Living to 100: Mortality Modelling

Modelling, Measurement and Management of Longevity Risk

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The Actuarial Research Centre (ARC) is the Institute and Faculty of Actuaries’ (IFoA) network of actuarial researchers around the world. The ARC seeks to deliver cutting-edge research programmes that address some of the significant, global challenges in actuarial science, through a partnership of the actuarial profession, the academic community and practitioners. The 'Modelling, Measurement and Management of Longevity and Morbidity Risk' research programme is being funded by the ARC, the SoA and the CIA.
ARC research program themes

- Improved models for mortality
- Key drivers of mortality
- Management of longevity risk
- Morbidity risk modelling for critical illness insurance
Outline

- Part 1: All cause mortality modelling
  - Introduction to stochastic mortality models
  - Why?
  - Example applications
- Part 2: Key drivers
  - Education level
  - Cause of death
  - Health inequalities
Part 1: All Cause Mortality Modelling
Graphical Diagnostics

- Mortality is falling
- Different improvement rates at different ages
- Different improvement rates over different periods
- Improvements are random
  - Short term fluctuations
  - Long term trends

- All *stylised facts*
- Other countries:
  - Some similarities
  - Some different patterns
Why do we need stochastic mortality models?

Data $\Rightarrow$ future mortality is **uncertain**

- Good risk management
- Setting risk reserves
- Regulatory capital requirements (e.g. Solvency II)
- Life insurance contracts with embedded options
- Pricing and hedging mortality-linked securities
Modelling

Aims:
- to develop the best models for forecasting future uncertain mortality;
  - general desirable criteria
  - complexity of model $\leftrightarrow$ complexity of problem;
- longevity versus brevity risk;
- measurement of risk;
- valuation of future risky cashflows.
Aims:
- active management of mortality and longevity risk;
  - internal (e.g. product design; natural hedging)
  - over-the-counter deals (OTC)
  - securitisation
- part of overall package of good risk management.
Stochastic Mortality Models

Two basic examples:
- Lee-Carter Model (1992)
- Cairns-Blake-Dowd Model (CBD) (2006)

Stochastic model:
- Central forecast
- Uncertainty around the central forecast

Good ERM ⇒ Use a combination of stochastic projections *plus* some deterministic scenarios or stress tests
The Lee-Carter Model

Death rate:

\[ m(t, x) = \frac{D(t, x)}{E(t, x)} = \frac{\text{deaths}(t, x)}{\text{average population}(t, x)} \]

Year \( t \); Age \( x \).

LC: \( \log m(t, x) = \alpha(x) + \beta(x)\kappa(t) \)

- \( \alpha(x) \) = base table; age effect
- \( \beta(x) \) = age effect
- \( \kappa(t) \) = period effect
The Lee-Carter Model

\[ \log m(t, x) = \alpha(x) + \beta(x) \kappa(t) \]

- Estimate \( \alpha(x) \), \( \beta(x) \), \( \kappa(t) \) from historical data
- “Traditional” model:
  - Fit a random walk model to historical \( \kappa(t) \)
  - Simulate future scenarios for \( \kappa(t) \)
  - Calculate future mortality scenarios given \( \kappa(t) \)
- Alternative models for \( \kappa(t) \) can be used
The CBD Model

\[ q(t, x) = \text{Probability of death in year } t \text{ given initially exact age } x. \]

\[ q(t, x) \approx 1 - \exp[-m(t, x)] \]

\[ \text{logit } q(t, x) = \log \left( \frac{q}{1 - q} \right) = \kappa_1(t) + \kappa_2(t)(x - \bar{x}) \]

- \( \kappa_1(t) = \) period effect; affects level
- \( \kappa_2(t) = \) period effect; affects slope
- \( \bar{x} = \) mean age
- Captures big picture at higher ages
Comparison

- LC $\Rightarrow$ all mortality rates dependent on a single $\kappa(t)$
  $\Rightarrow$ rates at all ages perfectly correlated
- CBD $\Rightarrow$ simpler age effects ($1$ and $x - \bar{x}$)
  but two period effects
  $\Rightarrow$ richer correlation structure
- CBD linearity $\Rightarrow$
  not good for younger ages
- Historical data:
  Different improvements at different ages over different time periods
  $\Rightarrow$ need more than one period effect
Applications: Scenario Generation

Example: the Lee Carter Model

- (Applied to a synthetic dataset)
- \( \log m(t, x) = \alpha(x) + \beta(x)\kappa(t) \)
- Choose a time series model for \( \kappa(t) \)
- Calibrate the time series parameters using data up to the current time (time 0)
- Generate \( j = 1, \ldots, N \) stochastic scenarios of \( \kappa(t) \)

\[ \kappa_1(t), \ldots, \kappa_N(t) \]
Applications: Scenario Generation

- Generate $N$ scenarios for the future $m(t, x)$
  $m_j(t, x)$ for $j = 1, \ldots, N$, $t = 0, 1, 2, \ldots$, $x = x_0, \ldots, x_1$
- Generate $N$ scenarios for the survivor index,
  $S_j(t, x)$
- Calculate financial functions

+ variations for some financial applications.
Applications: Scenario Generation, $\kappa(t)$

$\kappa(t)$: Generate scenario 1

Time

Period Effect: One Scenario

Historical

Simulated

Period Effect, $\kappa(t)$:

<table>
<thead>
<tr>
<th>Time</th>
<th>Historical</th>
<th>Simulated</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30</td>
<td>-1.0</td>
<td>-1.0</td>
</tr>
<tr>
<td>-20</td>
<td>-0.5</td>
<td>-0.5</td>
</tr>
<tr>
<td>-10</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>10</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>20</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>30</td>
<td>2.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Applications: Scenario Generation, $\kappa(t)$

Period Effect: Multiple Scenarios

Historical Simulated Period Effect: Multiple Scenarios

Time

Period Effect, $\kappa(t)$

Historical

Simulated

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Applications: Scenario Generation, $\kappa(t)$

Period Effect: Fan Chart

-30 -20 -10 0 10 20 30
-1.0 -0.5 0.0 0.5

Historical Simulated

Time

Period Effect, $\kappa(t)$
Applications: Scenario Generation, Future $m(t, x)$

Death Rates, Age 65: One Scenario

Death Rate (log scale)

Time

Death Rate

0.006

0.008

0.012

0.016

0 5 10 15 20 25 30
Applications: Scenario Generation, Future $m(t, x)$
Applications: Scenario Generation, Future $m(t, x)$
Annuity Pricing Requires Cohort Rates

Extract Cohort Death Rates, $m(t, x+t-1)$

Annuity valuation ⇒ follow cohorts

$m(0, x) \rightarrow m(1, x + 1) \rightarrow m(2, x + 2) \ldots$
Annuity Pricing Requires Cohort Rates

Cohort Death Rates From Age 65:

One Scenario

Cohort Age

Death Rate (log scale)

65 70 75 80 85 90 95 100

0.01 0.02 0.05 0.10 0.20

Annuity valuation ⇒ follow cohorts

\[ m(0, x) \rightarrow m(1, x + 1) \rightarrow m(2, x + 2) \ldots \]
Annuity Pricing Requires Cohort Rates

Cohort Death Rates From Age 65: Multiple Scenarios

Death Rate (log scale)

Cohort Age

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Annuity Pricing Requires Cohort Rates

Cohort Death Rates From Age 65:
Fan Chart

Death Rate (log scale)

Cohort Age
Cohort death rates $\rightarrow$ cohort survivorship
 Survivorship From Age 65: Multiple Scenarios

Cohort Age
Survivor Index (log scale)

Cohort Survivor Index

Survivorship From Age 65:
Multiple Scenarios

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Cohort Survivor Index

Survivorship From Age 65: Fan Chart

Survivor Index (log scale)

Cohort Age

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Cohort survivorship → ex post cohort life expectancy
Equivalent to a continuous annuity with 0% interest
Annuity Reserving

- Annuity of 1 per annum payable annually in arrears
- Interest rate: 2%
A Real Example: US Male Period Life Expectancy

Extract Period Death Rates, $m(t, x+t-1)$

Age

0 5 10 15 20 25 30
65 70 75 80 85 90

Time

0 5 10 15 20 25 30
A Real Example: US Male Period Life Expectancy

US Male Period Life Expectancy From Age 65
(Stochastic Model: CBD–X–K3–G)

Mortality improvement rate ≈ 1.7% p.a. at ages 65-85.
How to incorporate Expert Judgement?

- E.g. CBD model ⇒
  - $m^j_{CBD}(t, x)$ scenarios
  - $\bar{m}_{CBD}(t, x)$ central forecast
- Expert judgement ⇒
  - $\hat{m}(t, x)$ (central) forecast

- Blending ⇒ stochastic scenario $j$ becomes

$$m^j(t, x) = \frac{m^j_{CBD}(t, x)}{\bar{m}_{CBD}(t, x)} \times \hat{m}(t, x)$$

- Fully stochastic ⇒ full risk assessment
How to incorporate Expert Judgement?

- A variation on this is required by UK life insurance regulators

⇒ Don’t ignore stochastic models simply because you disagree with the central forecast!

- Additionally: new approaches to bring the two together are being developed
Part 2: Key Drivers
Drill into the Detail of US Data

- Level of **educational attainment** ⇒ predictor
- Individual **cause of death** ⇒ outcome

- Beware of grade inflation
- Help to understand trends in national data and subpopulations (e.g. white collar pension plan)
Data Sources

- Total Exposures: Human Mortality Database (smoothed to mitigate anomalies)
- CDC deaths: cause of death + education (+ ethnic group)
- CPS survey data: education proportions

Research ⇒

- smart synthesis of three data sources
- improved, less noisy, exposures by education level
Purpose of looking at cause of death data

- What are the key drivers of all-cause mortality?
- How are the key drivers changing over time?
- Which causes of death have high levels of inequality:
  - by education
  - other predictors
- Insight into mortality underpinning life insurance and pensions
- Insight into potential future mortality improvements

- Beware of
  - changes in ICD classification of deaths (e.g. 1999)
  - drift in how deaths are classified
  - changing education levels (grade inflation)
## Education Levels

<table>
<thead>
<tr>
<th>Education</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low education</td>
<td>Primary and lower secondary education</td>
</tr>
<tr>
<td>Medium education</td>
<td>Upper secondary education</td>
</tr>
<tr>
<td>High education</td>
<td>Tertiary education</td>
</tr>
<tr>
<td></td>
<td>Cause of Death Groupings</td>
</tr>
<tr>
<td>---</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>1</td>
<td>Infectious diseases incl. tuberculosis</td>
</tr>
<tr>
<td>2</td>
<td>Cancer: mouth, gullet, stomach</td>
</tr>
<tr>
<td>3</td>
<td>Cancer: gut, rectum</td>
</tr>
<tr>
<td>4</td>
<td>Cancer: lung, larynx, ..</td>
</tr>
<tr>
<td>5</td>
<td>Cancer: breast</td>
</tr>
<tr>
<td>6</td>
<td>Cancer: uterus, cervix</td>
</tr>
<tr>
<td>7</td>
<td>Cancer: prostate, testicular</td>
</tr>
<tr>
<td>8</td>
<td>Cancer: bones, skin</td>
</tr>
<tr>
<td>9</td>
<td>Cancer: lymphatic, blood-forming tissue</td>
</tr>
<tr>
<td>10</td>
<td>Benign tumours</td>
</tr>
<tr>
<td>11</td>
<td>Diseases: blood</td>
</tr>
<tr>
<td>12</td>
<td>Diabetes</td>
</tr>
<tr>
<td>13</td>
<td>Mental illness</td>
</tr>
<tr>
<td>14</td>
<td>Meningitis + nervous system (Alzh.)</td>
</tr>
<tr>
<td>15</td>
<td>Blood pressure + rheumatic fever</td>
</tr>
<tr>
<td>16</td>
<td>Ischaemic heart diseases</td>
</tr>
<tr>
<td>17</td>
<td>Other heart diseases</td>
</tr>
<tr>
<td>18</td>
<td>Diseases: cerebrovascular</td>
</tr>
<tr>
<td>19</td>
<td>Diseases: circulatory</td>
</tr>
<tr>
<td>20</td>
<td>Diseases: lungs, breathing</td>
</tr>
<tr>
<td>21</td>
<td>Diseases: digestive</td>
</tr>
<tr>
<td>22</td>
<td>Diseases: urine, kidney,..</td>
</tr>
<tr>
<td>23</td>
<td>Diseases: skin, bone, tissue</td>
</tr>
<tr>
<td>24</td>
<td>Senility without mental illness</td>
</tr>
<tr>
<td>25</td>
<td>Road/other accidents</td>
</tr>
<tr>
<td>26</td>
<td>Other causes</td>
</tr>
<tr>
<td>27</td>
<td>Alcohol → liver disease</td>
</tr>
<tr>
<td>28</td>
<td>Suicide</td>
</tr>
<tr>
<td>29</td>
<td>Accidental Poisonings</td>
</tr>
</tbody>
</table>
US Education Data

- Males and Females (2)
- Single ages 55-75 (21)
- Single years 1989-2015 (27)
- Causes of death (29)
- Low, medium & high education level (3)

Note: HMD’s *Human Cause of Death Database* ⇒
All ages (5’s), 1999-2015, No education
Male All Cause Death Rates by Education Group
For 1989 and 2015

Death Rate (log scale)

Low
High

Age

Low Ed 1989
Low Ed 2002
Low Ed 2015
High Ed 1989
High Ed 2002
High Ed 2015
US Education Data: Growing Inequality, Females

Female All Cause Death Rates by Education Group
For 1989 and 2015

Death Rate (log scale)

Age

55 60 65 70 75
0.002 0.005 0.010 0.020 0.050
1989
2015
High
Low
Low Ed 1989
Low Ed 2002
Low Ed 2015
High Ed 1989
High Ed 2002
High Ed 2015

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Proportion of Males with Low Education

US Males 1989–2015 Ages 55–75:
Proportion of Population with Low Education

Year 1989–2015
Age
55
56
57
58
59
60
61
62
63
64
65
66
67
68
69
70
71
72
73
74
75
30
40
50
60
70
80

Cohort diagonals ⇒ falling percentage
Widening gap
US Education Data: CoD Death Rates

Year 2000
Cancer: lung, larynx, ..

CoD Death Rate

Year 2015
Cancer: lung, larynx, ..

CoD Death Rate

Widening gap
US Education Data: CoD Death Rates

Widening gap; Mixed improvements

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US Education Data: CoD Death Rates

Widening gap; almost no improvements
US Education Data: CoD Death Rates

Year 2000
Accidental Poisonings

Year 2015
Accidental Poisonings

Case & Deaton (2015) ⇒ Accidental poisoning
US Education Data: CoD Death Rates

Widening gap
US Education Data: CoD Death Rates

Year 2000
Cancer: prostate, testicular

Year 2015
Cancer: prostate, testicular

Denmark ⇒ almost NO gap by education;
Denmark ⇒ small gap by affluence; smaller than US by education
Cause of Death Data: Health Inequalities

- Some causes of death have no obvious link to lifestyle/affluence/education
e.g. Prostate Cancer
  CancerUK: Prostate cancer is not clearly linked to any preventable risk factors.
- But education level ⇒ inequalities
- Possible explanations (a very non-expert view)
  - onset is not dependent on lifestyle/affluence/education
  - BUT lower educated ⇒
    - poorer health insurance coverage
    - later diagnosis
    - engage less well with treatment process
    - lower quality housing/diet etc.
Do Low and High education groups have the same CoD rate?

- Four $\times$ 5-year age groups
- 29 causes of death
- Signs Test (count low edu. $>$ high edu. mort.)
- $29 \times 4 = 116$ individual tests
- Reject equality hypothesis \textit{in all but one test}
- Accept $H_0$ ($p = 0.08$) for only one pairing: Meningitis + nervous system (Alzh.), 70-74
- Most $p$-values $< 10^{-6}$
Summary

Future work

- Analysis of sub-national datasets
  - e.g. SoA Group and Individual Annuity data
  - e.g. individual pension plan data
- Multiple population modelling

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Thank You!

Questions?

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Discussion Point

- Medicare kicks in after age 65
- But no obvious impact on inequality gap
- Although inequality gap naturally narrows with age
CoD Death Rates: Different Shapes & Patterns

Infectious diseases incl. tuberculosis

Meningitis + nervous system (Alzh.)

Ischaemic heart diseases

Diseases: circulatory

Diseases: lungs, breathing

Diseases: urine, kidney,...
CoD Death Rates: Different Shapes & Patterns

Cancer: gut, rectum

Death Rate (log scale)

Cancer: lung, larynx, ...

Death Rate (log scale)

Cancer: prostate, testicular

Death Rate (log scale)

Cancer: bones, skin

Death Rate (log scale)
Shapes: Conclusions

- Typically:
  - Non-cancerous diseases ⇒ approximately exponential growth
  - Neoplasms (cancers) ⇒ subexponential ??? polynomial

- What does this reveal about different disease mechanisms?