

Modelling Socio-Economic Differences in the Mortality of Danish Males Using a New Affluence Index

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Outline

- Introduction and motivation for multi-population modelling
- Constructing a new dataset
- Modelling Danish sub-population mortality
- Forecast correlations and basis risk
- Applications

1. Motivation: Stochastic Mortality

n lives, probability p of survival, N survivors

- Unsystematic mortality risk:

$$\Rightarrow N|p \sim \text{Binomial}(n, p)$$

\Rightarrow risk is diversifiable, $N/n \rightarrow p$ as $n \rightarrow \infty$

- Systematic mortality risk:

$\Rightarrow p$ is uncertain

\Rightarrow risk associated with p is not diversifiable



Motivation: Longevity Risk

Interested in *longevity risk*:

The risk that **in aggregate** people live longer **than anticipated**.

⇒ pension plan has insufficient cash to pay promised pensions



Multi-Population Challenges

- Why model multiple populations?
- Data availability
- Data quality and depth
- Model complexity
 - single population models can be complex
 - 2-population versions are more complex
 - multi-pop
- Multi-population modelling requires
 - (fairly) simple single-population models
 - simple dependencies between populations



2. A New Case Study and a New Model

- Sub-populations differ from national population
 - socio-economic factors
 - other factors
- Denmark
 - High quality data on ALL residents
 - 1981-2012 available
 - Can subdivide population using covariates on the database



- *What can we learn from Danish data that will inform us about other populations?*
- Key covariates (amongst others):
 - Wealth
 - Income



Problem

- High income \Rightarrow “affluent” *and low mortality*
BUT
- Low income \nRightarrow not affluent, high mortality
- High wealth \Rightarrow “affluent” *and low mortality*
BUT
- Low wealth \nRightarrow not affluent, high mortality

Empirical solution: use a combination

- Affluence, $A = \text{wealth} + K \times \text{income}$
- $K = 15$ seems to work well *statistically* as a predictor
- Low affluence, A , predicts poor mortality



Subdividing Data (after much experimentation!)

- Males resident in Denmark for the previous 12 months
- Divide population in year t
 - into 10 equal sized Groups (approx)
 - using *affluence, A*
- Individuals can change groups up to age 67
- Group allocations are locked down at age 67
(better than not locking down at age 67)

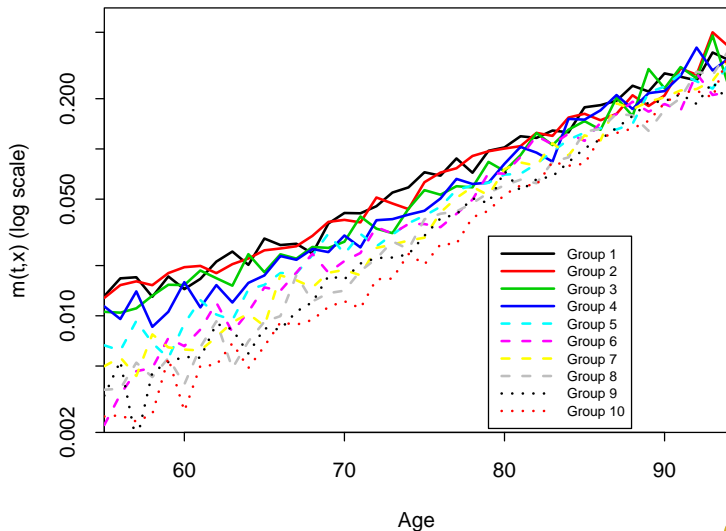


Subdivided Data

- Ages 55-94; Years 1985-2012
- Exposures $E^{(i)}(t, x)$ for groups $i = 1, \dots, 10$
range from over 4250 down to 13
- Deaths $D^{(i)}(t, x)$
range from 151 down to 4
- Crude age-specific death rates
 $\hat{m}^{(i)}(t, x) = D^{(i)}(t, x) / E^{(i)}(t, x)$
Population i , Year t , Age x
- Small groups \Rightarrow Poisson risk is important



Males Crude $m(t,x)$; 2012



Modelling the underlying death rates, $m^{(k)}(t, x)$

$m^{(k)}(t, x)$ = pop. k death rate in year t at age x

Population k , year t , age x

$$\log m^{(k)}(t, x) = \beta^{(k)}(x) + \kappa_1^{(k)}(t) + \kappa_2^{(k)}(t)(x - \bar{x})$$

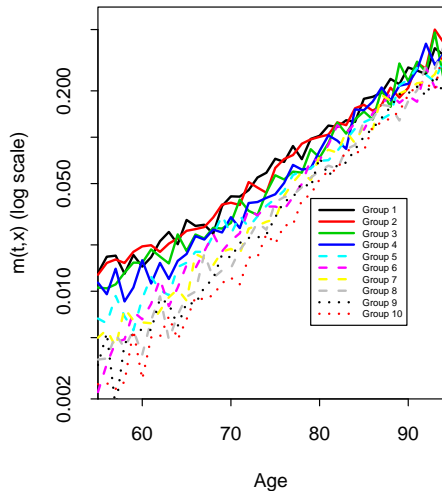
(Extended CBD with a non-parametric base table, $\beta^{(k)}(x)$)

- 10 groups, $k = 1, \dots, 10$ (low to high affluence)
- 28 years, $t = 1985, \dots, 2012$
- 40 ages, $x = 55, \dots, 94$

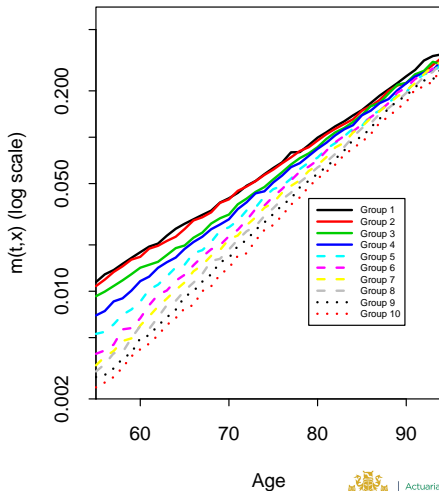


Model-Inferred Underlying Death Rates 2012

Males Crude $m(t,x)$; 2012



Males CBD-X Fitted $m(t,x)$; 2012 Point Estimates



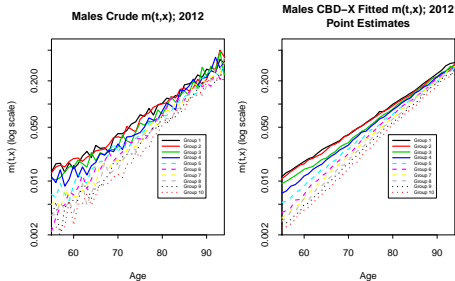
Modelling the death rates, $m_k(t, x)$

$$\log m^{(k)}(t, x) = \beta^{(k)}(x) + \kappa_1^{(k)}(t) + \kappa_2^{(k)}(t)(x - \bar{x})$$

- Model fits the 10 groups well without a cohort effect
- Non-parametric $\beta^{(k)}(x)$ is essential to preserve group rankings
 - Rankings are evident in crude data
 - *“Bio-demographical reasonableness”*:
more affluent \Rightarrow healthier



Model-Inferred Underlying Death Rates 2012

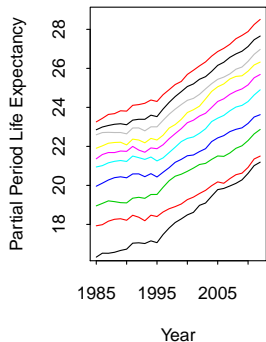


- Gap reduces from over $5\times$ to $1.3\times$
- Or **+14 years** difference for Group 1 \rightarrow 10, age 55; **+9** at 67.
- Convergence \Rightarrow way ahead for modelling very high ages???

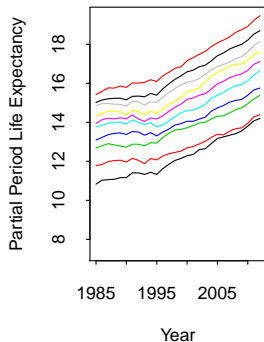


Partial Period Life Expectancy for Groups 1-10

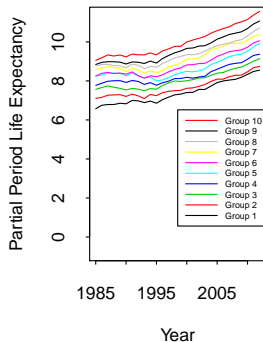
**Males Period EL:
Age 55**



**Males Period EL:
Age 65**



**Males Period EL:
Age 75**



Time series modelling

- $t \rightarrow t + 1$: Allow for correlation
 - between $\kappa_1^{(k)}(t + 1)$ and $\kappa_2^{(k)}(t + 1)$
 - between groups $k = 1, \dots, 10$

- Medium/long term, $t \rightarrow t + T$:
 - *group specific period effects gravitate towards the national trend (coherence)*
 - \Rightarrow Bio-demographical reasonableness:
 - *groups should not diverge*



A specific model

$$\begin{aligned}\kappa_1^{(i)}(t) &= \kappa_1^{(i)}(t-1) + \mu_1 + Z_{1i}(t) && \text{(random walk)} \\ &\quad - \psi \left(\kappa_1^{(i)}(t-1) - \bar{\kappa}_1(t-1) \right) && \text{(gravity between groups)} \\ \kappa_2^{(i)}(t) &= \kappa_2^{(i)}(t-1) + \mu_2 + Z_{2i}(t) \\ &\quad - \psi \left(\kappa_2^{(i)}(t-1) - \bar{\kappa}_2(t-1) \right)\end{aligned}$$

where

$$\bar{\kappa}_1(t) = \frac{1}{n} \sum_{i=1}^n \kappa_1^{(i)}(t) \quad \text{and} \quad \bar{\kappa}_2(t) = \frac{1}{n} \sum_{i=1}^n \kappa_2^{(i)}(t)$$



A specific model

$$\kappa_1^{(i)}(t) = \kappa_1^{(i)}(t-1) + \mu_1 + Z_{1i}(t) - \psi \left(\kappa_1^{(i)}(t-1) - \bar{\kappa}_1(t-1) \right)$$

$$\kappa_2^{(i)}(t) = \kappa_2^{(i)}(t-1) + \mu_2 + Z_{2i}(t) - \psi \left(\kappa_2^{(i)}(t-1) - \bar{\kappa}_2(t-1) \right)$$

Model structure \Rightarrow

- $(\bar{\kappa}_1(t), \bar{\kappa}_2(t)) \sim$ bivariate random walk
- Each $\kappa_1^{(i)}(t) - \bar{\kappa}_1(t) \sim AR(1)$ reverting to 0
- Each $\kappa_2^{(i)}(t) - \bar{\kappa}_2(t) \sim AR(1)$ reverting to 0
- $\beta^{(i)}(x)$ vs $\beta^{(j)}(x) \Rightarrow$ intrinsic group differences



Non-trivial correlation structure: between different ages and groups

$$\begin{aligned}\kappa_1^{(i)}(t) &= \kappa_1^{(i)}(t-1) + \mu_1 + Z_{1i}(t) - \psi \left(\kappa_1^{(i)}(t-1) - \bar{\kappa}_1(t-1) \right) \\ \kappa_2^{(i)}(t) &= \kappa_2^{(i)}(t-1) + \mu_2 + Z_{2i}(t) - \psi \left(\kappa_2^{(i)}(t-1) - \bar{\kappa}_2(t-1) \right)\end{aligned}$$

The Z_{ki} are multivariate normal, mean 0 and

$$\text{Cov}(Z_{ki}, Z_{lj}) = \begin{cases} v_{kl} & \text{for } i = j \\ \rho v_{kl} & \text{for } i \neq j \end{cases}$$

ρ = cond. correlation between $\kappa_1^{(i)}(t)$ and $\kappa_1^{(j)}(t)$ etc.



- Model is very simple
 - One gravity parameter, $0 < \psi < 1$
 - One between-group correlation parameter, $0 < \rho < 1$
- Many generalisations are possible
- But more parameters + more complex computing
- This simple model seems to fit quite well.
- Nevertheless \Rightarrow potential for further work



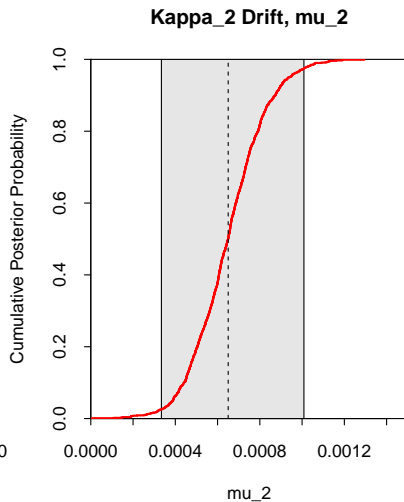
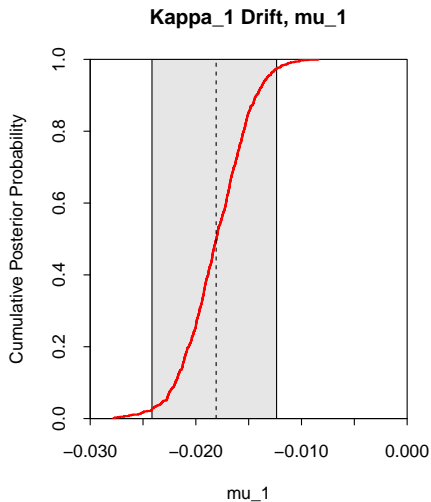
Prior distributions

- As uninformative as possible
- $\mu_1, \mu_2 \sim$ improper uniform prior
- $\{v_{ij}\} \sim$ Inverse Wishart
- $\rho \sim \text{Beta}(2, 2)$
- $\psi \sim \text{Beta}(2, 2)$

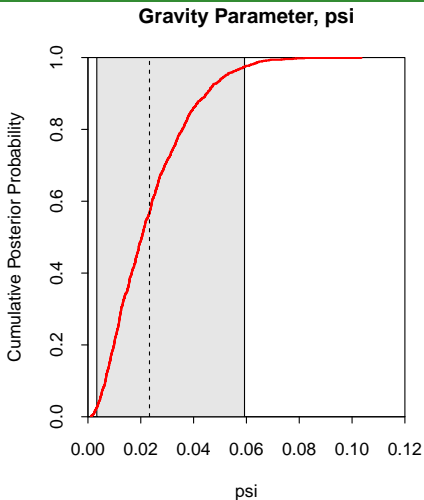
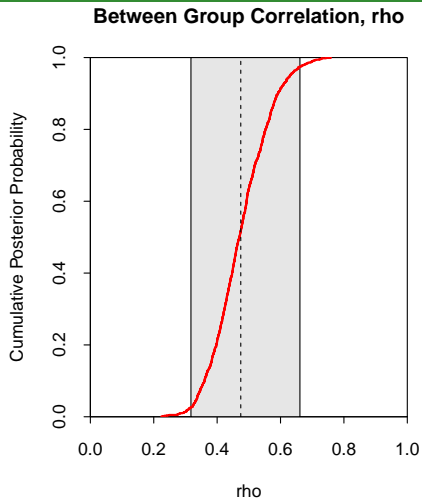
State variables and process parameters estimated using MCMC (Gibbs + Metropolis-Hastings)



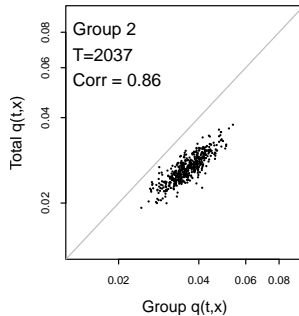
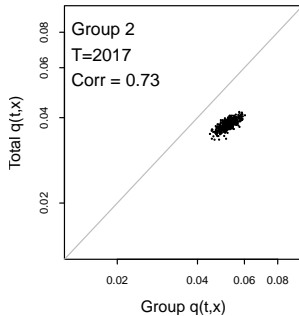
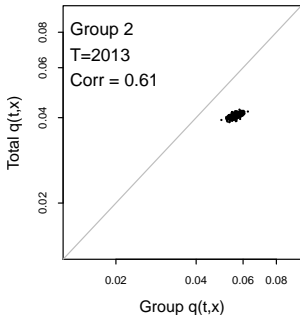
Posterior Distributions and 95% Credibility Intervals



Posterior Distributions and 95% Credibility Intervals



Simulated Group versus Population Mortality, $q(t, x)$



As T increases: +1 year; +5 years; +25years

- Scatterplots become more dispersed
- Shift down and to the left
- Correlation increases



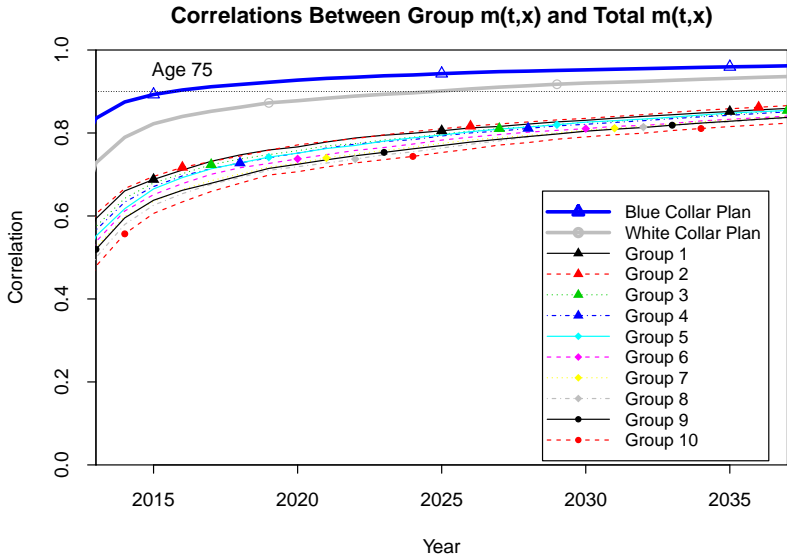
Deciles are quite narrow subgroups

More diversified e.g.

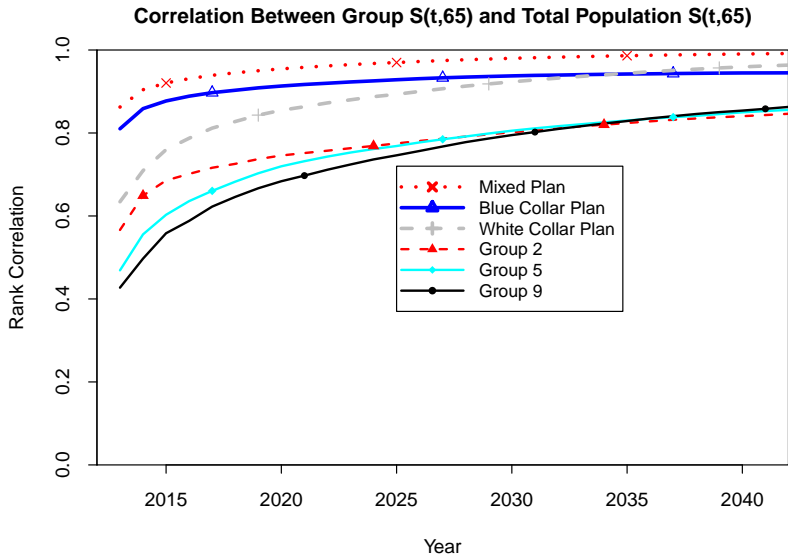
- **Blue collar pension plan**
⇒ equal proportions of groups 2, 3, 4
- **White collar pension plan**
⇒ equal proportions of groups 8, 9, 10
- **Mixed plan**
⇒ proportions $(0, 0, 1, 2, \dots, 7, 8)/36$ (e.g. amounts)



Forecast Correlations: Mortality Rates at Age 75

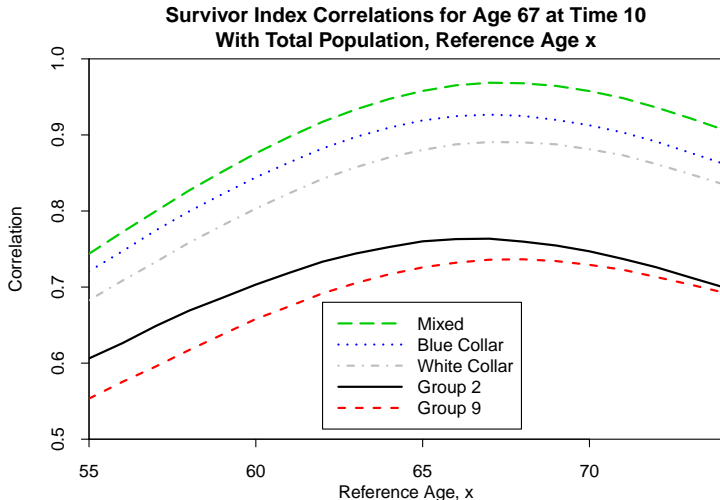


Forecast Correlations: Cohort Survivorship from 65



Forecast Correlations: Cohort Survivorship

Different reference ages: $cor(S_X(10, 67), S_{TOT}(10, x))$



Modelling Conclusions

- Development of a new multi-population dataset for Denmark
 - strong bio-demographically reasonable group rankings based on a new measure of affluence*
- Unlike multi-country data
 - a priori* ranking of affluence-related groups
- Proposal for a simple new multi-population model
- Mortality rates converge at high ages
- Strong correlations over medium to long term even allowing for parameter uncertainty
- Correlations depend strongly on diversity of sub-population



3. Postscript 1: Education as an Alternative Covariate

- **Level of Educational Attainment** also known to be a good predictor
 - Various US studies
 - Mackenbach et al. (2003) including Denmark: **Std. Mortality Rates**
 - Brønnum-Hansen and Baadsgaard (2012) Denmark: **$LE(x = 30)$**

- As close as possible on a *like for like* basis:

Crude death rates; age 30+; matching years.

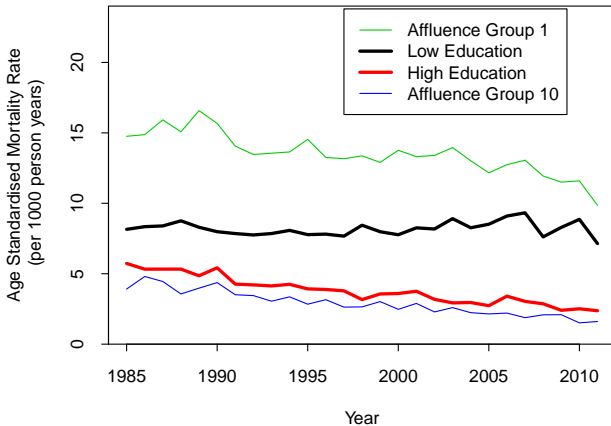
Affluence \Rightarrow

- Wider spread of SMR's than M. et al. (2003)
- Wider spread of $LE(30)$ than BHB (2012)
- Issue: “grade inflation” distorts results
- More to be done.



3. Postscript: Education as an Alternative Covariate

**Age Standardised Mortality Rates per 1000
Ages 45–54; European Standard Population (1976)**

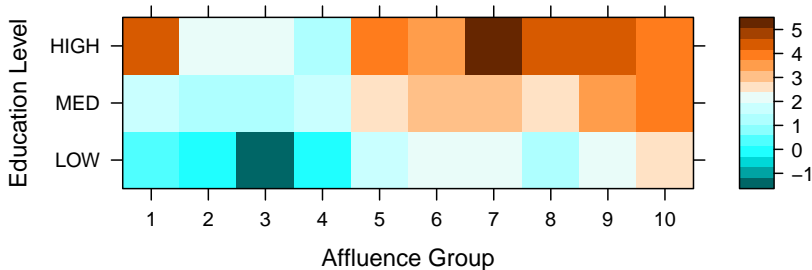


3. Postscript: Education as an Alternative Covariate

Dig a bit deeper:

Affluence + Education: average ASMR's over 5 years

Mortality Improvement Rates (%)
Period 1987–2009; Age Band 45–54
By Affluence and Education Group

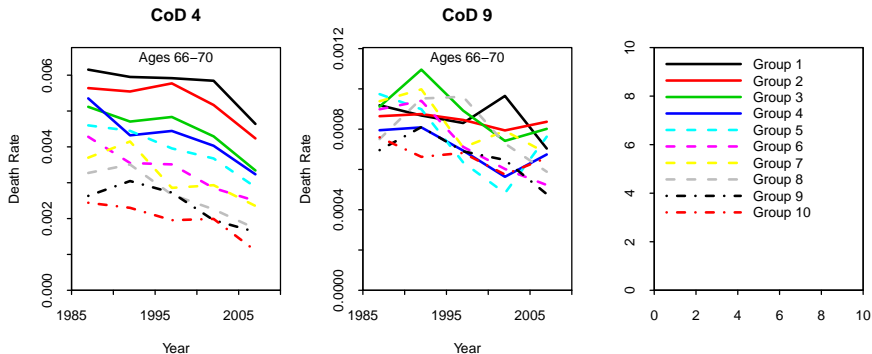


Postscript 2: Cause of Death Data – Health Inequalities

- Deaths subdivided into 29 CoD groups
- Compare affluence groups
- Biggest differences at younger age groups e.g. 51-55
- Causes of death linked to lifestyle
⇒ some CoD death rates are up to 20× higher for low affluence groups
- Growing gaps: liver diseases; diabetes
- *Almost all CoD groups have a strong statistically significant difference*



Denmark: Cause of Death Data – Health Inequalities



- 5×5 ages and years
- CoD4: Lung cancer and related cancers
- CoD9: Cancer of lymphatic or blood-forming tissues

- Some causes of death have **no obvious link** to lifestyle/affluence/education
- Possible explanations (a very non-expert view)
 - *onset* is not dependent on lifestyle/affluence/education
 - BUT less affluent/educated \Rightarrow
 - ??? later diagnosis
 - ??? engage less well with treatment process
 - ??? lower quality housing



4. Summary

- Danish data allows insight into relative mortality dynamics between socio-economic sub-populations
- Conclusions for other countries likely to be similar
- Additional education covariate
- Health inequalities

Working paper available on website.

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Thank You!

Questions?

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