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Index-Based Longevity Hedging: Calculating Capital Relief

Andrew Cairns

Joint work with Ghali El Boukfaoui (formerly Soc. Gen.)

Life Convention, Liverpool, November 2018

Proposed Approach for Calculating Capital Relief:

(Paper: Cairns and El Boukfaoui (2018) to appear in North American Actuarial Journal)

- Open up communication lines with the regulator
- Establish which method for calculating SCR (and RM)
- Discuss with the regulator
- Document fully all mortality forecasting models
- Discuss with the regulator
- Run the SCR calculations with and without hedge
- Sensitivity and other robustness tests
- Discuss with the regulator
- Agree capital relief for time 0 and times 1, 2,…

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Calculating the SCR Capital Relief

- $\boldsymbol{L}$ = PV of the full runoff: own liabilities
  - $S(t, x)$ = survivor index for cohort aged x at time 0
- $\boldsymbol{\tilde{L}}$ = PV of the full runoff: synthetic portfolio of liabilities; depends on:
  - $q_G(t, x)$ = general population mortality rates
  - $ER(0, t, x)$ = experience ratios hard coded at time 0
  - $\tilde{S}(t, x)$ = synthetic survivor index
- Experience ratios =>
  - $E[S(t, x)] \approx E[\tilde{S}(t, x)]$
  - $E[L] \approx E[\tilde{L}]$

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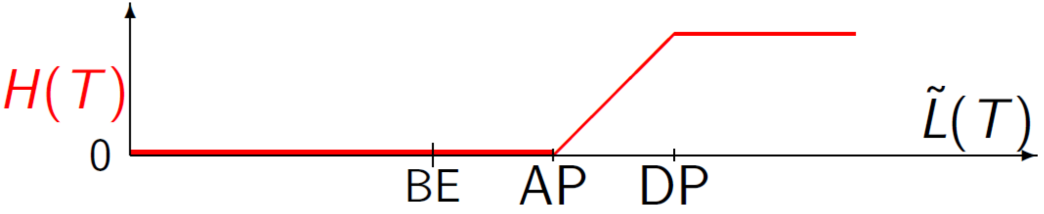
Values

- $L(0) \rightarrow \boldsymbol{L(1)} \rightarrow \boldsymbol{L(T)} \rightarrow \boldsymbol{L(\infty)} = \boldsymbol{L}$
- E.g. $L(T)$ = liability value given mortality experience up to T (point estimate)
- $\tilde{L}(0) \rightarrow \boldsymbol{\tilde{L}(1)} \rightarrow \boldsymbol{\tilde{L}(T)}$
- Hedge payoff $\boldsymbol{H(T)} = \boldsymbol{h(\tilde{L}(T))}$



- AP = Attachment Point
- DP = Detachment Point (or Exhaustion Point, EP)

“UK” approach

- SCR = 99.5% one-year VaR; based on $L(1) - H(1)$

Present Value at Time 0	Hedger’s liability	Synthetic liability	Hedge payoff
Full runoff	$\boldsymbol{L}$	$\tilde{\boldsymbol{L}}$	
Valued at Time T	$\boldsymbol{L(T)}$	$\tilde{\boldsymbol{L}}\boldsymbol{(T)}$	$\boldsymbol{H(T)}$
Valued at Time 1	$\boldsymbol{L(1)}$	$\tilde{\boldsymbol{L}}\boldsymbol{(1)}$	$\boldsymbol{H(1)}$
Valued at Time 0	$\boldsymbol{L(0)}$	$\tilde{\boldsymbol{L}}\boldsymbol{(0)}$	$\boldsymbol{H(0)}$

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Original Dutch Approach (incl. Cairns and El Boukfaoui)

- SCR = 99.5% T-year VaR; based on $L(T) - H(T)$

Present Value at Time 0	Hedger's liability	Synthetic liability	Hedge payoff
Full runoff	$\boldsymbol{L}$	$\boldsymbol{\tilde{L}}$	
Valued at Time T	$\boldsymbol{L(T)}$	$\boldsymbol{\tilde{L}(T)}$	$\boldsymbol{H(T)}$
Valued at Time 1	$\boldsymbol{L(1)}$	$\boldsymbol{\tilde{L}(1)}$	$\boldsymbol{H(1)}$
Valued at Time 0	$\boldsymbol{L(0)}$	$\boldsymbol{\tilde{L}(0)}$	$\boldsymbol{H(0)}$

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Alternative

- 95% VaR instead of 99.5% (approximately (?) the same as 1-year 99.5% VaR)
  (needs regulator engagement!)
- Full runoff: $L(\infty) - H(T) = L - H(T)$ instead of $L(T) - H(T)$

Present Value at Time 0	Hedger's liability	Synthetic liability	Hedge payoff
Full runoff	$\boldsymbol{L = L(\infty)}$	$\boldsymbol{\tilde{L}}$	
Valued at Time T	$\boldsymbol{L(T)}$	$\boldsymbol{\tilde{L}(T)}$	$\boldsymbol{H(T)}$
Valued at Time 1	$\boldsymbol{L(1)}$	$\boldsymbol{\tilde{L}(1)}$	$\boldsymbol{H(1)}$
Valued at Time 0	$\boldsymbol{L(0)}$	$\boldsymbol{\tilde{L}(0)}$	$\boldsymbol{H(0)}$

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Three Types of Basis Risk

- Population basis risk
  - Hedger’s experience and synthetic index are not perfectly correlated
- Structural basis risk
  - Non-linear payoff => no risk reduction beyond DP or below AP
- Tail basis risk
  - Cashflows after maturity, *T*:
    - Depend on risk emerging before *T* and after *T*
    - Risks before *T* are (can be) hedged
    - Additional risks emerging after maturity, T, are not hedged

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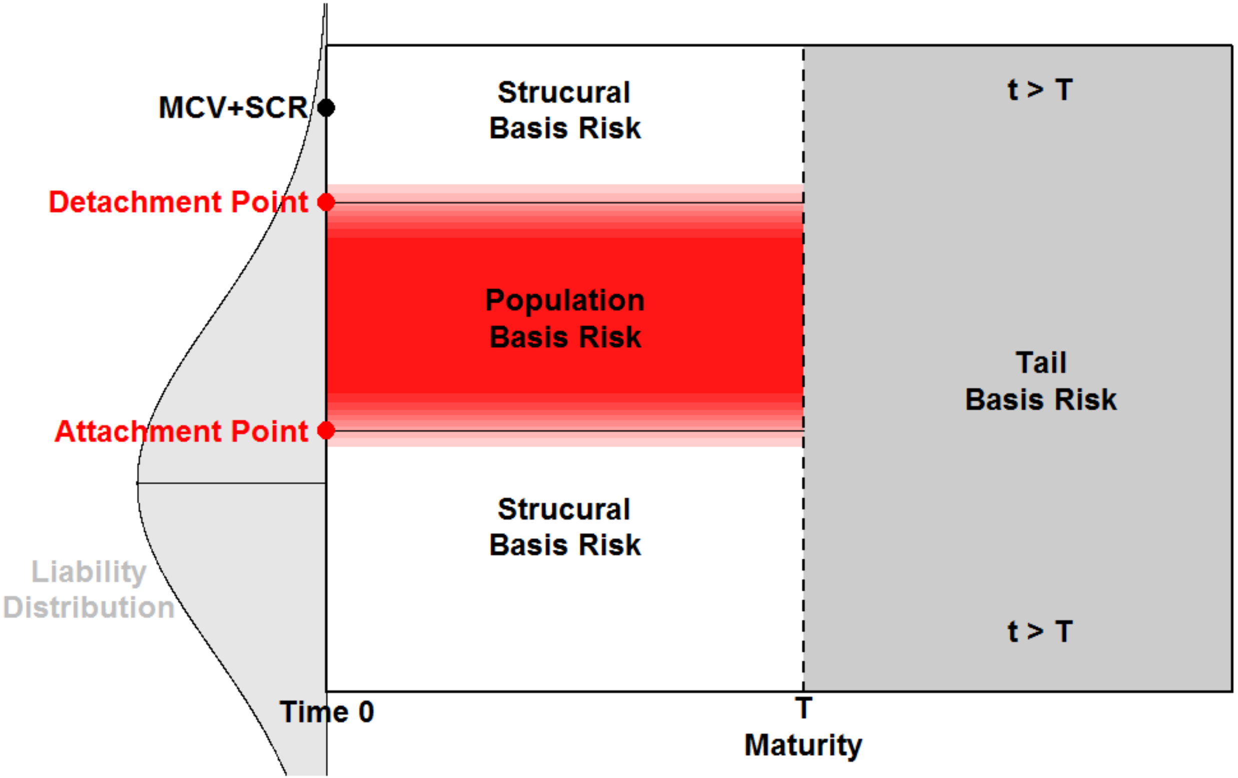
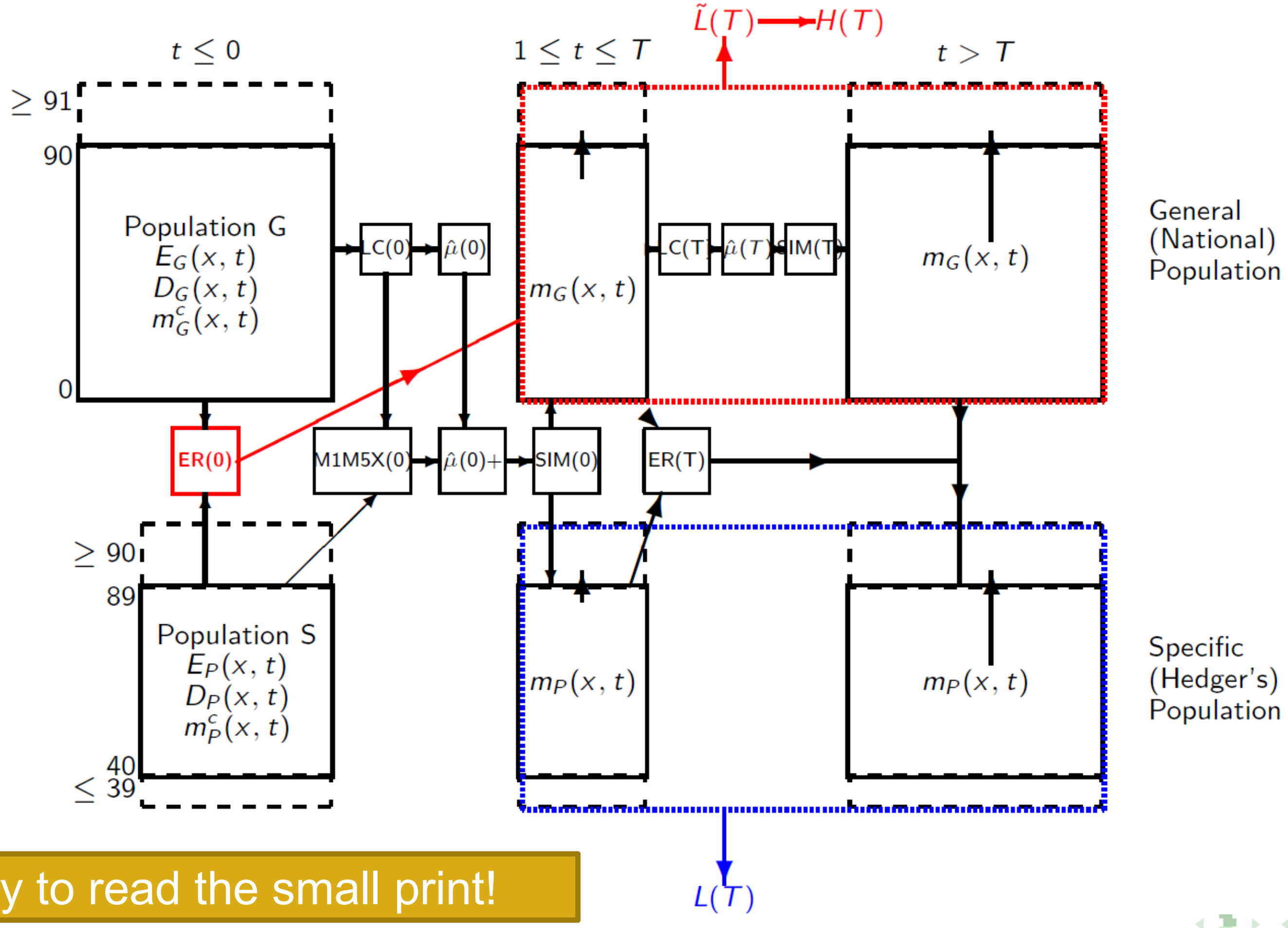
Three Types of Basis Risk: Stylised



Anatomy of a Hedging Calculation



Don't try to read the small print!

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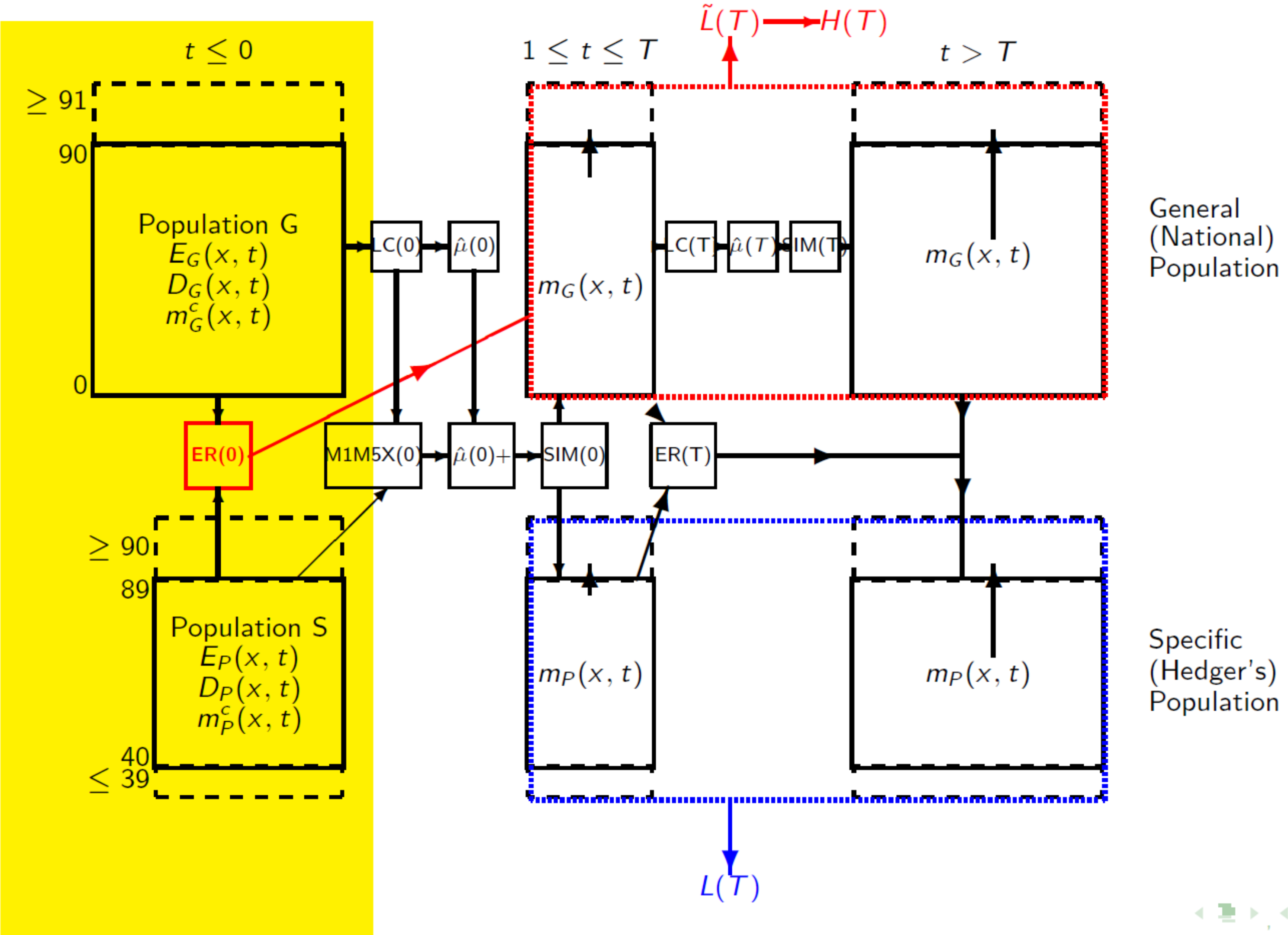

$\tilde{L}(T) \longrightarrow H(T)$
$t \leq 0$
$1 \leq t \leq T$
$t > T$
≥ 91
90
Population G
$E_G(x,t)$
$D_G(x,t)$
$m_G^c(x,t)$
0
LC(0)
$\hat{\mu}(0)$
$m_G(x,t)$
LC(T)
$\hat{\mu}(T)$
SIM(T)
$m_G(x,t)$
General
(National)
Population
ER(0)
M1M5X(0)
$\hat{\mu}(0)+$
SIM(0)
ER(T)
≥ 90
89
Population S
$E_P(x,t)$
$D_P(x,t)$
$m_P^c(x,t)$
40
≤ 39
$m_P(x,t)$
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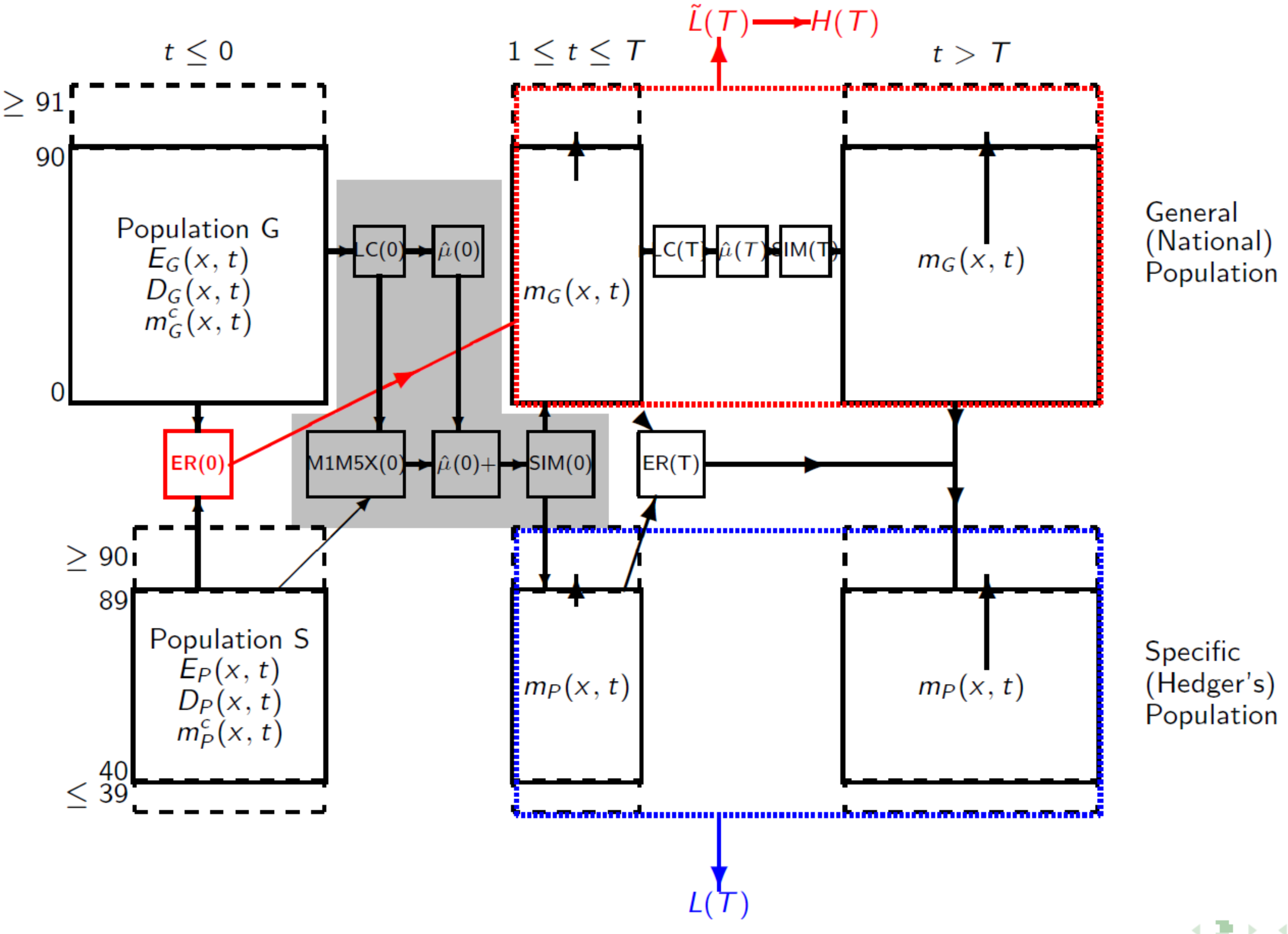
Specific
(Hedger's)
Population
$L(T)$



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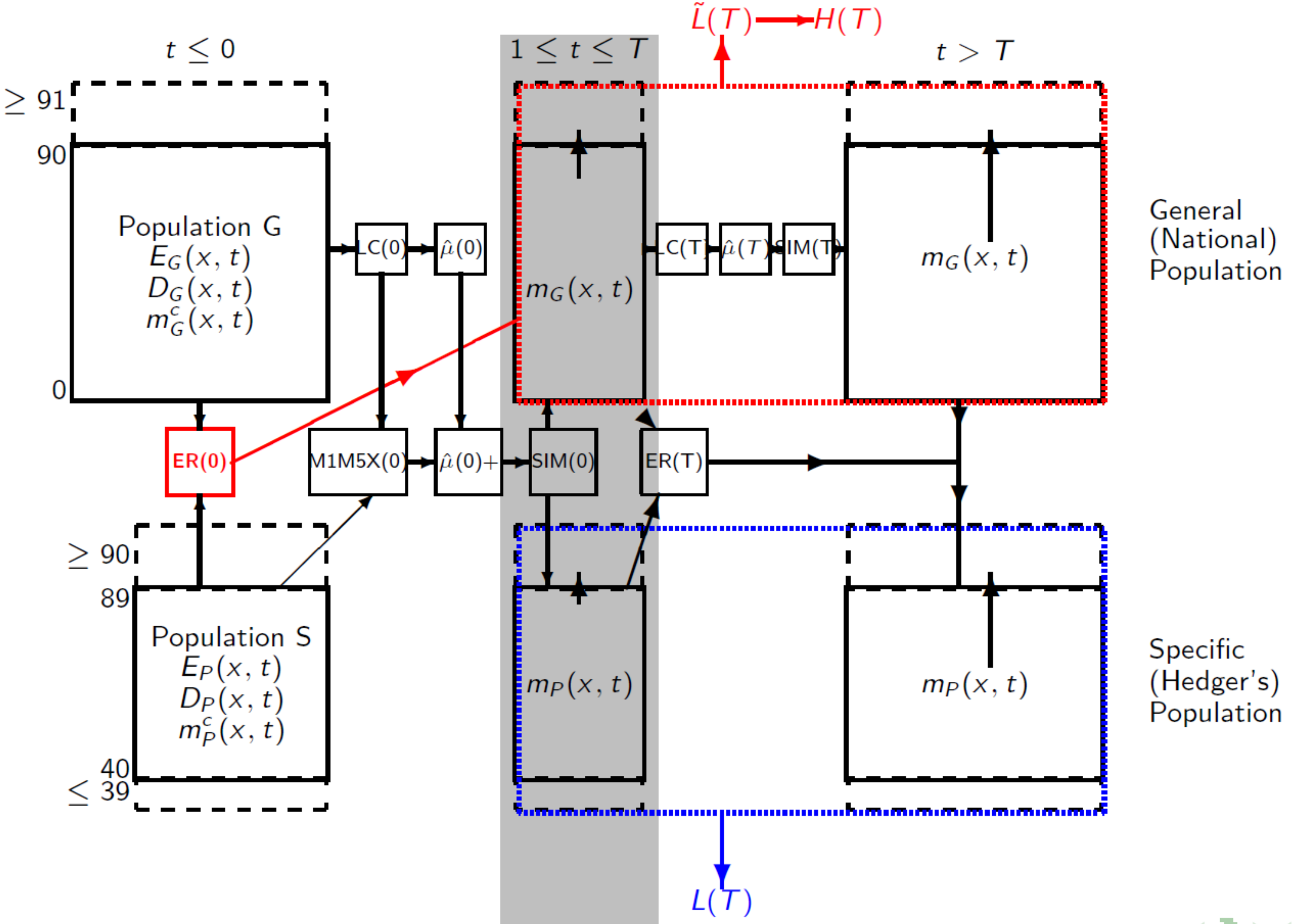

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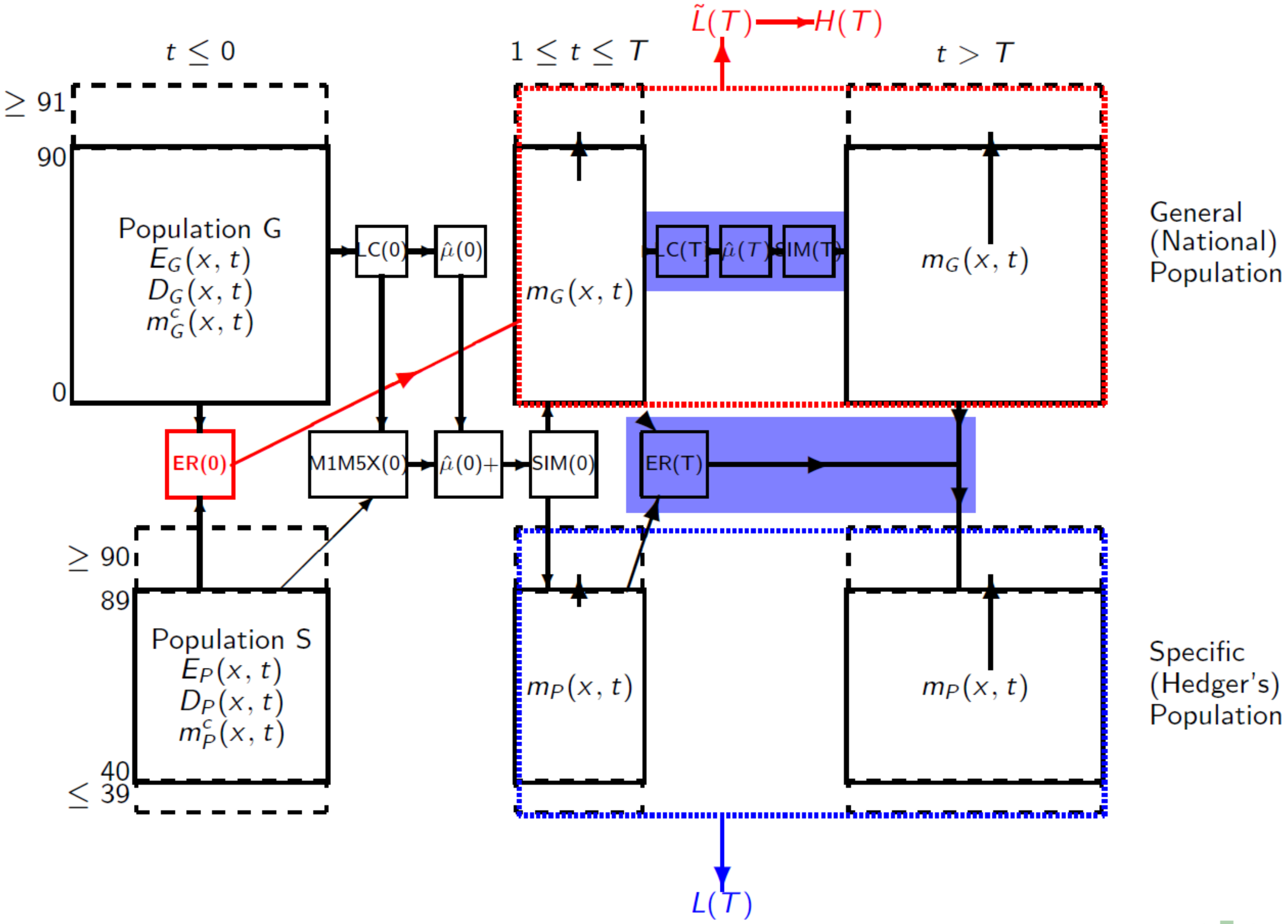
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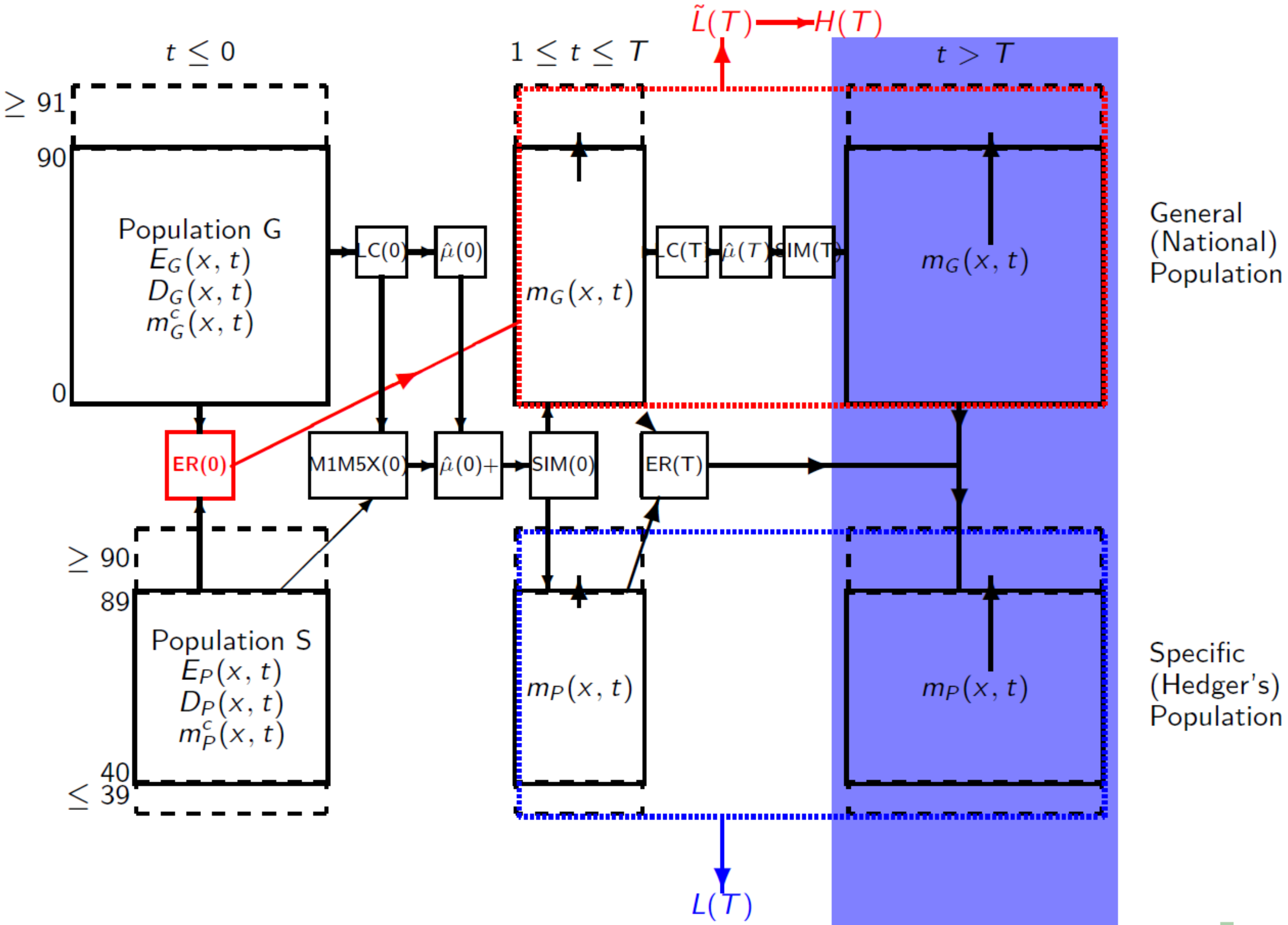
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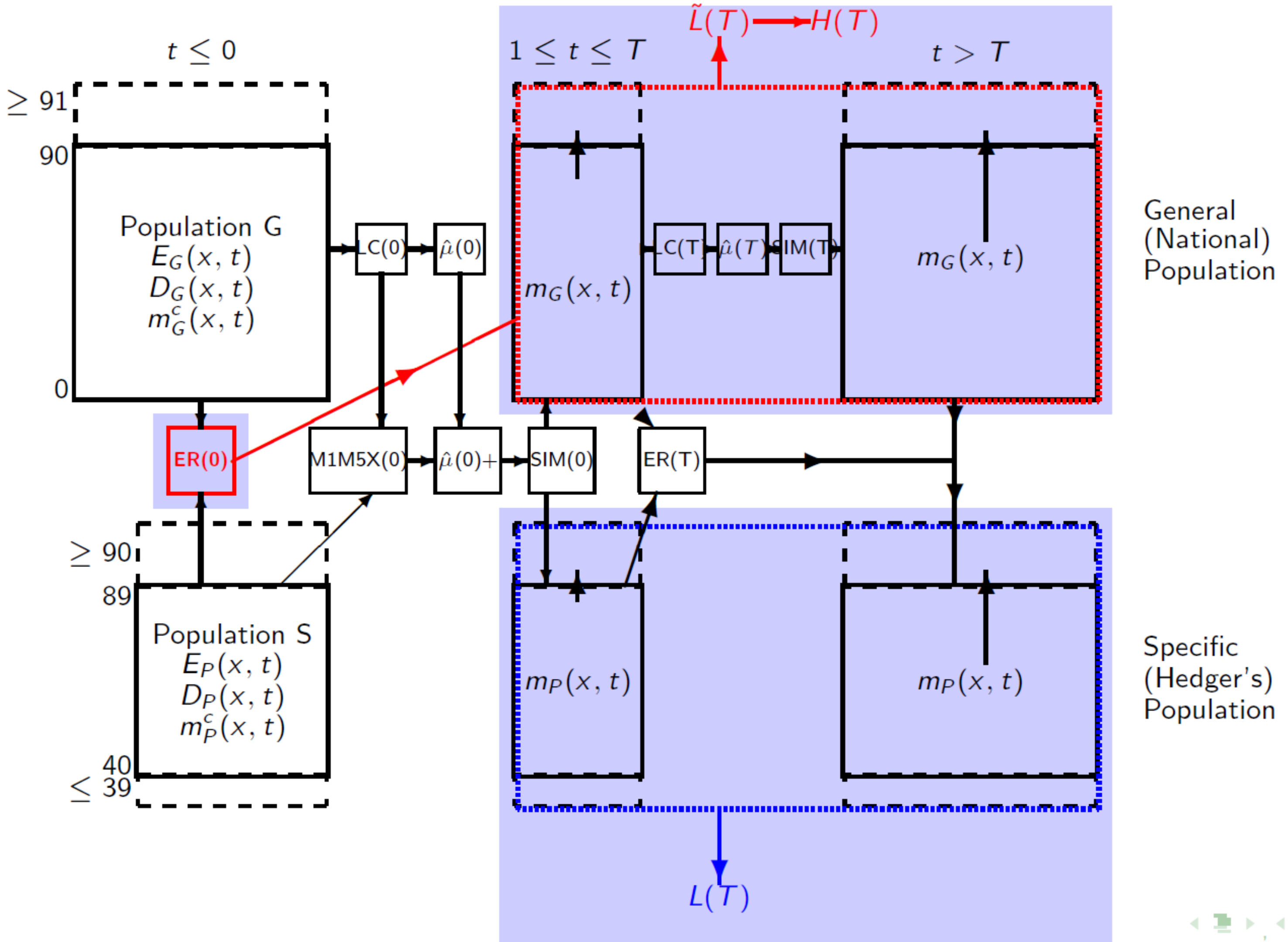

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Capital Relief Calculation Summary

- Simulated L(T), H(T) => hedged distribution => capital relief
- In advance, discuss and agree with the regulator the following:
  - Break down the process into a series of manageable steps
  - Document all of these steps carefully
  - Document clearly all of the models being used in each step

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Simulated Impact of Hedge at Time T

- Dutch insured lives dataset versus Dutch national mortality data
- Portfolio = mixture of deferred and immediate annuities
- Hedge: 10-year bull spread
- Case 1
  - AP=60% quantile of the synthetic liability
  - DP=95% quantile
- Case 2
  - AP=65% quantile
  - DP=99.5% quantile

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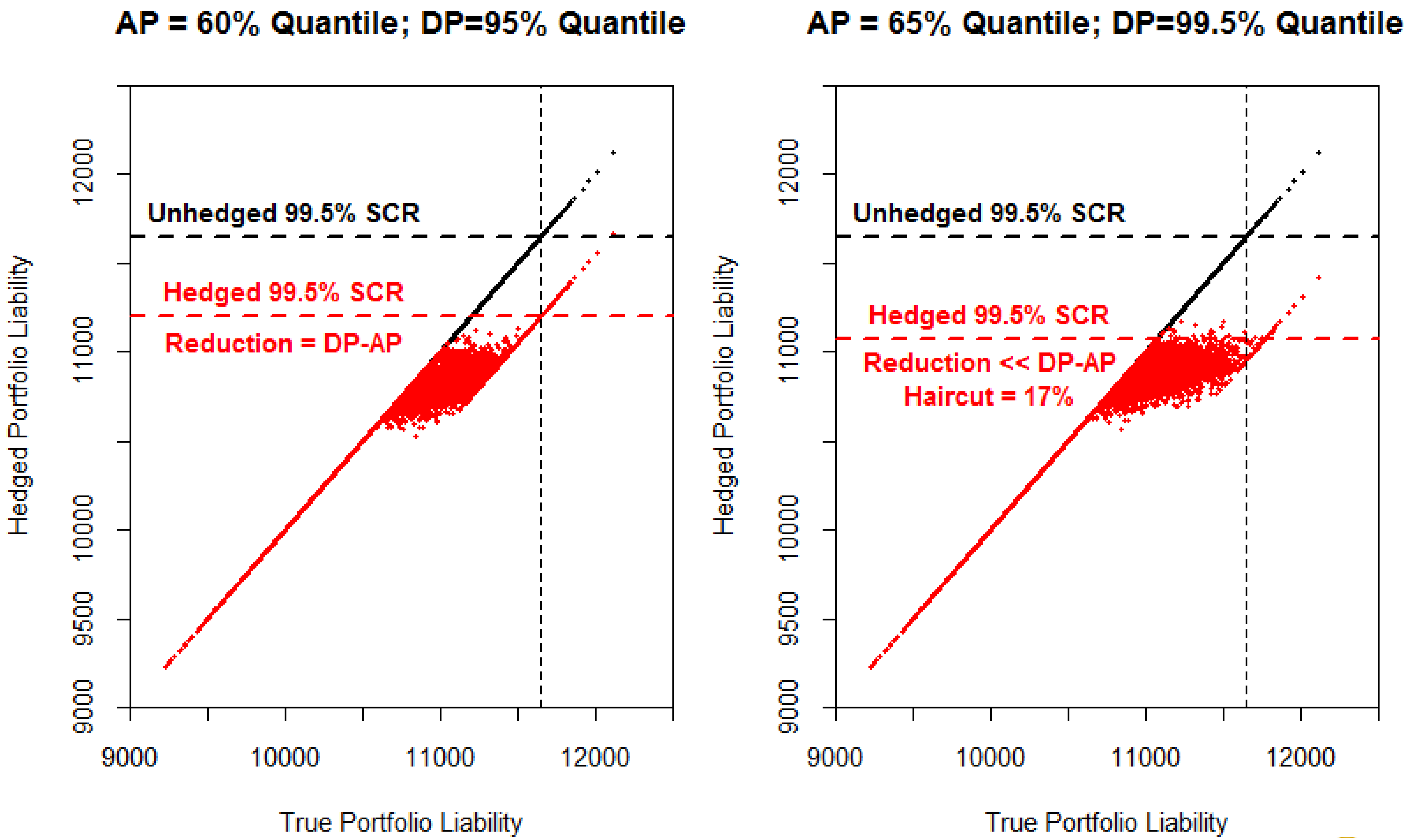

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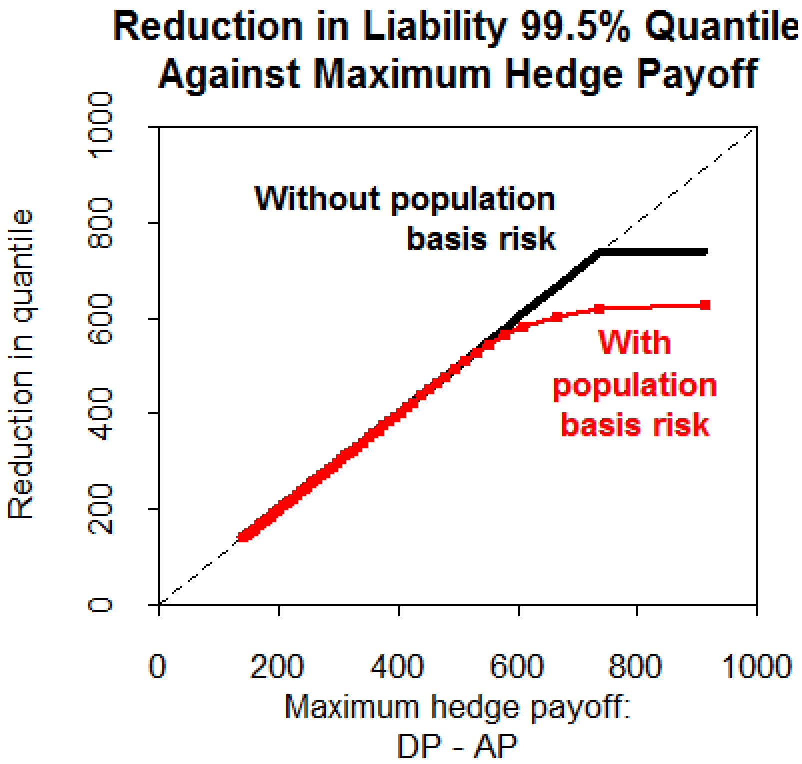

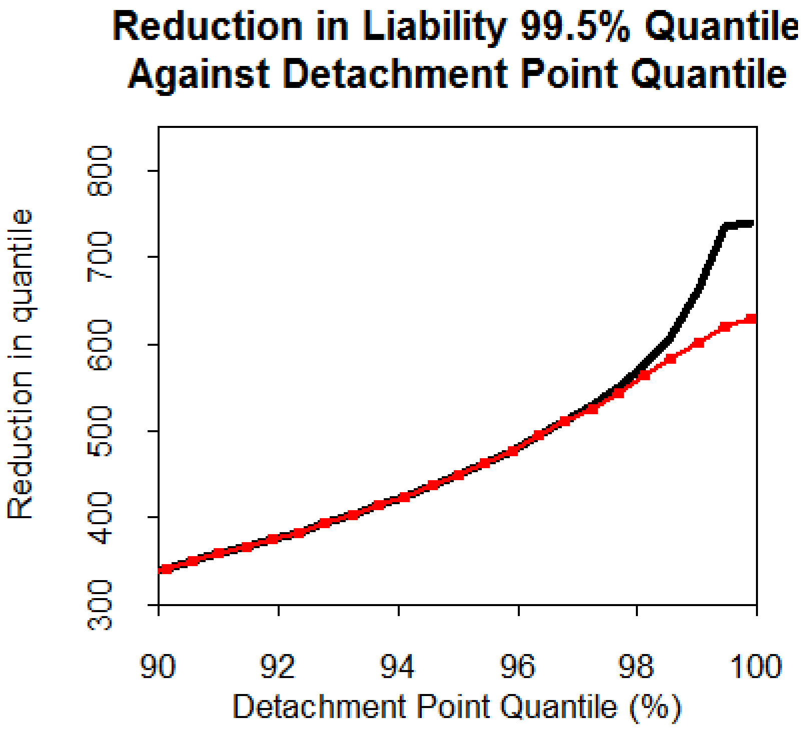

- “Liability” => not the excess over the best estimate
- Doesn’t account for the hedge price or the mean payoff

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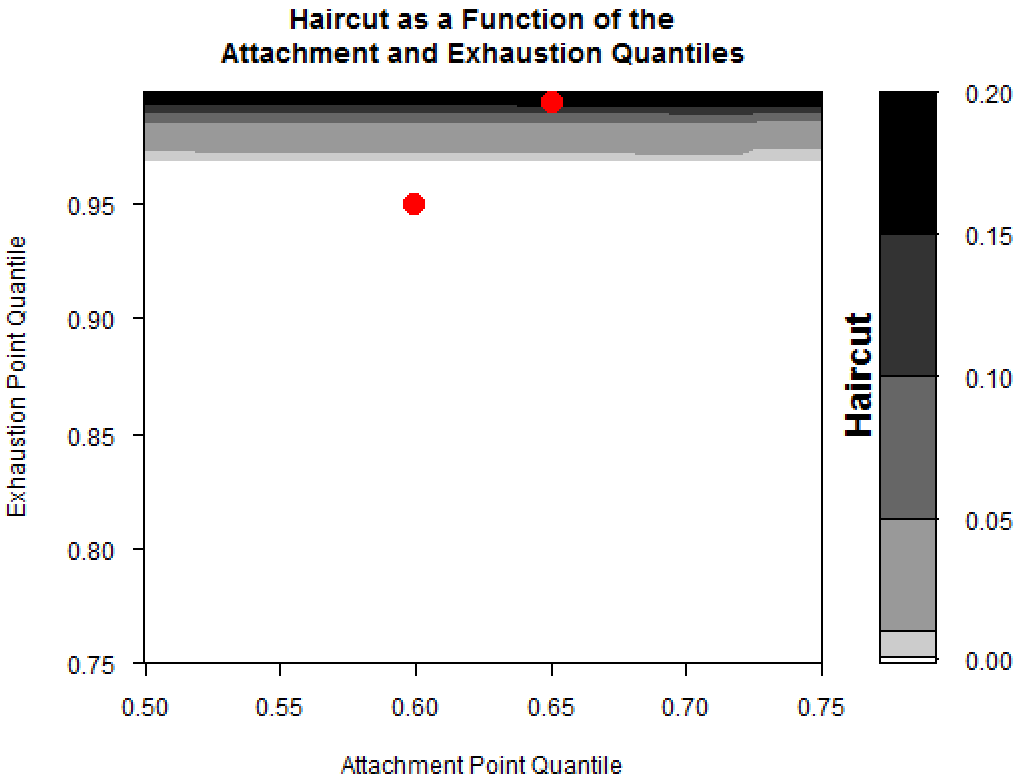

- Haircut = 1-
Capital relief *with* pop basis risk /
Capital relief *with no* pop basis risk

- Size of haircut is very dependent on the detachment point only

- DP near 99.5% gives poor results: haircut >> 0.

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Further reading

- Paper:
  - Cairns, A.J.G., and El Boukfaoui, G. (2017)
    Basis Risk in Index Based Longevity Hedges: A Guide For Longevity Hedgers
    To appear in *North American Actuarial Journal*
  - *Available at:*
    - *www.macs.hw.ac.uk/~andrewc/ARCresources*
    - *www.actuaries.org.uk/ARC*

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