Index Based Longevity Hedging as a Practical Risk Mitigation Tool for Deferred Pension Liabilities

Andrew J.G. Cairns

Heriot-Watt University, Edinburgh

and

Director, Actuarial Research Centre,

Institute and Faculty of Actuaries

Longevity 15, Washington, September 2019
Outline

- Why are deferred pensioners harder to hedge?
- Practical considerations
- Case study
- Conclusions
Intro: Why are deferred pensions difficult to hedge?

- Pension plan deferred pensioners
  - perceived as more risky than pensions in payment
  - purchase of *individual annuities* potentially expensive or not possible (no active insurers)
  - potential conversion options before/at retirement

- Pension plan active members
  ⇒ still accruing pension

- Potential solution:
  use index based longevity instruments to achieve a partial hedge

Counterparty: e.g. reinsurer; capital markets
Precedent: Hannover Re deal with NN Life
Practical Issues to Consider When Assessing a Hedge

- Hedging objective and risk appetite
- Hedge instrument maturity date, $T$
- Hedge instrument design and structure

Models:
- Practice versus academic ideal (one model)
- Valuation model at time 0: $ML(0)$
- Simulation model at time 0: $MS(0)$
- Hedge instrument payoff model at time $T$: $MH(T)$
- Liability valuation model at time $T$: $ML(T)$
Current UK Practice: models $ML(0)$ and $ML(T)$

- Valuations and buyouts etc. in the UK typically rely on Excel software for mortality improvements produced by the CMI (Continuous Mortality Investigation)

- Current version:
  *The CMI Mortality Projections Model CMI_2018*

- Data: historical *national* deaths and exposures (pop. 1)

- Model fitting: Fit the *Age Period Cohort Improvements* (APCI) model to historical data

\[
\log m_{APCI}(t, x) = \alpha(x) + \beta(x)(t - \bar{t}) + \kappa(t) + \gamma(t - x)
\]

- Minimise *Deviance + roughness penalty*
  \[\Rightarrow\] smooth curves for \(\alpha(x), \beta(x), \kappa(t), \gamma(t - x)\)

- The APCI model is recalibrated every year: CMI_YYYY
Year 0: final year in data
- \( IP(0, x) = \text{age-period improvement rate in final year} \)
- \( IC(0, x) = \text{cohort-linked improvement rate in final year} \)
- \( m_B(0, x) \): base table (not \( m_{APCI}(0, x) \))

Projections \( (t > 0) \):
- \( \text{Not the APCI model!} \)
- Starts from \( m_B(0, x) \), and \( IP(0, x) \) & \( IC(0, x) \)
- \( IP(t, x) \) glides smoothly from \( IP(0, x) \) to a \textit{long term rate}
- \( IC(t, x + t) \) follows cohorts and glides smoothly to 0
- Use \( IP, IC \) to generate future \( m(t, x) \)

Here:
build continued use of the CMI\_yyyy model into our model with automated recalibration
Population 1: national population
Population 2: sub-population (e.g. pension plan)

Experience ratios:

\[ ER(T, x) = \frac{m_2(T, x)}{m_1(T, x)} \]  
(or \( q_2/q_1 \))

or an average over the last \( K \) years

Valuation at \( T \):

- Project \( m_1(t, x) \) for \( t > T \) using recalibrated CMI_\( T \)
- Recalibrate experience ratios: \( ER(T, x) \)
- Project \( m_2(t, x) = m_1(t, x)ER(T, x) \) for \( t > T \)

In practice: experience ratios are stochastic

\[ \Rightarrow \text{build this into our model: population basis risk} \]
$ER(T, x)$: CMI SAPS experience vs Danish model

- CMI SAPS: pensioner data (i.e. no pre-retirement or deferred members)
- Denmark: Cairns et al. (2019) simulations
- Some differences; some similarities
Deferred pensions are perceived as more risky

- Deferred $\Rightarrow$ more risk: but this is complex
- Uncertainty in cashflow $\times$ discount factor
- Inclusion/exclusion of large, highly certain cashflows
Run-off risk: 95% Quantile versus Median

<table>
<thead>
<tr>
<th>(Real) Interest Rate</th>
<th>Deferred Annuity Age 50</th>
<th>Immediate Annuity Age 50</th>
<th>Immediate Annuity Age 65</th>
<th>Immediate Annuity Age 80</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>+6.6%</td>
<td>+4.0%</td>
<td>+5.4%</td>
<td>+4.9%</td>
</tr>
<tr>
<td>2%</td>
<td>+5.1%</td>
<td>+2.6%</td>
<td>+3.9%</td>
<td>+4.1%</td>
</tr>
</tbody>
</table>

- Deferred ⇒ more risky than immediate
- Lower interest rates ⇒ more risk
- Immediate annuity:
  - Younger ⇒ more risky in absolute terms
  - In relative terms: more complex
Non-hedgeable risks

- In deferment and at retirement
  - member might take cash transfer
  - conversion options at retirement
  - uncertainty over marital/cohabitation status at later date
  - uncertain accrual of further pension benefits
- Customised (member specific) transaction becomes more expensive or impossible
- Increased administration

- Index-based transaction reduces risk without the ongoing administrative hassle
Case Study

- Index: Danish males national mortality
- Pension plan: affluence deciles 7, 8, 9
- Cohort of males aged 50
- Pensions deferred to age 65
- Plan objective:
  to buy out the pensions when they vest in 15 years
Hedging Instrument

Index-based hedge (derivative)

- Synthetic $\tilde{L}(T) \approx$ true $L(T)$
- Call spread derived from underlying $\tilde{L}(T)$

Payoff at $T$, per unit

\[
H(T) = \begin{cases} 
0 & \text{if } \tilde{L}(T) < AP \\
\tilde{L}(T) - AP & \text{if } AP \leq \tilde{L}(T) < EP \\
EP - AP & \text{if } EP \leq \tilde{L}(T)
\end{cases}
\]

Call spread $\longleftrightarrow$ SPV $\longrightarrow$ Cat. bond
The Synthetic $\tilde{L}(T)$

- $\tilde{L} = \text{random PV at time 0 of a portfolio of synthetic liabilities}$
- \text{Synthetic mortality experience}
  - based on national population mortality
  - adjusted using \textit{experience ratios}, $ER(0, x)$

- $\tilde{L}(T) = \text{point estimate of } \tilde{L} \text{ based on info at } T$
  $\quad = \text{PV of actual } \textit{synthetic} \text{ cashflows up to } T$
  $\quad + \text{PV of estimated } \textit{synthetic} \text{ cashflows after } T$
  $\quad \text{using a model specified at time 0.}$
Historical Data + Initial Valuation

- Time 0 valuation model $ML(0)$
Methodology

- Identify and fit a suitable two/multi-population stochastic mortality model: \( MS(0) \)
- Use this model to generate stochastic scenarios
Methodology

- $ML(T)$ and possibly $MH(T)$:
- Scenario $j$: recalibrate the APCI model using data up to $T$
- Use the CMI projections model to project scenario $j$ mortality beyond time $T$
Methodology

Extrapolate to High Ages

$T \leq T_{max}$

$x_0$

$x_1$

$0$  $0$  $T$

$0$  $T$  $T_{max}$
Scenario $j \Rightarrow$ survivor indices $S(j, T, t, x)$ for cohorts aged $x$ at the start of year 1

Simulation scenario $j$ to $T$

Model recalibration and scenario $j$ projection beyond $T$
\[ S(j, T, t, x) \longrightarrow a(j, T, x) = \sum_{t=0}^{\infty} e^{-rt} B(t) S(j, T, t, x) \]

\[ B(t) = \begin{cases} 
0 & \text{in deferment (e.g. before age 65)} \\
1 & \text{in payment (e.g. 65 and older)} 
\end{cases} \]
Cairns, Kallestrup et al. (2019) ASTIN Bulletin

National population subdivided into *deciles* by *affluence*

1995-2016; ages 50-94 (updated data)

Decile $i$:

$$\log m(i, t, x) = \alpha(x) + \kappa_1(t) + \kappa_2(t)(x - \bar{x})$$

For this study:
- Simulate the 10 deciles
- Aggregate into
  - (a) national population
  - (b) white collar pension plan = deciles 7, 8, 9
- Simulation incorporates full *parameter uncertainty*
  - e.g. drift; average spread between groups
• Age 50; deferred pension from 65
• $T = 15$ hedge maturity
• Attachment point: $AP = \text{median of } \tilde{L}(T)$
• Exhaustion point: $EP = 90\% \text{ quantile of } \tilde{L}(T)$
• Assess the overall impact of the hedge and at the 95\% level
True liability versus synthetic liability: 1000 scenarios

Survivor Index Uncertainty
Population 2 versus Synthetic Index

Synthetic
Population 2

Correlation: 0.931
Hedged Liability, $L(T) - H(T)$

Impact of hedge on time $T = 15$ liability

Unheded 95% quantile: 14.11
Hedged 95% quantile: 13.76
Capital relief: 0.35
Max EP–AP: 0.39

Graph showing hedged liability vs. unhedged liability for different quantiles.
### Summary

All discounted to time 0:

<table>
<thead>
<tr>
<th></th>
<th>Unhedged:</th>
<th>Hedged:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean liability, $E[L(T)]$:</td>
<td>13.30</td>
<td></td>
</tr>
<tr>
<td>95% quantile:</td>
<td>14.11 (+6.1%)</td>
<td>13.76 (+3.5%)</td>
</tr>
<tr>
<td>Capital relief:</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>Max capital relief ($EP - AP$):</td>
<td>0.39 (e.g. no/lower pop. basis risk)</td>
<td></td>
</tr>
<tr>
<td>Hedge payoff</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean, $E[H(T)]$:</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>Hedge price:</td>
<td>??? &gt; $E[H(T)]$</td>
<td></td>
</tr>
</tbody>
</table>
Conclusions

- Deferred pensions are more difficult to hedge or buy out than pensions in payment
- Index based longevity hedges offer a possible solution
- Counterparties: reinsurers or capital markets
- Assessment requires careful specification of all models at time 0 and time $T$
Thank You!

Questions?

E: A.J.G.Cairns@hw.ac.uk

W: www.macs.hw.ac.uk/~andrewc/ARCresources
The Actuarial Research Centre (ARC) is the Institute and Faculty of Actuaries’ (IFoA) network of actuarial researchers around the world. The ARC seeks to deliver cutting-edge research programmes that address some of the significant, global challenges in actuarial science, through a partnership of the actuarial profession, the academic community and practitioners.

The ‘Modelling, Measurement and Management of Longevity and Morbidity Risk’ research programme is being funded by the ARC, the SoA and the CIA.

www.actuaries.org.uk/arc
Model $MS(0)$ for simulating future mortality

- Model used for simulating mortality in populations 1 and 2 at times $0 < t \leq T$, or all times $t > 0$ for assessment of full run-off
- Calibrated at time 0
- Step 1: fit the model to historical mortality data
- Step 2: choose a time series model for forecasting
- Step 3: estimate time series parameters for forecasting
- Generate $N$ stochastic scenarios $m_S(i, j, t, x)$ for populations $i = 1, 2$, scenario $j$, year $t$, age $x$. 
Alternatively, we might choose a time series model in advance and then calibrate the model to historical mortality rates and the forecasting parameters simultaneously (e.g. Bayesian).
Model $MH(T)$ for the hedge instrument payoff

- Scenario $j$
- Simulation scenario $j$ gives $m_S(i, j, t, x)$ for populations $i = 1, 2$ for $t = 1, \ldots, T$
- Used to calculate the hedge instrument payoff at $T$
- Calibrated at $T$ including central forecasts of improvement rates after $T$
- Calibration uses reference population (population 1) mortality up to $T$ (population 2 mortality is not used)
- Method of calibration is specified at time 0
Refer to this model and calibration as $MH(T)$.

Median projection for $T + 1, T + 2, \ldots$ using time $T$ calibration

Giving

$$m_H(1, j, t, x) = \begin{cases} 
m_S(1, j, t, x) & \text{for } t = 1, \ldots, T \\
\tilde{m}_H(T, 1, j, t, x) & \text{for } t = T + 1, T + 2, \ldots \end{cases}$$

Convert to

$$q_H(1, j, t, x) = 1 - \exp[-m_H(1, j, t, x)]$$
Model $MH(T)$ for the hedge instrument payoff (cont.)

- Calculate the synthetic mortality rates for the hedged population 2
  
  $$q_H(2, j, t, x) = q_H(1, j, t, x) \epsilon_H(0, x)$$

  where the $\epsilon_H(0, x)$ are the experience ratios embedded in the hedge contract at time 0

- Define $p_H(2, j, t, x) = 1 - q_H(2, j, t, x)$

- Calculate the synthetic cohort survival rates
  
  $$S_H(T, 2, j, t, x) = p_H(2, j, 1, x) \times \ldots \times p_H(2, j, t, x + t - 1)$$

- Calculate the synthetic liability $\tilde{L}(T)$ (discounted to time 0)

- Calculate the hedge instrument payoff $H(T)$. 
**Model** \( ML(T) \) for liability valuation at time \( T \)

- **Scenario \( j \)**
- **Simulation scenario \( j \) gives** \( m_S(i,j,t,x) \) **for populations** \( i = 1, 2 \) **for** \( t = 1, \ldots, T \)
- **Specify what the liability valuation model at time will be** (ML). For example: a combination of the CMI APCI model and experience ratios. Hence
- **Recalibrate ML at time \( T \):** \( ML(T) \).
- **Recalibrate the experience ratios** \( \epsilon_L(T,x) \)
- **Within** \( ML(T) \): calibrate the improvement rates for years \( T + 1, T + 2, \ldots \)
Model $ML(T)$ for liability valuation at time $T$ (cont.)

- Calculate the median (or best estimate) mortality projection at the core ages
- Project mortality rates to higher ages
- For $t = 1, \ldots, T$:
  - $q_L(T, 1, j, t, x) = q_S(1, j, t, x)$
  - $q_L(T, 2, j, t, x) = q_S(2, j, t, x)$
- For $t = T + 1, T + 2, \ldots$
  - $q_L(T, 1, j, t, x) = q_L(T, 1, j, t, x)$ (i.e. $ML(T)$ population 1 projection)
  - $q_L(T, 2, j, t, x) = q_L(T, 1, j, t, x)\epsilon_L(T, x)$

- Calculate the cohort survival rates $S_L(T, i, j, t, x)$ using the $q_L(T, i, j, t, x)$
- Calculate the liability $L(j, T)$. 
DK historical experience ratios

DK males 1995–2016 Experience Ratios
Whole pop versus Groups 7–9