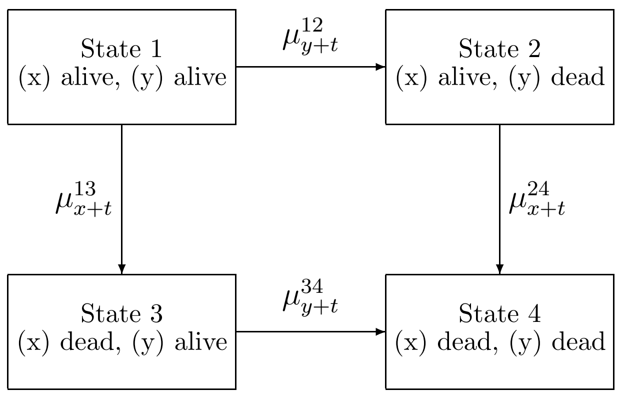
Heriot-Watt University

M.Sc. in Actuarial Science

Life Insurance Mathematics I

Tutorial 6

1. A life office uses the following Markov model for pricing and valuation of joint-life insurances and annuities of all types, sold to two lives (x) and (y).



If a benefit is paid for at outset by means of a single premium, the policy value $V(t)$ is just its expected present value of the benefit at time t. Hence state Thiele's equation(s) for the following benefits.

(a) An assurance of £1 payable immediately upon the first death of (x) and (y).

(b) An assurance of £1 payable immediately upon the second death of (x) and (y).

(c) An assurance of £1 payable immediately upon the death of (x), provided (y) is then still alive.

(d) An assurance of £1 payable immediately upon the death of (x), provided (y) is then dead.

(e) An annuity payable continuously at rate £1 per annum, while (x) and (y) are both alive.

(f) An annuity payable continuously at rate £1 per annum, while at least one of (x) and (y) is still alive.

(g) An annuity payable continuously at rate £1 per annum to (y), while (y) is alive but provided (x) is dead. .

2. (Excel exercise): The office in Q.1 uses the following mortality basis:

$$\begin{aligned} \mu^{13}_{x+t} &= \mu^{24}_{x+t} = 0.0004 \times 1.09^{x+t} \\ \mu^{12}_{y+t} &= \mu^{34}_{y+t} = 0.0004 \times 1.09^{y+t} \end{aligned}$$

with force of interest $\delta = 0.05$ per annum and no expenses.

Using an Euler scheme with step size $h = 0.01$ years, find $V^1(0)$, if the office sells each of the following contracts to (40) and (35).

(a) A term assurance with sum assured £1 payable immediately on the first death of (40) and (35), if this occurs within 10 years.

(b) A term assurance with sum assured £1 payable immediately on the second death of (40) and (35), if this occurs within 10 years.

(c) A contingent assurance with sum assured £1 payable immediately on the death of (40), if this occurs within 10 years *and* (35) is still alive.

(d) An annuity payable continuously at rate £1 per annum while (40) and (35) are alive, but for a maximum term of 10 years.

(e) An annuity payable continuously at rate £1 per annum while at least one of (40) and (35) is alive, but for a maximum term of 10 years.

(f) A reversionary annuity payable continuously at rate £1 per annum while (40) is alive and (35) is dead, but for a maximum term of 10 years.

3. Calculate the following assuming the mortality of the AM92 table, and describe each expression in words:

(a) ${}_{10}p_{30:40}$

(b) $q_{30:40}$

(c) $\mu_{40:50}$

(d) ${}_{10}p_{[30]:[40]}$

(e) $q_{[30]:[40]}$

(f) $\mu_{[40]:[50]}$

(g) $\mu_{[40]+1:[60]+1}$

(h) ${}_{3|}q_{[30]+1:[40]+1}$

4. Let T_x and T_y be the independent random future lifetimes of two lives age x and y, and define $T_{\min} = \min[T_x, T_y]$ and $T_{\max} = \max[T_x, T_y]$.

(a) Derive an expression for the density of $T_{\max}$.

(b) Define $\overset{\circ}{e}_{xy} = \mathrm{E}[T_{\min}]$. Show that:

$$\overset{\circ}{e}_{xy} = \int_0^\infty {}_tp_{xy}dt.$$

(c) Show that $\mathrm{Cov}\,(T_{\min}, T_{\max}) = \left(\overset{\circ}{e}_x - \overset{\circ}{e}_{xy}\right)\left(\overset{\circ}{e}_y - \overset{\circ}{e}_{xy}\right)$

(d) Further let K_{min} be the integer part of $T_{\min}$ and define $e_{xy} = \mathrm{E}[K_{\min}]$. Show that

i. $e_{xy} = \sum_{t=1}^{\infty} {}_tp_{xy}$. and

ii. $\overset{\circ}{e}_{xy} \approx e_{xy} + \frac{1}{2}$.

(e) Derive an expression for the 'force of mortality' associated with T_{max}, denoted $\mu_{\overline{x:y}}(t)$. What is its value at $t = 0$? Explain this result.

5. Given that $l_{xy} = 10,000$, $l_{x+10:y} = 9,600$, and $l_{x:y+10} = 9,200$ calculate the probability that, of the two independent lives aged x and y, exactly one will survive for 10 years.

6. Show that:

(a) $\ddot{a}_{xy} = \sum_{k=0}^{\infty} v^k {}_kp_{xy}$.

(b) $\ddot{a}_{xy:\overline{n}|} = \ddot{a}_{x:\overline{n}|} + \ddot{a}_{y:\overline{n}|} - \ddot{a}_{\overline{xy}:\overline{n}|}$.

(c) $A_{\overline{xy}} = A_x + A_y - A_{xy}$.

7. Derive an expression for the variance of the random variable $v^{K_{max}+1}$.

8. For a male aged 70 exact and a female aged 67 exact, who are subject to the mortality of the PMA92 and PFA92 tables respectively, with interest of 4% per annum, find:

(a) $\ddot{a}_{70:67}$

(b) $\ddot{a}^{(12)}_{70:67}$

(c) $\ddot{a}_{70:67:\overline{10}|}$

(d) $\ddot{a}^{(12)}_{70:67:\overline{10}|}$

(e) $\ddot{a}_{\overline{70:67}}$

(f) $\ddot{a}^{(12)}_{\overline{70:67}}$

9. State in words the meanings of the symbols A_{xy}, $A_{\overline{xy}:\overline{n}|}$ and $\bar{A}_{xy:\overline{n}|}$. Prove that:

(a) $A_{\overline{xy}:\overline{n}|} = 1 - d\ddot{a}_{\overline{xy}:\overline{n}|}$

(b) $\bar{A}_{xy:\overline{n}|} = 1 - \delta\bar{a}_{xy:\overline{n}|}$.