The Impact of Longevity Risk Hedging on Economic Capital

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The Actuarial Research Centre (ARC) is the Institute and Faculty of Actuaries’ (IFoA) network of actuarial researchers around the world. The ARC seeks to deliver cutting-edge research programmes that address some of the significant, global challenges in actuarial science, through a partnership of the actuarial profession, the academic community and practitioners. The ’Modelling, Measurement and Management of Longevity and Morbidity Risk’ research programme is being funded by the ARC, the SoA and the CIA.

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Actuarial Research Centre (ARC): funded research arm of the Institute and Faculty of Actuaries

Three major programmes started in 2016, including

**Modelling, Measurement and Management of Longevity and Morbidity Risk**
- New/improved models for modelling longevity
- Management of longevity risk
- Underlying drivers of mortality
- Modelling morbidity risk for critical illness insurance
Outline

- Introduction and motivation
- Hedging longevity risk with an index-based call-spread option contract
- Anatomy of a hedging calculation in 22 easy steps!
- Numerical example
- Discussion
Motivation

- Longevity risk
- Measurement
  - e.g. **Capital Requirement**
  - Best estimate + *extra for risk*
- Longevity risk management
  - customised hedges
  - index-based hedges
Motivation

• Why use **General Population Longevity Index** based risk transfer instruments?
  → **Capacity and Price**

• Pros/cons
  • Transferred risk is efficiently priced
  • But hedger left with **basis risk**

• Thus we need
  • a clear and rigorous approach to quantify basis risk
  • hedger and regulator agreement on approach
  • to quantify properly the **Capital Relief**
Introduction

Underlying problem:

- Life insurer
- Aim 1: measure mortality/longevity risk
- Aim 2: manage mortality/longevity risk
- e.g. to *reduce* regulatory capital
- e.g. to *reduce* economic capital
- e.g. to *increase* economic value
Regulatory Capital Requirements: Annuity Portfolio

- Solvency II options:
  - Solvency Capital Requirement,
    \[ \text{SCR} = \text{difference between } \text{Best estimate of annuity liabilities (BE)} \text{ and } \text{Annuity liabilities following an immediate 20\% reduction in mortality} \]
  - or \( \text{SCR} = \text{extra capital required at time 0 to ensure solvency at time 1 with 99.5\% probability} \)
  - or \( \text{SCR} = \text{extra capital at time 0 to ensure solvency at time } T \text{ with } x\% \text{ probability} \)
Liability to be Hedged

- $L = \text{random PV at time 0 of liabilities}$
- $L(0) = \text{point estimate of } L \text{ based on time 0 info}$
- $L(T) = \text{point estimate of } L \text{ based on info at } T$
  
  - $= \text{PV of actual cashflows up to } T$
  - $+ \text{PV of estimated cashflows after } T$

- Risk $\Rightarrow$ capital requirements

What type of hedge to modify capital requirements and manage risk?
Hedging Options

- Index-based hedge
  - Synthetic $\tilde{L}(T) \approx \text{true } L(T)$
  - Call spread derived from underlying $\tilde{L}(T)$
    Payoff at $T$, per unit

$$H(T) = \begin{cases} 
0 & \text{if } \tilde{L}(T) < AP \text{ (Attachment Point)} \\
\tilde{L}(T) - AP & \text{if } AP \leq \tilde{L}(T) < EP \text{ (Exhaustion Point)} \\
EP - AP & \text{if } EP \leq \tilde{L}(T)
\end{cases}$$
The Synthetic $\tilde{L}(T)$

- $\tilde{L} = \text{random PV at time 0 of a portfolio of synthetic liabilities}$
- Synthetic mortality experience
  - based on general population mortality
  - adjusted using experience ratios

- $\tilde{L}(T) = \text{point estimate of } \tilde{L} \text{ based on info at } T$
  - $= \text{PV of actual } \textit{synthetic} \text{ cashflows up to } T$
  - $+ \text{PV of estimated } \textit{synthetic} \text{ cashflows after } T$
Questions and Observations

- What impact $L(T) \rightarrow L(T) - H(T)$?
- Need a two population mortality model
- Practical reality: calculation is more complex than academic ‘ideal world’
- What are good choices of $AP$, $EP$, $T$?
Anatomy of a Hedging Calculation in 22 Easy Steps!

- **General (National) Population**
  - Population G
    - $E_G(x, t)$
    - $D_G(x, t)$
    - $m_G(x, t)$

- **Specific (Hedger’s) Population**
  - Population S
    - $E_P(x, t)$
    - $D_P(x, t)$
    - $m_P(x, t)$

- **Hedger’s Calculation**
  - $L(T)$
  - $H(T)$

- **Equations**
  - $t \leq 0$
  - $1 \leq t \leq T$
  - $t > T$

- **Variables**
  - $G$ (General Population)
  - $S$ (Specific Population)
  - $E_G$, $D_G$, $m_G$ (Population G)
  - $E_P$, $D_P$, $m_P$ (Population S)
  - $L(T)$, $H(T)$

- **Additional Notes**
  - $\hat{\mu}(0)$
  - $\hat{\mu}(T)$
  - $\hat{\mu}(0) + \hat{\mu}(T)$
  - $SIM(0)$, $SIM(T)$
  - $LC(0)$, $LC(T)$
  - $ER(0)$, $ER(T)$

- **Author**
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Anatomy of a Hedging Calculation: Steps 1, 2

- **General (National) Population**
  - Population G
    - \( E_G(x, t) \)
    - \( D_G(x, t) \)
    - \( m_G(x, t) \)
  - \( \mu(0) \)
  - \( \hat{\mu}(0) \)
  - \( E(0) \)
  - \( M1M5X(0) \)
  - \( C(0) \)  
  - \( \hat{C}(0) \)
  - \( \hat{\mu}(0)+ \)
  - \( \hat{\mu}(t) \)
  - \( \hat{\mu}(T) \)
  - \( SIM(0) \)
  - \( SIM(T) \)
  - \( E(T) \)
  - \( m_G(x, t) \)
  - \( L(T) \)  
  - \( H(T) \)

- **Specific (Hedger’s) Population**
  - Population S
    - \( E_P(x, t) \)
    - \( D_P(x, t) \)
    - \( m_P(x, t) \)
  - \( m_P(x, t) \)
  - \( L(T) \)
Anatomy of a Hedging Calculation: Steps 3-5

[Diagram showing the calculation process with different populations and variables]
Anatomy of a Hedging Calculation: Steps 6, 7, 14, 15, 17

General (National) Population

Population G
\( E_G(x, t) \)
\( D_G(x, t) \)
\( m_G(x, t) \)

Specific (Hedger’s) Population

Population S
\( E_P(x, t) \)
\( D_P(x, t) \)
\( m_P(x, t) \)

\( \tilde{L}(T) \rightarrow H(T) \)

\( t \leq 0 \)

\( m_G(x, t) \)

\( t > T \)

\( m_P(x, t) \)
Anatomy of a Hedging Calculation: Steps 8, 9, 12

$\begin{align*}
&\text{Population } G \\
&\quad E_G(x, t) \\
&\quad D_G(x, t) \\
&\quad m_G(x, t) \\
&\text{Population } S \\
&\quad E_P(x, t) \\
&\quad D_P(x, t) \\
&\quad m_P(x, t) \\
&\quad \mu(0) \\
&\text{M1M5X(0)} \\
&\text{ER(0)} \\
&\text{L(T)} \\
&\quad \mu(T) \\
&\quad \hat{\mu}(T) \\
&\quad \text{SIM(T)} \\
&\text{ER(T)} \\
&\quad \tilde{L}(T) \\
&\quad H(T) \\
&\quad m_G(x, t) \\
&\quad m_P(x, t) \\
&\text{t} \leq 0 \\
&1 \leq t \leq T \\
&t > T \\
\end{align*}$
Anatomy of a Hedging Calculation: Steps 10, 11, 13, 14, 16, 18

General (National) Population

Population $G$
$E_G(x, t)$
$D_G(x, t)$
$m_G(x, t)$

$\hat{\mu}(0)$
$\hat{\mu}(T)$

$C(0)$
$C(T)$

$M1M5X(0)$
$SIM(0)$

$ER(0)$
$ER(T)$

$t \leq 0$

$t > T$

Specific (Hedger's) Population

Population $S$
$E_P(x, t)$
$D_P(x, t)$
$m_P(x, t)$

$\hat{\mu}(0) + SIM(T)$

$\hat{\mu}(T)$

$t \leq T$

$\bar{L}(T) \rightarrow H(T)$

$L(T)$

$m_P(x, t)$

$m_G(x, t)$
Anatomy of a Hedging Calculation: Steps 19-22

General (National) Population

Population G

$E_G(x, t)$

$D_G(x, t)$

$m_G(x, t)$

$t \leq 0$

$t \leq 0$

$t \leq T$

$t > T$

$\mu(0)$

$\tilde{m}(T)$

$H(T)$

$\hat{\mu}(T)$

$\hat{\mu}(0)$

$m_G(x, t)$

$m_P(x, t)$

$m_P(x, t)$

$\ell(T)$

$\hat{\mu}(0) + \text{SIM}(0)$

$\text{M1M5X}(0)$

$\mu(0)$

$\text{ER}(0)$

$\text{ER}(T)$

$\text{SIM}(T)$

$\mu(0)$

$\mu(0)$

$\text{ER}(0)$

$\text{ER}(T)$

$\text{SIM}(T)$

Specific (Hedger's) Population

Population S

$E_P(x, t)$

$D_P(x, t)$

$m_P(x, t)$

$\geq 39$

$\geq 40$

$\geq 89$

$\geq 90$

$\geq 91$
How many models do you need?

*Academic ‘ideal’: One model*

*In practice:*

- **Time 0:**
  - Liability valuation model (BE + SCR)
  - Simulation model (0 $\rightarrow$ $T$)

- **Time $T$:**
  - Hedge instrument valuation model
  - Liability valuation model

- ‘Models’ for extrapolating to high (and low) ages
**Time 0 Models**

- **Unhedged Liabilities:**
  Deterministic BE + 20% stress

- **Simulation:** (by way of example)
  - General population: (Lee-Carter/M1)

    \[
    \ln m_{gen}(x, t) = A(x) + B(x)K(t) \quad \text{(Lee-Carter/M1)}
    \]

- Hedger’s own population: (M1-M5X)

    \[
    \ln m_{pop}(x, t) = \ln m_{gen}(x, t) + a(x) + k_1(t) + k_2(t)(x - \bar{x})
    \]
Hedge instrument:
- Lee-Carter (M1) for general population
- Recalibration: *on basis specified at time 0*

\[ q_{\text{pop}}^H(x, t) = q_{\text{gen}}^H(x, t) \times ER(x, 0) \rightarrow \tilde{L}(T) \rightarrow H(T) \]

Liability: specific (hedger’s) population
- Lee-Carter (M1) for general population
- Possibly different calibration from the hedge instrument

\[ q_{\text{pop}}^L(x, t) = q_{\text{gen}}^L(x, t) \times ER(x, T) \rightarrow L(T) \]
- Approach must mimic local practice
Hedging Example

- Data: Netherlands
  - CBS national data
  - CVS insurance data (Dutch aggregated industry experience data)

- Hedge instrument maturity: $T = 10$
- Attachment and exhaustion points at 60% and 95% quantiles of $\tilde{L}(T)$
- Key point: $EP \ll 99.5\%$ quantile of $\tilde{L}(T)$
Hedging Example

- Portfolio of deferred and immediate annuities
- Current ages 40 to 89
- Weights (≡ pension amounts):

Before and after: Compare $L(T)$ with $L(T) - H(T)$

$SCR = 99.5\%$ quantile − mean
Hedging Example \((n = 10,000\) scenarios\)
Simulated Annuity Portfolio Present Values

Hedging Example: Unhedged VaR = 11,649

EP(0.95)

AP(0.6)

Unhedged 99.5% VaR

PV(0) Synthetic Portfolio, \( \tilde{L}(T) \)

PV(0) Specific Portfolio Liability, \( L(T) \)
Hedging Example: Hedged VaR = 11,199

Plot shows kinked contours of $L(T) - H(T)$. 
Hedged VaR = 11,119 with no Pop. Basis Risk

Simulated Annuity Portfolio Present Values

Plot shows kinked contours of $L(T) - H(T)$. 
Hedging Example: VaR Calculations

Liability Distribution Functions

No Basis Risk
No Hedge
With Basis Risk

Note: CDF makes no allowance for the price of the hedge.
Hedging Example: Higher AP (0.65) and EP (0.995)

Liability Distribution Functions

Liability With/Without Hedge

Cumulative Probability

Liability With/Without Hedge

Cumulative Probability

No Hedge
Hedge with Basis Risk
Hedge with NO Basis Risk

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Longevity Risk Hedging
### Numerical Example: AP, EP = 60% and 95% quantiles

<table>
<thead>
<tr>
<th>Scenario</th>
<th>SCR(_{20%}) stress</th>
<th>SCR(_{10})</th>
<th>SCR(_{11})</th>
<th>SCR(_{20})</th>
<th>SCR(_{21})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L(0)):</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\tilde{L}(T)):</td>
<td>(SCR_{10})</td>
<td>840</td>
<td></td>
<td></td>
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<tr>
<td>(\tilde{L}(T) - H(T)):</td>
<td>(SCR_{11})</td>
<td></td>
<td>478</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(L(T)):</td>
<td>(SCR_{20})</td>
<td></td>
<td></td>
<td>960</td>
<td></td>
</tr>
<tr>
<td>(L(T) - H(T)):</td>
<td>(SCR_{21})</td>
<td></td>
<td></td>
<td></td>
<td>598</td>
</tr>
</tbody>
</table>

(\text{Pop 1; no hedge})

(\text{Pop 1; with } \tilde{L}(T) \text{ hedge})

(\text{Pop 2; no hedge})

(\text{Pop 2; with } \tilde{L}(T) \text{ hedge})

**Table:** SCR values in excess of the mean liability. For the hedging instrument \(AP = 10779\) (60% quantile) and \(EP = 11228\) (95% quantile). Pop 1: synthetic \(\tilde{L}(T)\). Pop 2: true \(L(T)\).
How good is the hedge?

- “Good” ⇒ price and risk reduction
- “Good” ↔ Types of basis risk
  - Structural (e.g. non-linear payoff)
  - Population basis risk
    - Within population (e.g. linkage to different cohort)
    - Different population

- **Hedge effectiveness** ⇒ % reduction in required capital
- **Haircut** ⇒ impact on capital relief as a result of population basis risk
- **EIOPA Solvency II guidelines** ⇒ regulatory approval should focus on the haircut
Numerical Example: AP, EP = 60% and 95% quantiles

| $L(0)$: | $SCR_{20\%stress}$ | 840 |
| $\tilde{L}(T)$: | $SCR_{10}$ | 840 | (Pop 1; no hedge) |
| $\tilde{L}(T) - H(T)$: | $SCR_{11}$ | 478 | (Pop 1; with $\tilde{L}(T)$ hedge) |
| $L(T)$: | $SCR_{20}$ | 960 | (Pop 2; no hedge) |
| $L(T) - H(T)$: | $SCR_{21}$ | 598 | (Pop 2; with $\tilde{L}(T)$ hedge) |

Table: SCR values in excess of the mean liability. For the hedging instrument $AP = 10779$ (60% quantile) and $EP = 11228$ (95% quantile). Pop 1: synthetic $\tilde{L}(T)$. Pop 2: true $L(T)$.

What is the impact of Population basis risk on hedge effectiveness?

Haircut $HC = 1 - \frac{SCR_{20} - SCR_{21}}{SCR_{10} - SCR_{11}} = 0.000$. 
Haircut $\approx 0$: Interpretation

- Here $EP$ ”$<<$” 99.5% quantile
- Above the 99.5% quantile the call spread (almost) always pays off in full
- So population basis risk $\Rightarrow$ little impact
- Structural basis risk prevails

- More detailed analysis $\Rightarrow$
  Haircut is *worst* (highest) when $EP$ is close to the 99.5% quantile.
Reduction in SCR: Dependence on AP and EP

Reduction in SCR with Hedge as a Percentage of SCR without Hedge

Attachment Point Quantile
Exhaustion Point Quantile

●
●
Discussion

- Rigorous approach: practical assessment of the impact of a longevity hedge
- Call spread: choice of EP ⇒ impact on haircut ⇒ impact on regulatory approval
- Choice of AP and EP ⇒ impact on SCR reduction
- Interaction: SCR reduction ↔ price ⇒ tradeoff
- Applies equally well to economic capital
- Index option → Cat Bond for receivers
Thank You!

Questions?

Paper online at:

www.macs.hw.ac.uk/~andrewc/ARCresources
How to choose Maturity, AP and EP?

- Reduction in SCR ➽
- Cat Bond nominal ➼
- Bull spread price ➼
- Shareholder value added ➽
- Insurer risk appetite, hedging objectives etc.
Theory vs Practice: Bridging the Gap

Try to avoid this:

OR
Theory vs Practice: Bridging the Gap

Where we are now?
Sensitivity to Hedge Maturity, $T$

- e.g. $T = 20$

- % reduction in SCR is *slightly* higher
- Haircut is *slightly* worse
- Haircut is still $\approx 0$ for $EP \leq 99.5\%$ quantile

- The longer the maturity:
  - less liquid market
  - less confidence in future reserving method
  - more future capital relief (everything else held constant)