline{25}|} = 0.48039 \quad {}_{20}V_{30:\overline{25}|} = 0.71399$$

The policy values should be lower if we use a higher interest rate.

4. The bonus declared in year 1 is $0.03(60,000) = 1,800$. The bonus declared in year 2 comprise of $0.045(1,800) = 81$ which is earned on the existing bonus and another $1,800$ earned on the basic benefit. In year three the bonus on the existing bonuses is $0.045\,[2(1,800)+81] = 165.65$ and another $1,800$ on the basic benefit. Therefore the total bonuses declares to date is,

$$3(1,800) + 81 + 165.66 = 5,646.65$$

(Alternatively, the declared bonuses are $1,800(1.045^2 + 1.045 + 1) = 5,646.65$). The policy value is

$${}_3V_{40:\overline{25}|} = 65,646.65A_{43:\overline{22}|} - 60,000P_{[40]:\overline{25}|} \cdot \ddot{a}_{43:\overline{22}|} = 7,062.81.$$

5. The policy value is

$$\begin{aligned}
{}_{15}V_{40} &= 33,000\left[1.02913v_0|q_{55} + 1.02913^2v^2{}_1|q_{55} + 1.02913^3v^3{}_2|q_{55} + \cdots\right] - 350\ddot{a}_{55} \\
&= 33,000\left[\frac{1.02913}{1.06}{}_0|q_{55} + \left(\frac{1.02913}{1.06}\right)^2{}_1|q_{55} + \left(\frac{1.02913}{1.06}\right)^3{}_2|q_{55} + \cdots\right] - 350\ddot{a}_{55} \\
&= 33,000A_{55} - 350\ddot{a}_{55} = 13,603.75
\end{aligned}$$

since A_{55} is evaluated at 3% and $\ddot{a}_{55}$ is evaluated at 6%.

6. (a) Let P be the annual premium, then

$$P\ddot{a}_{[62]:\overline{3}|} = 2,000A_{[62]:\overline{3}|} + 150 + 20\ddot{a}_{[62]:\overline{3}|} - 20$$

at 6% p.a. interest and A1967–70 select mortality. Therefore $P = 665.07$.

(b) Let P' be the net premium that would be paid at the outset, on the valuation basis (A1967–70 ultimate mortality and 3% interest). From the Tables
$P' = 2,000(0.32036) = 640.72$.
The policy value at $t = 0$ is 0.
At $t = 1$, the policy value is $2,000A_{63:\overline{2}|} - 640.72\ddot{a}_{63:\overline{2}|} = 635.61$.
At $t = 2$ the policy value is $2,000A_{64:\overline{1}|} - 640.72\ddot{a}_{64:\overline{1}|} = 1,301.02$.

7. (a) The recursive relationship between pure endowment policy values is:

$$({}_tV_{x:\overset{1}{\overline{n}|}} + P_{x:\overset{1}{\overline{n}|}})(1+i) = p_{x+t} \cdot {}_{t+1}V_{x:\overset{1}{\overline{n}|}}.$$

To prove this, write:

$$({}_tV_{x:\overset{1}{\overline{n}|}} + P_{x:\overset{1}{\overline{n}|}}) = A_{x+t:\overset{1}{\overline{n-t}|}} - P_{x:\overset{1}{\overline{n}|}}(\ddot{a}_{x+t:\overline{n-t}|} - 1)$$

and note that:

$$A_{x+t:\overline{n-t}|}^{\;1} = q_{x+t}v \times 0 + p_{x+t} \cdot v \cdot A_{x+t+1:\overline{n-t-1}|}^{\;1}$$

and:

$$\ddot{a}_{x+t:\overline{n-t}|} = 1 + p_{x+t} \cdot v \cdot \ddot{a}_{x+t+1:\overline{n-t-1}|}$$

so:

$$\begin{aligned}(A_{x+t:\overline{n-t}|}^{\;1} - P_{x:\overline{n}|}^{\;1} \cdot \ddot{a}_{x+t:\overline{n-t}|})(1+i) &= p_{x+t}A_{x+t+1:\overline{n-t-1}|}^{\;1} - P_{x:\overline{n}|}^{\;1} \cdot p_{x+t} \cdot \ddot{a}_{x+t+1:\overline{n-t-1}|} \\ &= p_{x+t} \cdot {}_{t+1}V_{x:\overline{n}|}^{\;1}.\end{aligned}$$

(b) The policy value for a level annuity-due payable for life is just $\ddot{a}_{x+t}$. The recursive relationship between policy values is therefore:

$$(\ddot{a}_{x+t} - 1)(1+i) = p_{x+t} \cdot \ddot{a}_{x+t+1}.$$

The statement stands as its own proof.