Index-Based Longevity Hedging: Calculating Capital Relief

Andrew Cairns

Joint work with Ghali El Boukfaoui (formerly Soc. Gen.)

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Proposed Approach for Calculating Capital Relief:

• Open up communication lines with the regulator
• Establish which method for calculating SCR (and RM)
• Discuss with the regulator
• Document fully all mortality forecasting models
• Discuss with the regulator
• Run the SCR calculations with and without hedge
• Sensitivity and other robustness tests
• Discuss with the regulator
• Agree capital relief for time 0 and times 1, 2,…
Calculating the SCR Capital Relief

- \( L = PV \) of the full runoff: own liabilities
  - \( S(t, x) = \) survivor index for cohort aged \( x \) at time 0

- \( \tilde{L} = PV \) of the full runoff: synthetic portfolio of liabilities; depends on:
  - \( q_G(t, x) = \) general population mortality rates
  - \( ER(0, t, x) = \) experience ratios hard coded at time 0
  - \( \tilde{S}(t, x) = \) synthetic survivor index

Experience ratios =>
- \( E[S(t, x)] \approx E[\tilde{S}(t, x)] \)
- \( E[L] \approx E[\tilde{L}] \)
Values

- $L(0) \rightarrow L(1) \rightarrow L(T) \rightarrow L(\infty) = L$
- E.g. $L(T)$ = liability value given mortality experience up to $T$ (point estimate)
- $\tilde{L}(0) \rightarrow \tilde{L}(1) \rightarrow \tilde{L}(T)$
- Hedge payoff $H(T) = h(\tilde{L}(T))$
- AP = Attachment Point
- DP = Detachment Point (or Exhaustion Point, EP)
"UK" approach

- SCR = 99.5% one-year VaR; based on $L(1) - H(1)$

<table>
<thead>
<tr>
<th>Present Value at Time 0</th>
<th>Hedger’s liability</th>
<th>Synthetic liability</th>
<th>Hedge payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full runoff</td>
<td>$L$</td>
<td>$\tilde{L}$</td>
<td></td>
</tr>
<tr>
<td>Valued at Time $T$</td>
<td>$L(T)$</td>
<td>$\tilde{L}(T)$</td>
<td>$H(T)$</td>
</tr>
<tr>
<td>Valued at Time 1</td>
<td>$L(1)$</td>
<td>$\tilde{L}(1)$</td>
<td>$H(1)$</td>
</tr>
<tr>
<td>Valued at Time 0</td>
<td>$L(0)$</td>
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Original Dutch Approach (incl. Cairns and El Boukfaoui)

- SCR = 99.5% $T$-year VaR; based on $L(T) - H(T)$

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Alternative

- 95% VaR instead of 99.5% (approximately (?) the same as 1-year 99.5% VaR) (needs regulator engagement!)

- Full runoff: \( L(\infty) - H(T) = L - H(T) \) instead of \( L(T) - H(T) \)

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<td>( L = L(\infty) )</td>
<td>( \tilde{L} )</td>
<td></td>
</tr>
<tr>
<td>Valued at Time ( T )</td>
<td>( L(T) )</td>
<td>( \tilde{L}(T) )</td>
<td>( H(T) )</td>
</tr>
<tr>
<td>Valued at Time 1</td>
<td>( L(1) )</td>
<td>( \tilde{L}(1) )</td>
<td>( H(1) )</td>
</tr>
<tr>
<td>Valued at Time 0</td>
<td>( L(0) )</td>
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Three Types of Basis Risk

• Population basis risk
  – Hedger’s experience and synthetic index are not perfectly correlated

• Structural basis risk
  – Non-linear payoff ⇒ no risk reduction beyond DP or below AP

• Tail basis risk
  – Cashflows after maturity, $T$:
    • Depend on risk emerging before $T$ and after $T$
    • Risks before $T$ are (can be) hedged
    • Additional risks emerging after maturity, $T$, are not hedged
Three Types of Basis Risk: Stylised
Anatomy of a Hedging Calculation

Don’t try to read the small print!
Capital Relief Calculation Summary

• Simulated L(T), H(T) => hedged distribution => capital relief

• In advance, discuss and agree with the regulator the following:
  – Break down the process into a series of manageable steps
  – Document all of these steps carefully
  – Document clearly all of the models being used in each step
Simulated Impact of Hedge at Time T

- Dutch insured lives dataset versus Dutch national mortality data
- Portfolio = mixture of deferred and immediate annuities
- Hedge: 10-year bull spread
- Case 1
  - AP=60% quantile of the synthetic liability
  - DP=95% quantile
- Case 2
  - AP=65% quantile
  - DP=99.5% quantile
• “Liability” => not the excess over the best estimate
• Doesn’t account for the hedge price or the mean payoff
• Haircut = 1-
  Capital relief \textit{with} pop basis risk / Capital relief \textit{with no} pop basis risk

• Size of haircut is very dependent on the detachment point only

• DP near 99.5% gives poor results: haircut $>> 0$. 
Further reading

• Paper:
    Basis Risk in Index Based Longevity Hedges: A Guide For Longevity Hedgers
    To appear in *North American Actuarial Journal*
  – Available at:
    • [www.macs.hw.ac.uk/~andrewc/ARCresources](http://www.macs.hw.ac.uk/~andrewc/ARCresources)
    • [www.actuaries.org.uk/ARC](http://www.actuaries.org.uk/ARC)