

Hotel Park Inn, Amsterdam

The many uses of VaR for longevity trend risk

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21st September 2018



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1 About Longevity

- Founded 2006.
- Based in Edinburgh.
- Clients in UK, USA, Canada and Switzerland.
- Research partnership with Heriot-Watt.

- Experience analysis and mis-estimation:



- Stochastic mortality projections and capital:



- Rating pension schemes:



2 Some questions

2 Some questions:

- How do you put a multi-year trend risk into a one-year view?
- How do different product types behave?
- How do VaR and CTE regimes compare?
- How do you value an index-based hedge?

VaR Value-at-Risk

CTE Conditional Tail Expectation

LC Model from Lee and Carter [1992]

APC Age-Period-Cohort model

M5/CBD Model from Cairns et al. [2006]

M6 Model from Cairns et al. [2009]

(S) Smoothing as per Eilers and Marx [1996]

2DAC Model from Richards et al. [2006]

3 Trend risk v. one-year view

“Whereas a catastrophe can occur in an instant, longevity risk takes decades to unfold”

The Economist [2012]

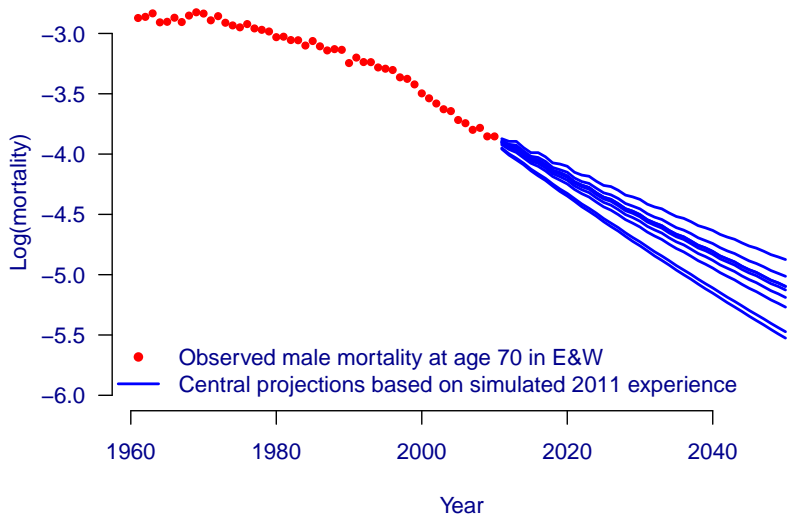
- Longevity trend risk unfolds over many years.
- Insurance regulations have a one-year view of risk.
- How do you reconcile the two?

Solution from Richards et al. [2014]:

1. Simulate next year's experience data.
2. Refit the projection model.
3. Value liabilities.
4. Discard simulated experience data.

Repeat (1)–(4) a few thousand times...

3 Sensitivity of forecast



Source: Lee-Carter example from Richards et al. [2014].

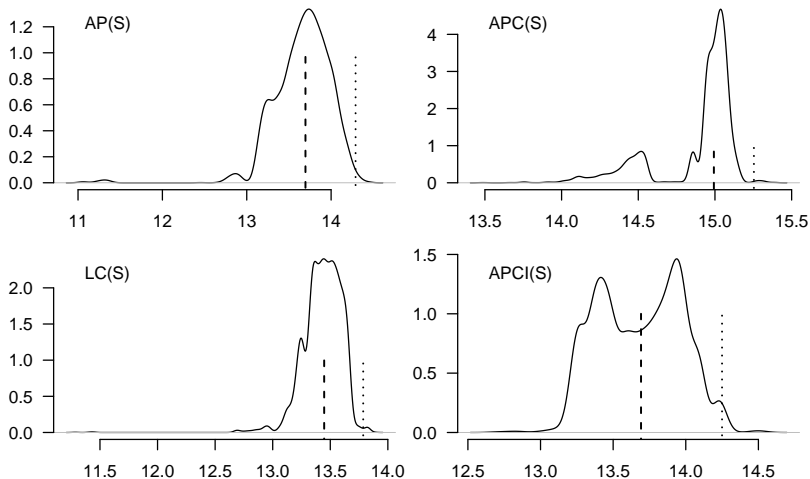
- Our unknown liability is X (say).
- VaR-style solvency capital:

$$\left(\frac{Q_\alpha}{\mathbb{E}[X]} - 1 \right) * 100\%$$

where Q_α is α -quantile of X , i.e. $\Pr(X < Q_\alpha) = \alpha$.

- We don't know the distribution of X ...
... but we do have a sample of simulations.
- Estimate $\mathbb{E}[X]$ from mean of sample.
- Estimate Q_α from sample using Harrell and Davis [1982].

3 One-year liability densities



Annuities payable to male aged 70. Means marked with dashed line and $Q_{99.5\%}$ marked with dotted line.

Source: Richards et al. [2017, Table 4].

- Variety of density shapes.
 - ⇒ not all unimodal
 - ... and not all symmetric.
- Considerable variability between models.
 - ⇒ need to use multiple models
 - ... and exercise *actuarial judgement*.

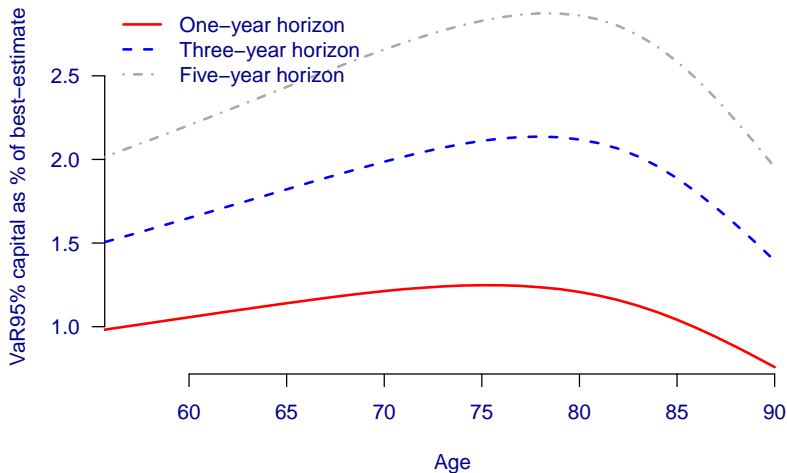
4 Trend risk v. multi-year view LONGEVITAS

- Richards et al. [2014] was for one-year insurer solvency.
- The same methodology has other applications...

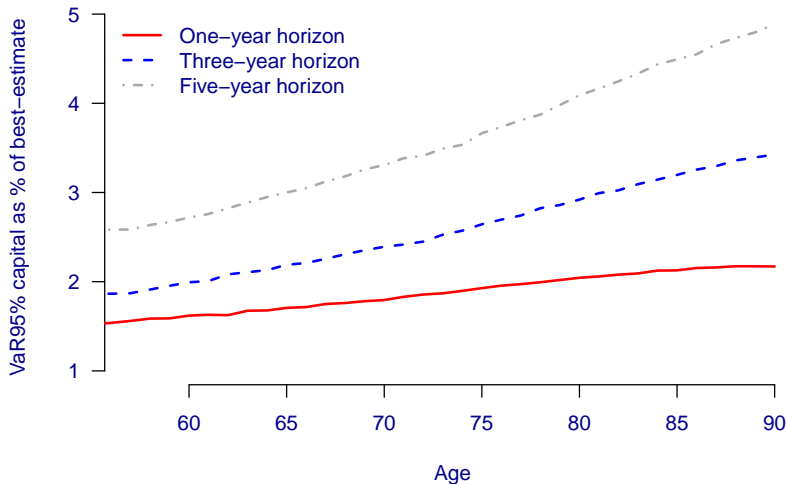
Medium-term business planning:

- 3–5 years for insurer ORSA.
- Ten-year “glide path” to buy-out for pension schemes.

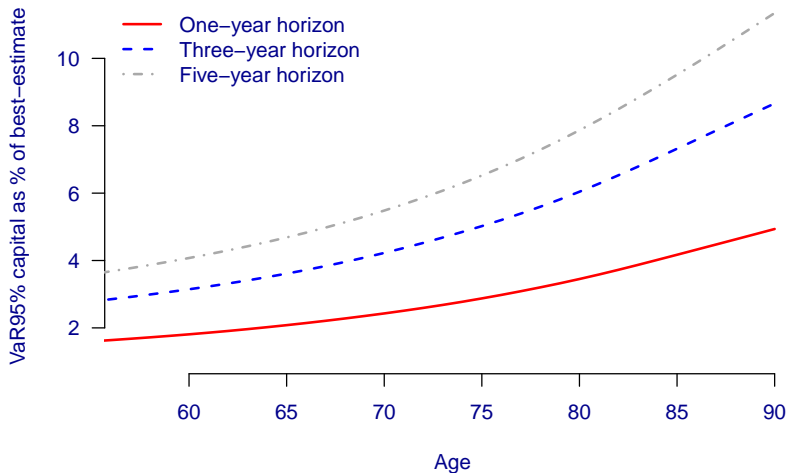
- Take one-year framework from Richards et al. [2014].
- Extend time horizon to 3–5 years.
- Reduce p-value to, say, 95%...



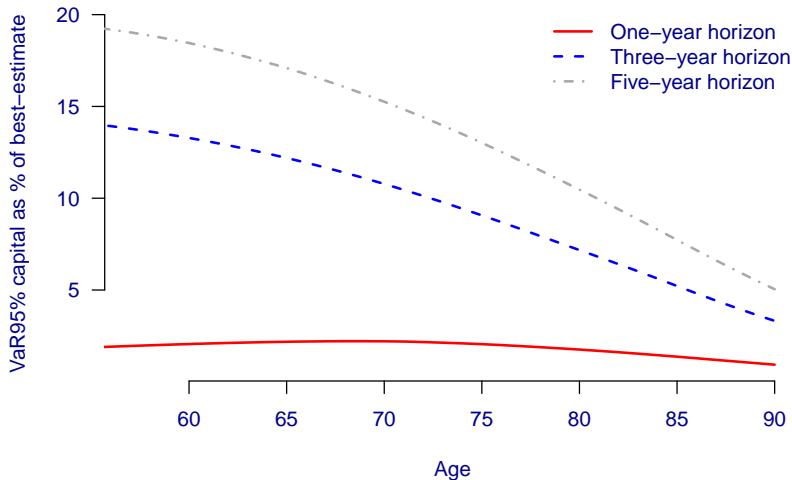
Immediate annuities under Lee-Carter model. UK data ages 50–104, 1971–2016



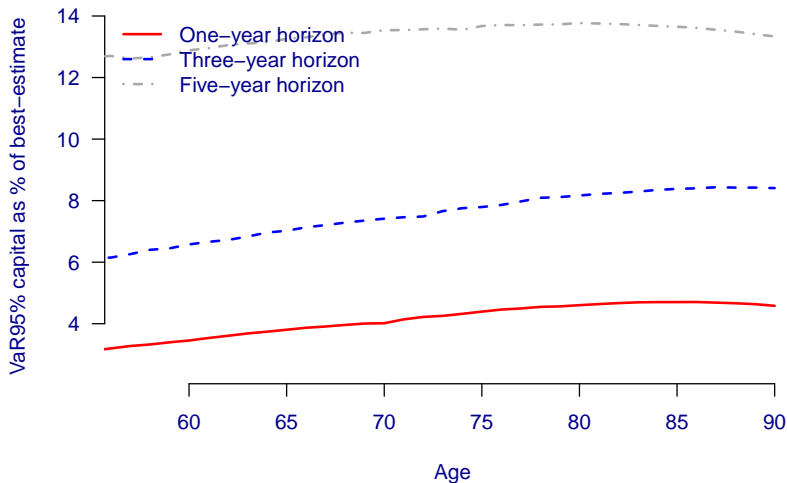
Immediate annuities under APC(S) model. UK data ages 50–104, 1971–2016



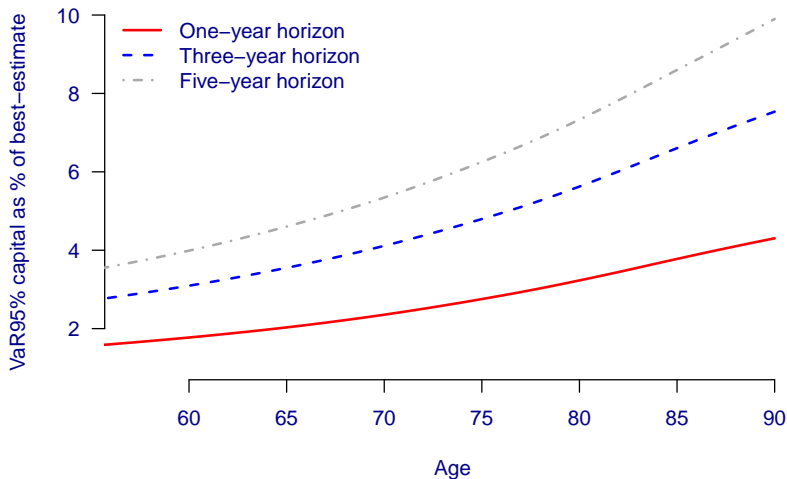
Immediate annuities under M5(S) model. UK data ages 50–104, 1971–2016



Immediate annuities under Lee-Carter model. UK data ages 50–104, 1971–2016



Immediate annuities under APC(S) model. UK data ages 50–104, 1971–2016



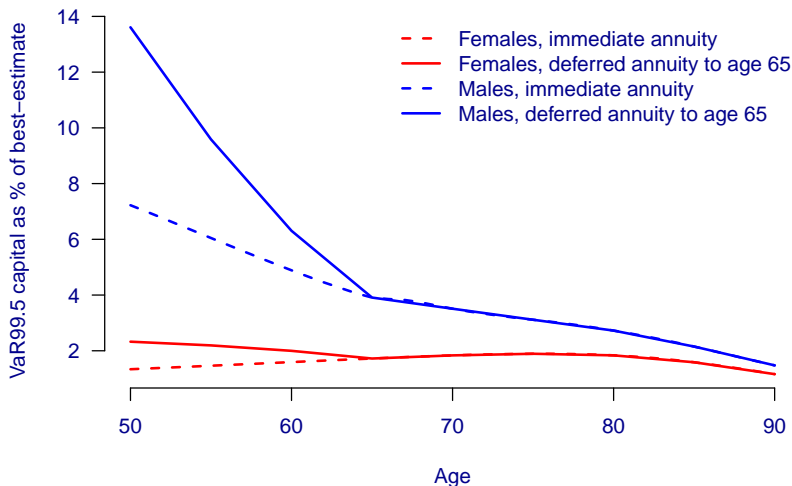
Immediate annuities under M5(S) model. UK data ages 50–104, 1971–2016

- No consistent pattern in capital by term.
- Considerable variability between models.
 - ⇒ need to use multiple models
 - ... and exercise *actuarial judgement* (again).

5 Deferred annuities

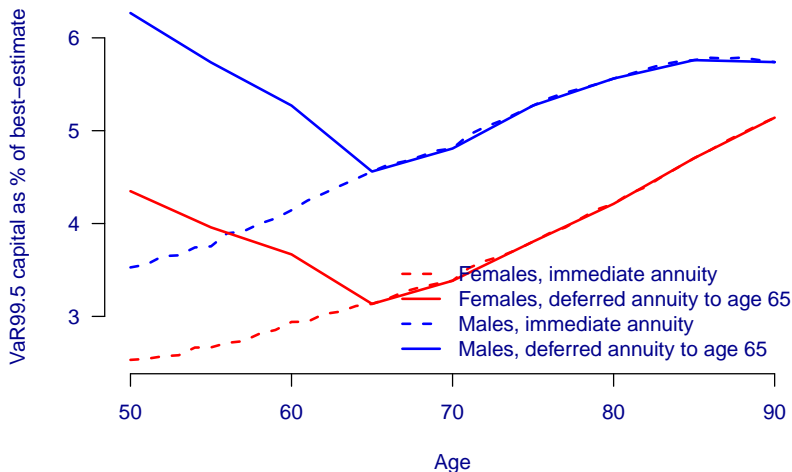
- Most published work concerns immediate annuities and pensions in payment.
- What about deferred annuities and pensions?
- Assume payment from age 65.
- Compare VaR99.5% solvency capital for immediate and deferred annuities.

5 Solvency capital, LC(S)

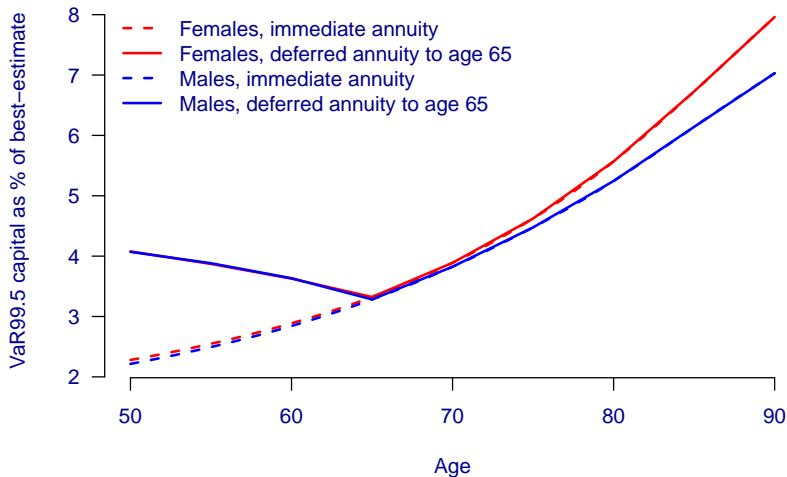


Deferred and immediate annuities under Lee-Carter model. UK data ages 50–104, 1971–2016

5 Solvency capital, APC(S)



Deferred and immediate annuities under APC(S) model. UK data ages 50–104, 1971–2016



Deferred and immediate annuities under M5(S) model. UK data ages 50–104, 1971–2016

- Depending on age, solvency capital for deferred annuities can be double that of annuities in payment.
- Sharp differences in solvency capital by gender.

- Our unknown liability is X (say).
- VaR-style solvency capital:

$$\left(\frac{Q_\alpha}{\mathbb{E}[X]} - 1 \right) * 100\%$$

where Q_α is α -quantile of X , i.e. $\Pr(X < Q_\alpha) = \alpha$.

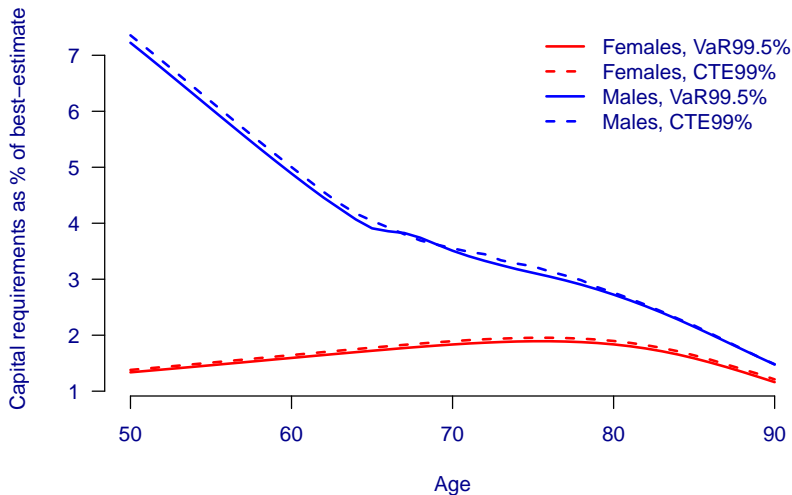
- Our unknown liability is X (say).
- CTE-style solvency capital:

$$\left(\frac{\mathbb{E}[X|X \geq Q_\alpha]}{\mathbb{E}[X]} - 1 \right) * 100\%$$

where Q_α is α -quantile of X , i.e. $\Pr(X < Q_\alpha) = \alpha$.

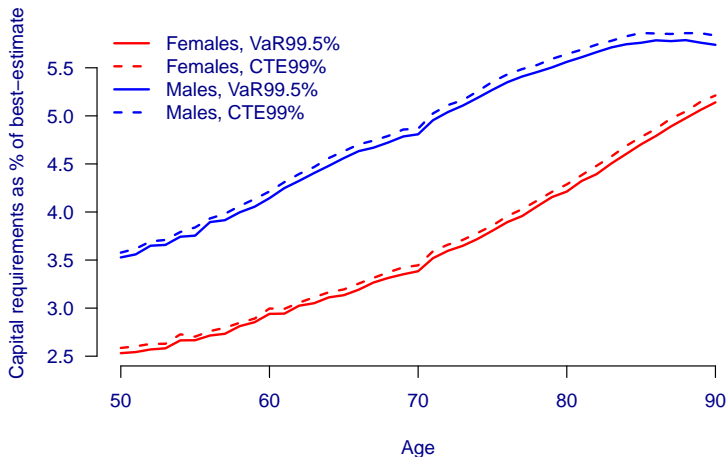
- How does VaR capital compare to CTE capital?
- $CTE_{\alpha} > VaR_{\alpha}$ (obviously!)
- But how does VaR99.5% compare to CTE99%?
- Can calculate both from same sample...

6 VaR v. CTE — LC(S) model

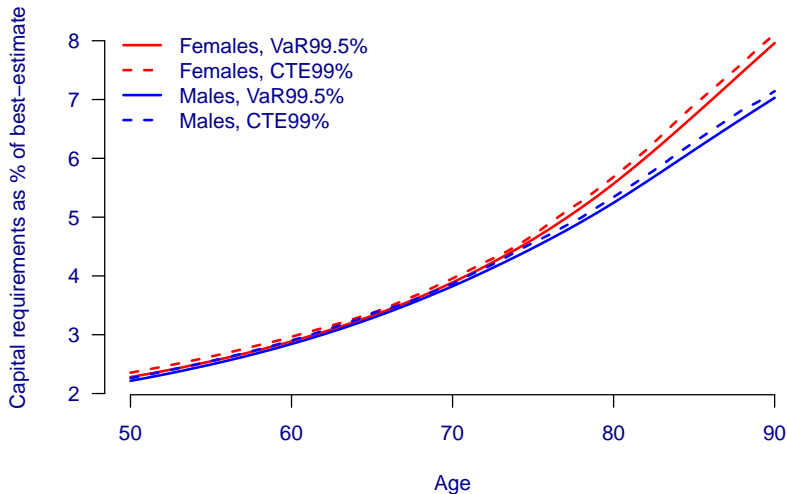


Annuities in payment under Lee-Carter model. UK data ages 50–104, 1971–2016

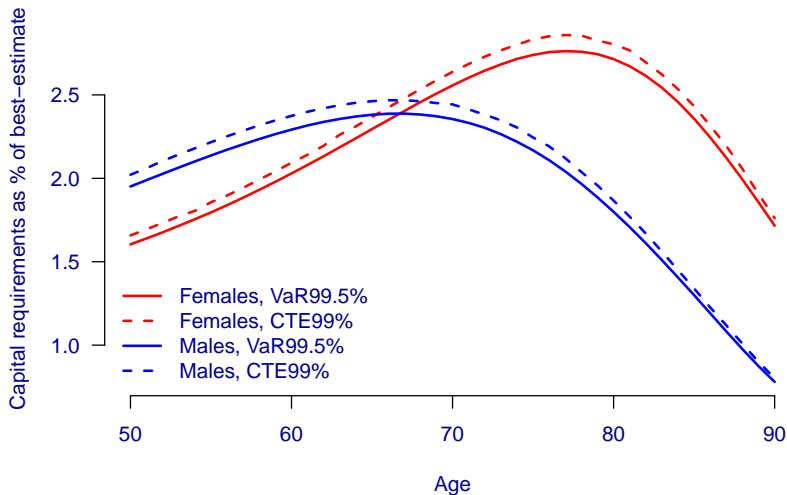
6 VaR v. CTE — APC(S) model LONGEVITAS



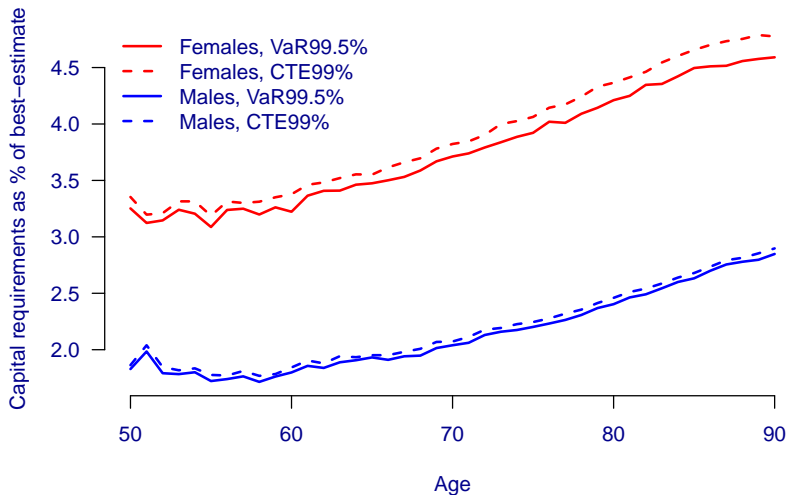
Annuities in payment under APC(S) model. UK data ages 50–104, 1971–2016



Annuities in payment under M5(S) model. UK data ages 50–104, 1971–2016

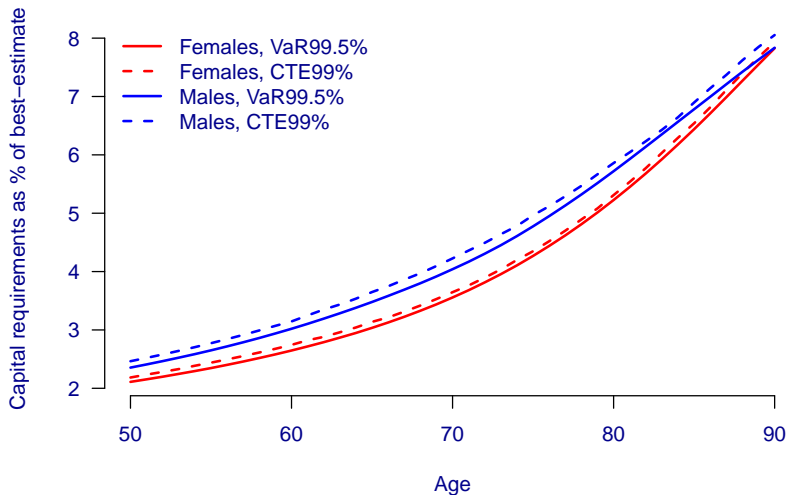


Annuities in payment under LC(S) model. UK data ages 50–104, 1971–2016



Annuities in payment under APC(S) model. UK data ages 50–104, 1971–2016

6 VaR v. CTE — M5(S)



Annuities in payment under M5(S) model. UK data ages 50–104, 1971–2016

- Longevity trend-risk capital very comparable between VaR99.5% and CTE99%.
- CTE99% usually slightly more prudent than VaR99.5%.
- Difference usually under 0.1%.

7 Index-based hedges

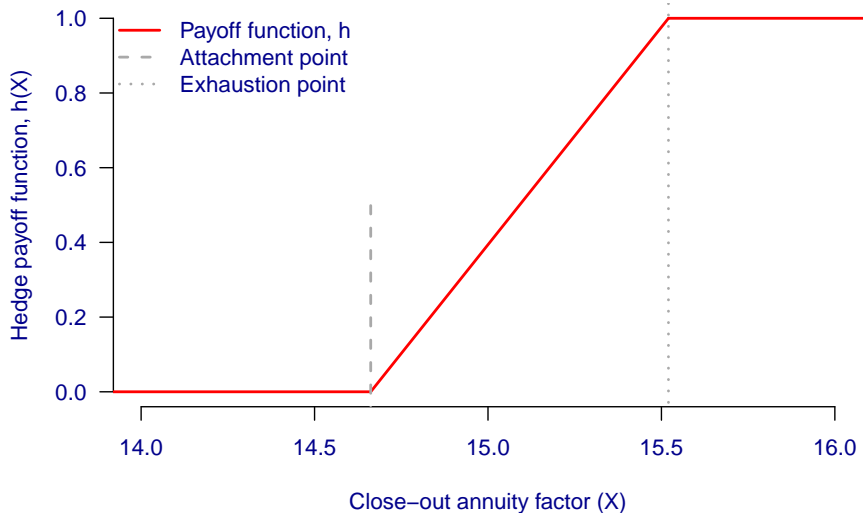
- Population mortality (basis risk).
- Term n years.
- At end of term, fit Lee-Carter model (say) and use to value annuity with unknown value X .
- Use a function of X to close out the contract.
⇒ This is just another multi-year VaR calculation.

- Risk metric (annuity value) is X .
- Only pay above attachment point, AP .
- Pay no more than exhaustion point, EP .
- Standardise payoff, h , as:

$$h(X) = \max \left(0, \min \left(\frac{X - AP}{EP - AP}, 1 \right) \right)$$

- See Cairns and El Boukfaoui [2017] for detailed discussion.

7 Hedge payoff function



- Set $AP = Q_{\alpha_1}$ and $EP = Q_{\alpha_2}$ ($\alpha_1 < \alpha_2$).
- Q_α set with reference to Lee-Carter sample paths over n years, i.e. an n -year VaR simulation.
- Probability of payoff is $1 - \alpha_1$.
- Mean payoff can be estimated from VaR results.

- $n = 15$ years.
- Use Lee-Carter model for close-out calculation.
- Follow Cairns and El Boukfaoui [2017] and set $AP = Q_{60\%}$ and $EP = Q_{95\%}$.
- Probability of a payoff is 0.4.
- Average payoff is 0.375 (from 5,000 simulations).

- Lee-Carter model used for both sample paths over n years **and** for payoff calculation.
- Assume we keep the Lee-Carter model for payoff calculation and also keep the same attachment and exhaustion points.
- What happens if the sample paths follow a *different* model?

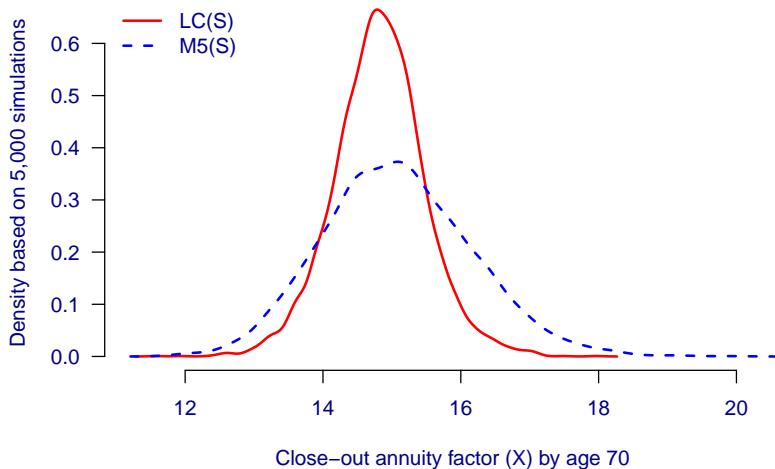
Impact of different sample-path models on payoff:

Model	Payoff prob.	Mean payoff
LC(S)	0.40	0.375
M5(S)	0.53	0.592
2DAC	0.80	0.434
M6	0.82	0.710

Source: own calculations using population data for males in Netherlands, ages 50–104, 1971–2016.
Annuity values discounted at 2% p.a.

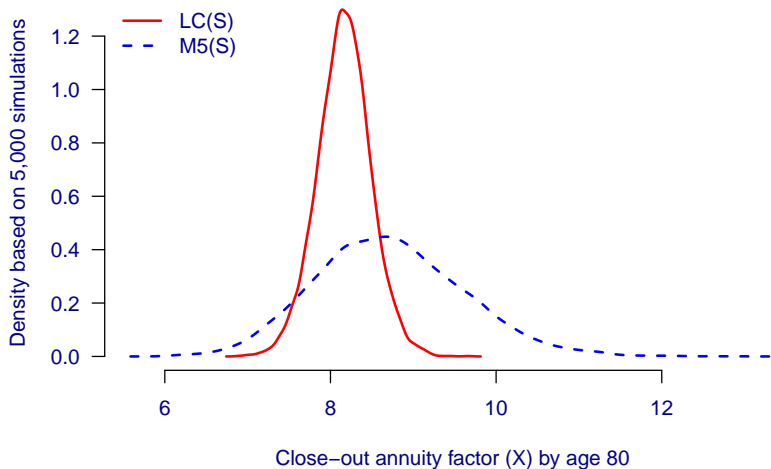
- Model for future mortality is unknowable (model risk).
 - So payoff probability and expected payoff are also unknowable.
 - ▶ What value should the hedge contract have on the balance sheet?
 - ▶ What solvency capital relief should be given?
- ⇒ Actuarial judgement required on both counts.

- How different can the answers get?
- Consider the spread at various ages under CBD model (M5)...



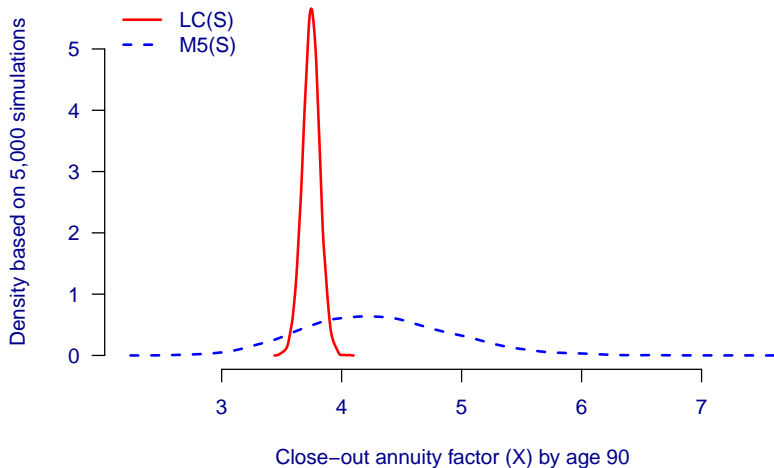
VaR annuity factors valued under Lee-Carter model after 15 years of mortality following the M5(S) model.

Netherlands data for males aged 50–104, 1971–2016. Annuity cashflows discounted at 2% p.a.



VaR annuity factors valued under Lee-Carter model after 15 years of mortality following the M5(S) model.

Netherlands data for males aged 50–104, 1971–2016. Annuity cashflows discounted at 2% p.a.



VaR annuity factors valued under Lee-Carter model after 15 years of mortality following the M5(S) model.

Netherlands data for males aged 50–104, 1971–2016. Annuity cashflows discounted at 2% p.a.

Variations to explore in future research:

- Different payoff functions.
- Valuing options to close out early.

If you are interested in the above, let me know!

- Longevity trend risk can be put into a one-year framework.
- Same outputs can be used for both VaR- and CTE-style solvency regimes.
- Framework extends to ORSA for insurers...
...and “glide paths” to buy-outs
...and assessing index-based hedges.
- Model risk is critical throughout.
- Expert judgement required for solvency capital...
...and valuation of index-based hedges.

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More on longevity risk at www.longevity.co.uk