Age Heaping in Population Data of Emerging Countries

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Overview

- Motivation
- Main Objective
- MLE and Bayesian approaches
- Model and Notation
- Results
- Conclusions
- Forthcoming research
Age Heaping occurs when people misreport age.
Motivation.

Population and Deaths data

Mexico, Females
1990, Death rates

Reported $\hat{m}_{t,x}$

Age heaping

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Age Heaping in Population Data of Emerging Countries
Motivation.

Mortality analyses \(\xrightarrow{\text{Good quality mortality data}}\) HMD.

However, in many other countries population and deaths data can be somewhat unreliable.

Population \(\xrightarrow{\text{Misreporting of age}}\) census.

Deaths \(\xrightarrow{\text{Misreporting of age}}\) deaths data.
Main Objective

- Develop mortality models for countries where their population data is affected by age heaping.

Application: Reported data $\rightarrow$ Smoothed HMD $\rightarrow$ International Reinsurance.
We design a model taking into account two dimensional data. Hence, we consider the data by age $x$ and across cohorts $y = t - x$.

- First approach $\rightarrow$ MLE
- Second approach $\rightarrow$ Bayesian framework,
MLE and Bayesian approaches

For any cohort $y$ we denote by $|y|$ the number of ages available for this cohort, that is, $n_y = |y|$ is the length of cohort $y$ in our data set. The corresponding set of ages $x$ is denoted by $\mathcal{X}_y$.

$$
\begin{align*}
E_{x,y} \xrightarrow{\text{Age heaping}} & \hat{E}_{x,y} \\
D_{x,y} \xrightarrow{\text{Age heaping}} & \hat{D}_{x,y}
\end{align*}
$$
Approximate log-likelihood function

\[ D_{x,y} \sim \text{Poisson} \left( m_{x,y} E_{x,y} \right), \]

\[ m_{x,y} = \exp \left[ a_y + b_y (x - \bar{x}) + c_y \left( (x - \bar{x})^2 - \sigma_x^2 \right) \right], \]

\[ \ell(\theta) = \sum_{x,y} \hat{D}_{x,y} \log \left( m_{x,y} \hat{E}_{x,y} \right) - m_{x,y} \hat{E}_{x,y} + C. \]

where,

\[ \theta = \{ a, b, c \} \]
Penalised log-likelihood function

$$\ell\ell p(\theta) = \ell(\theta) - \lambda_1 p(a) - \lambda_2 p(b) - \lambda_3 p(c),$$

$$p(\xi_y) = \sum_{\tilde{y}=2}^{n_y-1} \left( \Delta^2 \xi_y \right)^2,$$

where $\Delta^2 \xi_y$ is the second order difference of $\xi_y$, and $\lambda_1$, $\lambda_2$ and $\lambda_3$ are the smoothing parameters.
Directed Acyclic Graph
Bayesian approach

\[ \ell(\theta) = \sum_{x,y} \hat{D}_{x,y} \log \left( m_{x,y} \hat{E}_{x,y} \right) - m_{x,y} \hat{E}_{x,y} + C. \]

where, \( \theta = \{a, b, c, \delta_a, \mu_b, \mu_c\} \)

Prior distributions

\[ a_{y+1|a_y} \sim N(a_y + \delta_a, \sigma_a^2), \quad a_1 \sim N(0, 0.01), \quad \delta_a \sim N(\mu_{\delta_a}, \sigma_{\delta_a}^2) \]

\[ b_y \sim N(\mu_b, \sigma_b^2) \text{ iid}, \quad \mu_b \sim N(\mu_{\mu_b}, \sigma_{\mu_b}^2) \text{ iid}, \]

\[ c_y \sim N(\mu_c, \sigma_c^2) \text{ iid}, \quad \mu_c \sim N(\mu_{\mu_c}, \sigma_{\mu_c}^2) \text{ iid}. \]
Full log posterior log \( \pi(\theta) \)

\[
\propto \left[ \sum_{x,y} \hat{D}_{x,y} \log \left( m_{x,y} \hat{E}_{x,y} \right) - m_{x,y} \hat{E}_{x,y} \right]
\]

\[
- \frac{1}{2} \left[ \left( \sum_{y=1}^{n_y-1} \log \left( 2\pi \sigma_{a}^2 \right) + \frac{(a_{y+1} - (a_{y} + \delta_{a}))^2}{\sigma_{a}^2} \right) + \log \left( 2\pi \sigma_{a_1}^2 \right) + \frac{a_{1}^2}{\sigma_{a_1}^2} \right]
\]

\[
- \frac{1}{2} \left[ \sum_{y=1}^{n_y} \log \left( 2\pi \sigma_{b}^2 \right) + \frac{(b_{y} - \mu_{b})^2}{\sigma_{b}^2} \right] - \frac{1}{2} \left[ \sum_{y=1}^{n_y} \log \left( 2\pi \sigma_{c}^2 \right) + \frac{(c_{y} - \mu_{c})^2}{\sigma_{c}^2} \right]
\]

\[
- \frac{1}{2} \left[ \log \left( 2\pi \sigma_{\delta_{a}}^2 \right) + \frac{(\delta_{a} - \mu_{\delta_{a}})^2}{\sigma_{\delta_{a}}^2} \right] - \frac{1}{2} \left[ \log \left( 2\pi \sigma_{\mu_{b}}^2 \right) + \frac{(\mu_{b} - \mu_{\mu_{b}})^2}{\sigma_{\mu_{b}}^2} \right]
\]

\[
- \frac{1}{2} \left[ \log \left( 2\pi \sigma_{\mu_{c}}^2 \right) + \frac{(\mu_{c} - \mu_{\mu_{c}})^2}{\sigma_{\mu_{c}}^2} \right].
\]
Parameters $a_y$, $b_y$ M-H, Mexico.

Mexico, Cohort Females, $a_y$

- Observed $\hat{a}_y$
- Fitted $\hat{a}_y$, $\sigma_a=0.001$, $\sigma_c=0.00015$
- Fitted $\hat{a}_y$, $\sigma_a=0.001$, $\sigma_c=0.00009$
- Fitted $\hat{a}_y$, $\sigma_a=0.001$, $\sigma_c=0.00002$

Mexico, Cohort Females, $b_y$

- Observed $\hat{b}_y$
- Fitted $\hat{b}_y$, $\sigma_b=0.001$, $\sigma_{\mu_b}=1$
- Fitted $\hat{b}_y$, $\sigma_b=0.0005$, $\sigma_{\mu_b}=0.00005$
- Fitted $\hat{b}_y$, $\sigma_b=0.0003$, $\sigma_{\mu_b}=0.000075$
Mexico, Cohort Females, $c_y$

- Observed $\hat{c}_y$
- Fitted $\hat{c}_y$, $\sigma_c=0.00015$, $\sigma_{\mu_c}=1$
- Fitted $\hat{c}_y$, $\sigma_c=0.00009$, $\sigma_{\mu_c}=0.000005$
- Fitted $\hat{c}_y$, $\sigma_c=0.00002$, $\sigma_{\mu_c}=0.0000075$

Mexico, Females 1990, Death rates

- $\log(\hat{m}_{t,x})$ (observed)
- $\hat{m}_{t,x}$ (fitted)
- Credible Interval
- $\hat{c}_y$ (fitted)
Fitted exposures $\tilde{E}_{x,y}$, Mexico.

Mexico, Females, Cohorts 1933 and 1948

Mexico, Females, Cohorts 1935 and 1955

Fitted exposures $\tilde{E}_{x,y} = \frac{D_{x,y}}{\tilde{m}_{x,y}} = \frac{\text{Reported deaths}}{\text{Fitted Force of Mortality}}$

where $\tilde{m}_{x,y} = \exp \left[ \tilde{a}_y + \tilde{b}_y (x - \bar{x}) + \tilde{c}_y \left( (x - \bar{x})^2 - \sigma_x^2 \right) \right]$. 
Fitted exposures $\tilde{E}_{t,x}$, Mexico 1990.
Conclusions

- Smooth time series $c \xrightarrow{\text{Reduce age heaping}} m_{t,x}$ and population. However, we do not want to smooth too much because it would destroy the natural volatility from the data.
- This model improves the quality of the Mexican data by reducing age heaping across all cohorts.
- The remaining volatility in the fitted exposures comes from the death counts.
Forthcoming research

- Include constraints on death counts to reduce the volatility in the fitted exposures.
- We will collaborate with HMD to see how their approach can be adapted to Mexican data for producing complete life table series, which is also relevant to international reinsurance.
Thank You!

Questions?