

# Index Based Longevity Hedging as a Practical Risk Mitigation Tool for Deferred Pension Liabilities

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Longevity 15, Washington, September 2019



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- Why are deferred pensioners harder to hedge?
- Practical considerations
- Case study
- Conclusions

## Intro: Why are deferred pensions difficult to hedge?

- Pension plan **deferred pensioners**
  - perceived as more risky than pensions in payment
  - purchase of *individual annuities* potentially expensive or not possible (no active insurers)
  - potential conversion options before/at retirement
- Pension plan **active members**
  - ⇒ still accruing pension
- Potential solution:
  - use **index based longevity instruments to achieve a partial hedge**
  - Counterparty: e.g. reinsurer; capital markets
  - Precedent: Hannover Re deal with NN Life

# Practical Issues to Consider When Assessing a Hedge

- Hedging objective and risk appetite
- Hedge instrument maturity date,  $T$
- Hedge instrument design and structure
  
- Models:
  - Practice versus academic ideal (one model)
  
  - Valuation model at time 0:  $ML(0)$
  - Simulation model at time 0:  $MS(0)$
  
  - Hedge instrument payoff model at time  $T$ :  $MH(T)$
  - Liability valuation model at time  $T$ :  $ML(T)$

## Current UK Practice: models $ML(0)$ and $ML(T)$

- Valuations and buyouts etc. in the UK typically rely on Excel software for **mortality improvements** produced by the **CMI (Continuous Mortality Investigation)**
- Current version:  
*The CMI Mortality Projections Model CMI\_2018*
- **Data:** historical *national* deaths and exposures (pop. 1)
- **Model fitting:** Fit the *Age Period Cohort Improvements (APCI)* model to historical data

$$\log m_{\text{APCI}}(t, x) = \alpha(x) + \beta(x)(t - \bar{t}) + \kappa(t) + \gamma(t - x)$$

- Minimise *Deviance + roughness penalty*  
 $\Rightarrow$  smooth curves for  $\alpha(x), \beta(x), \kappa(t), \gamma(t - x)$
- The APCI model is recalibrated every year: CMI\_YYYY

## Current UK Practice: models $ML(0)$ and $ML(T)$ (cont.)

- Year 0: final year in data
  - $IP(0, x)$  = age-period *improvement rate* in final year
  - $IC(0, x)$  = cohort-linked *improvement rate* in final year
  - $m_B(0, x)$ : base table (not  $m_{APCI}(0, x)$ )
- Projections ( $t > 0$ ):
  - *Not the APCI model!*
  - Starts from  $m_B(0, x)$ , and  $IP(0, x)$  &  $IC(0, x)$
  - $IP(t, x)$  glides smoothly from  $IP(0, x)$  to a *long term rate*
  - $IC(t, x + t)$  follows cohorts and glides smoothly to 0
  - Use  $IP, IC$  to generate future  $m(t, x)$
- Here:  
build continued use of the CMI\_yyyy model into our model with automated recalibration

## Current UK Practice: Population 2

- Population 1: national population
- Population 2: sub-population (e.g. pension plan)
- Experience ratios:

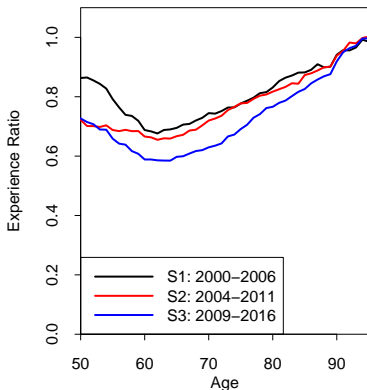
$$ER(T, x) = \frac{m_2(T, x)}{m_1(T, x)} \quad (\text{or } q_2/q_1)$$

or an average over the last  $K$  years

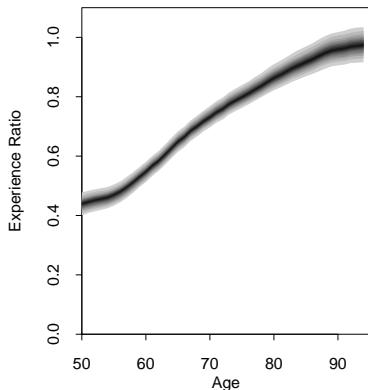
- Valuation at  $T$ :
  - Project  $m_1(t, x)$  for  $t > T$  using recalibrated  $CMI_T$
  - Recalibrate experience ratios:  $ER(T, x)$
  - Project  $m_2(t, x) = m_1(t, x)ER(T, x)$  for  $t > T$
- In practice: experience ratios are stochastic  
⇒ build this into our model: population basis risk

# $ER(T, x)$ : CMI SAPS experience vs Danish model

**CMI SAPS (UK Pension Fund)  
Pensioner Experience Ratios**



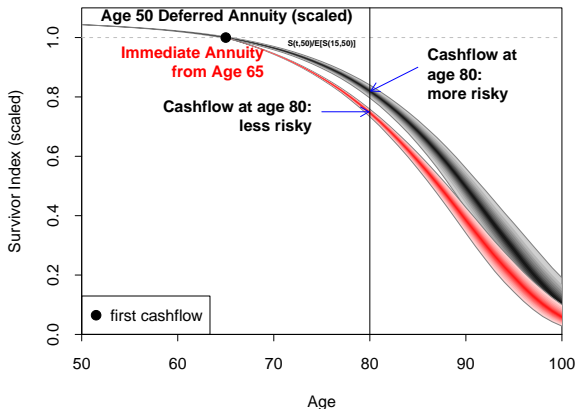
**Simulated Experience Ratios in 2026  
Synthetic Danish Pension Plan**



- CMI SAPS: *pensioner* data (i.e. no pre-retirement or deferred members)
- Denmark: Cairns et al. (2019) simulations
- Some differences; some similarities



# Deferred pensions are perceived as more risky



- Deferred  $\Rightarrow$  more risk: but this is complex
- Uncertainty in cashflow  $\times$  discount factor
- Inclusion/exclusion of large, highly certain cashflows

## Reasons why a deferred pension is more difficult to hedge

Run-off risk: 95% Quantile versus Median

(Real) Interest Rate	Deferred Annuity Age 50	Immediate Annuity Age 50	Immediate Annuity Age 65	Immediate Annuity Age 80
0%	+6.6%	+4.0%	+5.4%	+4.9%
2%	+5.1%	+2.6%	+3.9%	+4.1%

- Deferred  $\Rightarrow$  more risky than immediate
- Lower interest rates  $\Rightarrow$  more risk
- Immediate annuity:  
Younger  $\Rightarrow$  more risky in absolute terms  
In relative terms: more complex

## Non-hedgeable risks

- In deferment and at retirement
  - member might take cash transfer
  - conversion options at retirement
  - uncertainty over marital/cohabitation status at later date
  - uncertain accrual of further pension benefits
- Customised (member specific) transaction becomes more expensive or impossible
- Increased administration
  
- Index-based transaction *reduces* risk without the ongoing administrative hassle

## Case Study

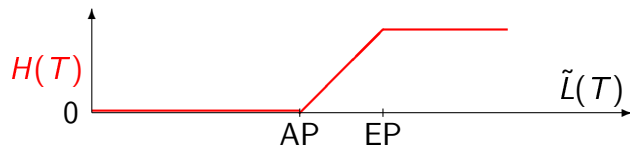
- Index: Danish males national mortality
- Pension plan: affluence deciles 7, 8, 9
- Cohort of males aged 50
- Pensions deferred to age 65
- Plan objective:  
to buy out the pensions when they vest in 15 years

# Hedging Instrument

Index-based hedge (derivative)

- Synthetic  $\tilde{L}(T) \approx$  true  $L(T)$
- Call spread derived from underlying  $\tilde{L}(T)$

Payoff at  $T$ , per unit



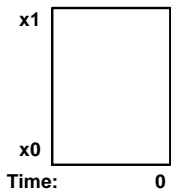
$$H(T) = \begin{cases} 0 & \text{if } \tilde{L}(T) < AP & \text{(Attachment Point)} \\ \tilde{L}(T) - AP & \text{if } AP \leq \tilde{L}(T) < EP & \text{(Exhaustion Point)} \\ EP - AP & \text{if } EP \leq \tilde{L}(T) \end{cases}$$

Call spread  $\longleftarrow$  SPV  $\longrightarrow$  Cat. bond

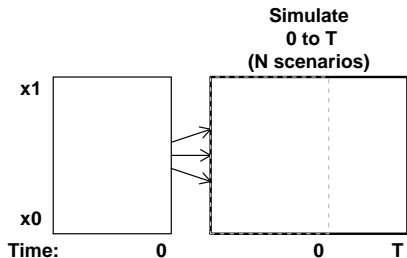
## The Synthetic $\tilde{L}(T)$

- $\tilde{L}$  = random PV at time 0 of a portfolio of synthetic liabilities
- Synthetic mortality experience
  - based on national population mortality
  - adjusted using **experience ratios**,  $ER(0, x)$
- $\tilde{L}(T)$  = point estimate of  $\tilde{L}$  based on info at  $T$   
= PV of actual *synthetic* cashflows up to  $T$   
+ PV of estimated *synthetic* cashflows after  $T$   
using a model specified at time 0.

## Historical Data + Initial Valuation

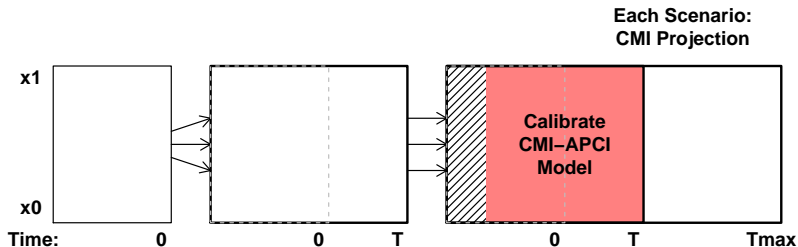


- Time 0 valuation model  $ML(0)$



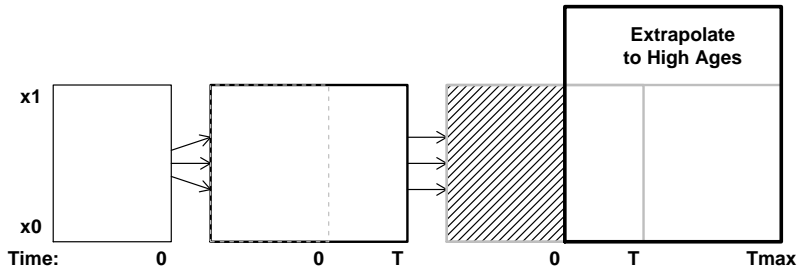
- Identify and fit a suitable two/multi-population stochastic mortality model:  $MS(0)$
- Use this model to generate stochastic scenarios

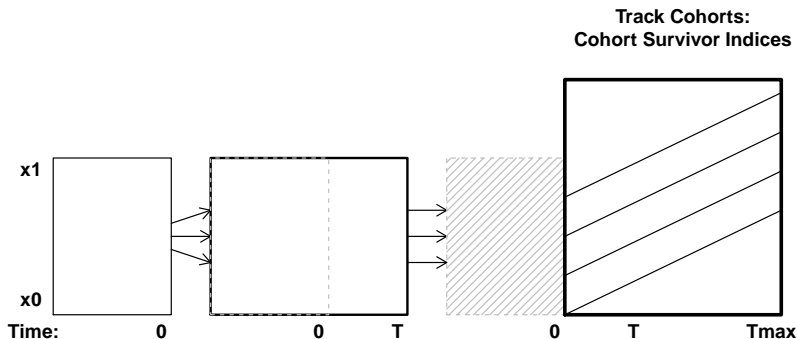




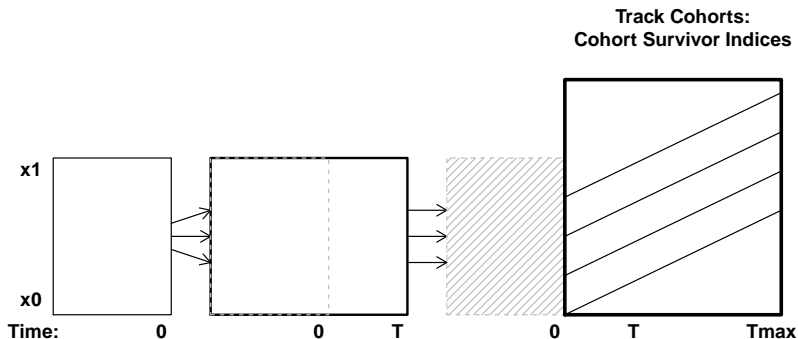
- $ML(T)$  and possibly  $MH(T)$ :
- Scenario  $j$ : recalibrate the APCI model using data up to  $T$
- Use the CMI projections model to project scenario  $j$  mortality beyond time  $T$

# Methodology





- Scenario  $j \Rightarrow$  survivor indices  $S(j, T, t, x)$  for cohorts aged  $x$  at the start of year 1
- Simulation scenario  $j$  to  $T$
- Model recalibration and scenario  $j$  projection beyond  $T$



- $$S(j, T, t, x) \longrightarrow a(j, T, x) = \sum_{t=0}^{\infty} e^{-rt} B(t) S(j, T, t, x)$$

$$B(t) = \begin{cases} 0 & \text{in deferment (e.g. before age 65)} \\ 1 & \text{in payment (e.g. 65 and older)} \end{cases}$$

## Case Study: Danish Males Mortality + Stochastic Model

- Cairns, Kallestrup et al. (2019) ASTIN Bulletin
- National population subdivided into *deciles* by *affluence*
- 1995-2016; ages 50-94 (updated data)
- Decile  $i$ :

$$\log m(i, t, x) = \alpha(x) + \kappa_1(t) + \kappa_2(t)(x - \bar{x})$$

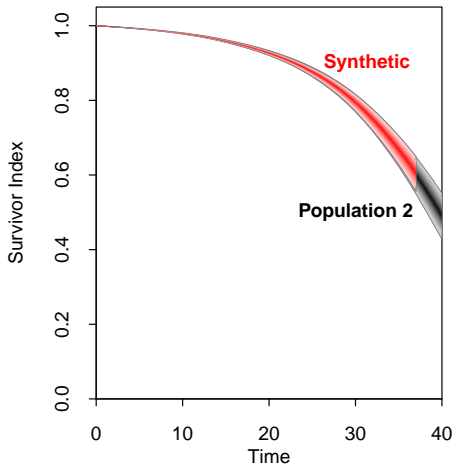
- For this study:
  - Simulate the 10 deciles
  - Aggregate into
    - (a) national population
    - (b) white collar pension plan = deciles 7, 8, 9
  - Simulation incorporates full *parameter uncertainty*  
e.g. drift; average spread between groups

## Results

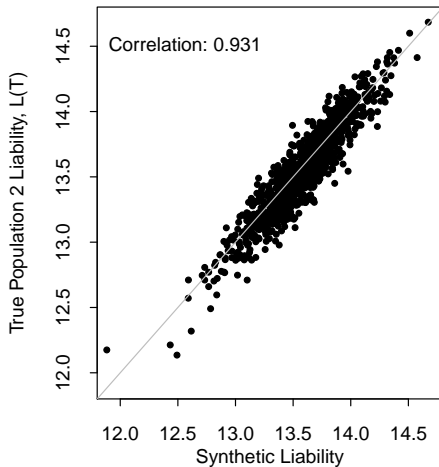
- Age 50; deferred pension from 65
- $T = 15$  hedge maturity
- Attachment point:  $AP = \text{median}$  of  $\tilde{L}(T)$
- Exhaustion point:  $EP = 90\%$  quantile of  $\tilde{L}(T)$
- Assess the overall impact of the hedge and at the 95% level

# True liability versus synthetic liability: 1000 scenarios

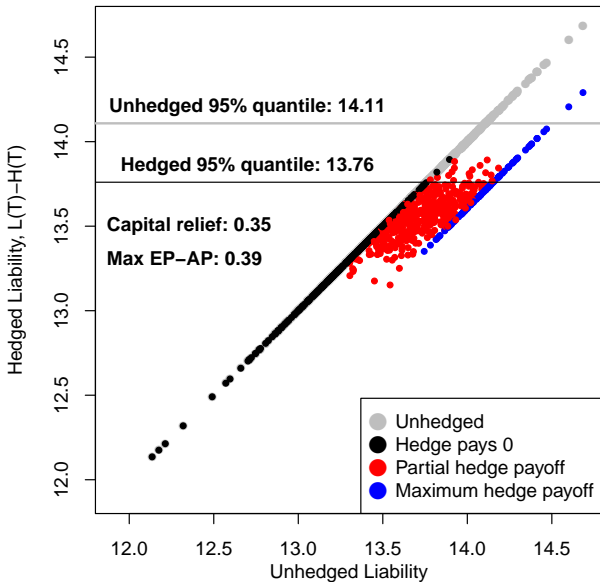
Survivor Index Uncertainty  
Population 2 versus Synthetic Index



Synthetic versus True Liability



# Impact of hedge on time $T = 15$ liability





## Summary

All discounted to time 0:

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Unhedged:

Mean liability, $E[L(T)]$ :	13.30	
95% quantile:	14.11	(+6.1%)

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Hedged:

95% quantile:	13.76	(+3.5%)
Capital relief:	0.35	
Max capital relief ( $EP - AP$ ):	0.39	(e.g. no/lower pop. basis risk)

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Hedge payoff

Mean, $E[H(T)]$ :	0.11	
Hedge price:	???	$> E[H(T)]$

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# Conclusions

- Deferred pensions are more difficult to hedge or buy out than pensions in payment
- Index based longevity hedges offer a possible solution
- Counterparties: reinsurers or capital markets
- Assessment requires careful specification of all models at time 0 and time  $T$



# Thank You!

## Questions?

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The 'Modelling, Measurement and Management of Longevity and Morbidity Risk' research programme is being funded by the ARC, the SoA and the CIA.

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# Bonus slides

## Model $MS(0)$ for simulating future mortality

- Model used for simulating mortality in populations 1 and 2 at times  $0 < t \leq T$ , or all times  $t > 0$  for assessment of full run-off
- Calibrated at time 0
- Step 1: fit the model to historical mortality data
- Step 2: choose a time series model for forecasting
- Step 3: estimate time series parameters for forecasting
- Generate  $N$  stochastic scenarios  $m_S(i, j, t, x)$  for populations  $i = 1, 2$ , scenario  $j$ , year  $t$ , age  $x$ .

## Model $MS(0)$ for simulating future mortality (cont.)

- Alternatively, we might choose a time series model in advance and then calibrate the model to historical mortality rates and the forecasting parameters simultaneously (e.g. Bayesian).

## Model $MH(T)$ for the hedge instrument payoff

- Scenario  $j$
- Simulation scenario  $j$  gives  $m_S(i, j, t, x)$  for populations  $i = 1, 2$  for  $t = 1, \dots, T$
- Used to calculate the hedge instrument payoff at  $T$
- Calibrated at  $T$  including central forecasts of improvement rates after  $T$
- Calibration uses reference population (population 1) mortality up to  $T$  (population 2 mortality is not used)
- Method of calibration is specified at time 0



## Model $MH(T)$ for the hedge instrument payoff (cont.)

- Refer to this model and calibration as  $MH(T)$ .
- Median projection for  $T + 1, T + 2, \dots$  using time  $T$  calibration
- Giving

$$m_H(1, j, t, x) = \begin{cases} m_S(1, j, t, x) & \text{for } t = 1, \dots, T \\ \tilde{m}_H(T, 1, j, t, x) & \text{for } t = T + 1, T + 2, \dots \end{cases}$$

- Convert to  
 $q_H(1, j, t, x) = 1 - \exp[-m_H(1, j, t, x)]$

## Model $MH(T)$ for the hedge instrument payoff (cont.)

- Calculate the synthetic mortality rates for the hedged population 2

$$q_H(2, j, t, x) = q_H(1, j, t, x) \epsilon_H(0, x)$$

where the  $\epsilon_H(0, x)$  are the experience ratios embedded in the hedge contract at time 0

- Define  $p_H(2, j, t, x) = 1 - q_H(2, j, t, x)$
- Calculate the synthetic cohort survival rates

$$S_H(T, 2, j, t, x) = p_H(2, j, 1, x) \times \dots \times p_H(2, j, t, x + t - 1)$$

- Calculate the synthetic liability  $\tilde{L}(T)$  (discounted to time 0)

- Calculate the hedge instrument payoff  $H(T)$

## Model $ML(T)$ for liability valuation at time $T$

- Scenario  $j$
- Simulation scenario  $j$  gives  $m_S(i, j, t, x)$  for populations  $i = 1, 2$  for  $t = 1, \dots, T$
- Specify what the liability valuation model at time will be (ML).  
For example: a combination of the CMI APCI model and experience ratios. Hence
- Recalibrate ML at time  $T$ :  $ML(T)$ .
- Recalibrate the experience ratios  $\epsilon_L(T, x)$
- Within  $ML(T)$ : calibrate the improvement rates for years  $T + 1, T + 2, \dots$

## Model $ML(T)$ for liability valuation at time $T$ (cont.)

- Calculate the median (or best estimate) mortality projection at the core ages
- Project mortality rates to higher ages
- For  $t = 1, \dots, T$ :
  - $q_L(T, 1, j, t, x) = q_S(1, j, t, x)$
  - $q_L(T, 2, j, t, x) = q_S(2, j, t, x)$
- For  $t = T + 1, T + 2, \dots$ 
  - $q_L(T, 1, j, t, x) = q_L(T, 1, j, t, x)$  (i.e.  $ML(T)$  population 1 projection)
  - $q_L(T, 2, j, t, x) = q_L(T, 1, j, t, x) \epsilon_L(T, x)$
- Calculate the cohort survival rates  $S_L(T, i, j, t, x)$  using the  $q_L(T, i, j, t, x)$
- Calculate the liability  $L(j, T)$ .

# DK historical experience ratios

