ROBUST HEDGING OF LONGEVITY RISK

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Plan

- Intro $+$ model
- Recalibration risk – introduction
- Robustness questions – index hedging
- Discussion

Background

- Annuity providers and pension plans
- Exposure to longevity risk
  - systematic risk (underlying mortality rates)
  - binomial risk (lives)
  - concentration risk (amounts)
- Alongside: interest rate risk, equity risk ....

What is longevity risk?

the risk that *in aggregate* a group of lives live longer *than anticipated*

Simple example:

- n lives; probability p of survival to T
- $N|p \sim$ Binomial(n, p) survivors at T
- If p is known: $N/n \rightarrow$ constant p
- if p is not known: then N/n contains systematic risk

Hedging problem 1

Annuity provider seeks to hedge its exposure to longevity risk

- Large cohort aged 65 at time 0
- Equal, level annuities payable for life
- $S(t,65) =$ proportion still alive at t
- $PV = \sum_{t=1}^{\infty} e^{-rt} S(t,65)$
- Objective: Hedge longevity risk in PV

Hedging problem 2

Annuity provider seeks to hedge its exposure to longevity risk

- Large cohort aged 65 at time 0
- Equal, level annuities payable for life
- $S(t, 65)$ = proportion still alive at t
- Deficit $D(t) = MCV_{Liabs}(t) - MCV_{Assets}(t)$
- Objective: Hedge longevity risk in $D(T)$
  e.g. $T = 1$ under Solvency II

Hedging problem 3

Pension plan

- Cohort now aged 55
- Plan will buy annuities at age 65
- Objective: hedge the longevity risk in the annuity price

Options for hedging

- Customised hedges:
  - e.g. longevity swap
  - floating leg linked to OWN cashflows
  - indemnification
- Index-based hedges:
  - Standardised contracts
  - e.g. Linked to a national index

    $\Rightarrow$ basis risk

Focus of this talk: Index-based hedges

- Customised hedges only available to very large pension plans
- Index-based hedges
  - smaller schemes
  - better value for money for large plans ???
  - Quantity of hedging instrument
    Hedge effectiveness
    Price
    How confident are we in these quantities? $\Rightarrow$ ROBUSTNESS
- Here: Hedge Effectiveness $:=$ % reduction in Variance of Deficit

Simple Example: Data

- Population 1: Index
  - England &Wales males, 1961-2005, ages 50-89
- Population 2: Hedger
  - CMI assured lives, 1961-2005, ages 50-89
  - CMI: proxy for a typical white-collar pension plan

CMI data not available after 2006

Simple example

- Static *value* hedge: $t = 0 \longrightarrow T$
- $a_k(T, x) =$ population k annuity value at T
- Liability value $L(T) = a_2(T, 65)$
- Hedging instrument: q-forward (www.LLMA.com)

$$H(T) = q_k(T, x) - q_k{}^{\text{fxd}}(0, T, x)$$

$$q_k^{\text{fxd}}(0, T, x) = \text{value at } T \text{ of swap fixed leg}$$

- $k = 2$ (CMI) $\Rightarrow$ CUSTOMISED hedge
- $k = 1$ (E&W) $\Rightarrow$ INDEX hedge

Simple example: APC model

$m_k(t,x) =$ population k death rate

$$\log m_k(t,x) = \beta^{(k)}(x) + \kappa^{(k)}(t) + \gamma^{(k)}(t-x)$$

$\beta^{(1)}(x),\ \ \beta^{(2)}(x)$ population 1 and 2 age effects

$\kappa^{(1)}(t),\ \ \kappa^{(2)}(t)$ period effects

$\gamma^{(1)}(c),\ \ \gamma^{(2)}(c)$ cohort effects

Realism: valuation model $\neq$ simulation model

- (Re-)calibration using data up to T $\Rightarrow$ realistic!
- Valuers just observe historical mortality plus

  one future sample path of mortality from 0 to T

  $\Rightarrow$ do not know the “true” simulation/true model
- Using true model $\Rightarrow$ too optimistic (??) c.f. Black-Scholes
- Valuation model $+$ calibration window

  $\Rightarrow$ *Knightian Uncertainty*

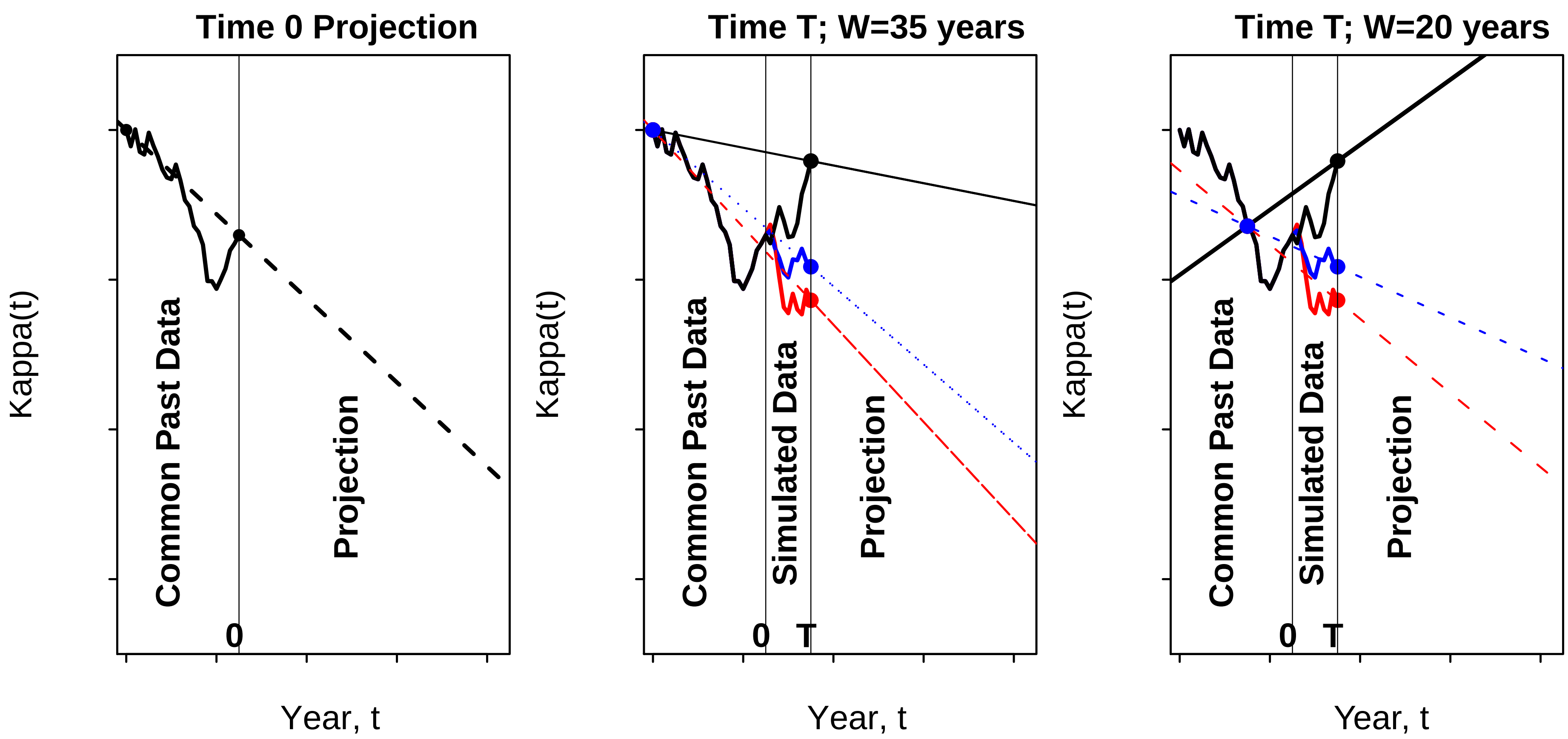
Key observation

- Critical parameter: $\nu_\kappa =$ long term trend in $\kappa^{(1)}(t),\ \kappa^{(2)}(t)$
- Recalibration $\Rightarrow$ ν_κ recalibrated at T
- Recalibration $\Rightarrow$ (assessment of) risk $\nearrow$
- BUT (assessment of) hedge effectiveness also $\nearrow$ for some hedges
- WHY?

  Additional trend risk is common to both populations.

  $$a_k(T,x) \approx f(\beta_{[x]}^{(k)}, \kappa_T^{(k)}, \gamma_{T-x+1}^{(k)}, \nu_\kappa)$$

Recalibration risk – example (random walk)



- You *will* recalibrate at T
- Recalibration depends on as yet unknown experience from 0 to T
- Recalibration depends on length of lookback window

How robust are estimates of:

- Optimal hedge ratios
- Hedge effectiveness
- Initial hedge instrument prices

relative to:

- Treatment of parameter risk
- Treatment of population basis risk
- Valuation model: recalibration risk
- Poisson risk?

Modelling Variants

- PC: Full parameter certainty (PC);

  Valuation Model NOT recalibrated in 2015

- PC-R: As full PC

  Except: Valuation Model recalibrated in 2015

- PU: Full parameter uncertainty with recalibration

- PU-Poi: Full PU with recalibration $+$ Poisson risk

Data

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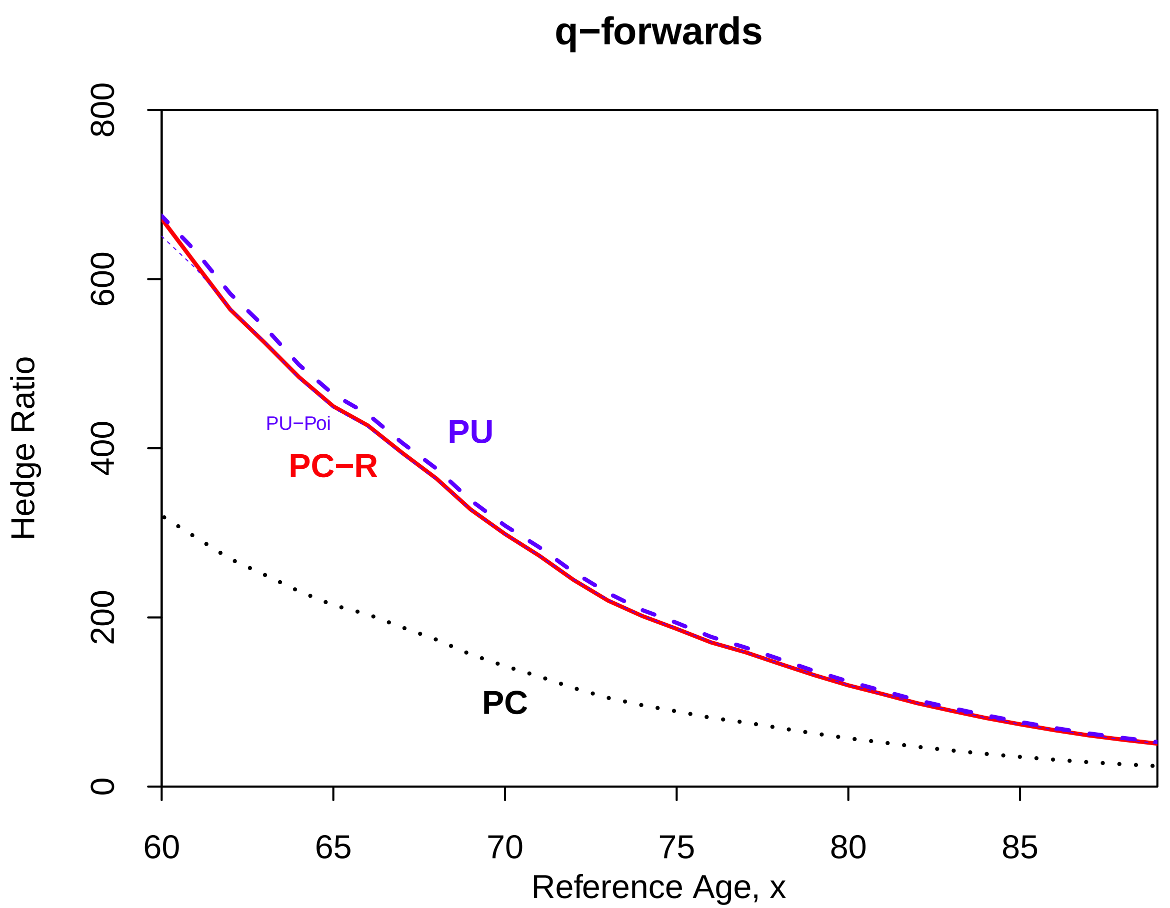
Hedging options

- Recall: Liability, $L = a_2(T, 65)$ (CMI)
- Hedging instrument (ref England & Wales):
  - q-Forward maturing at T

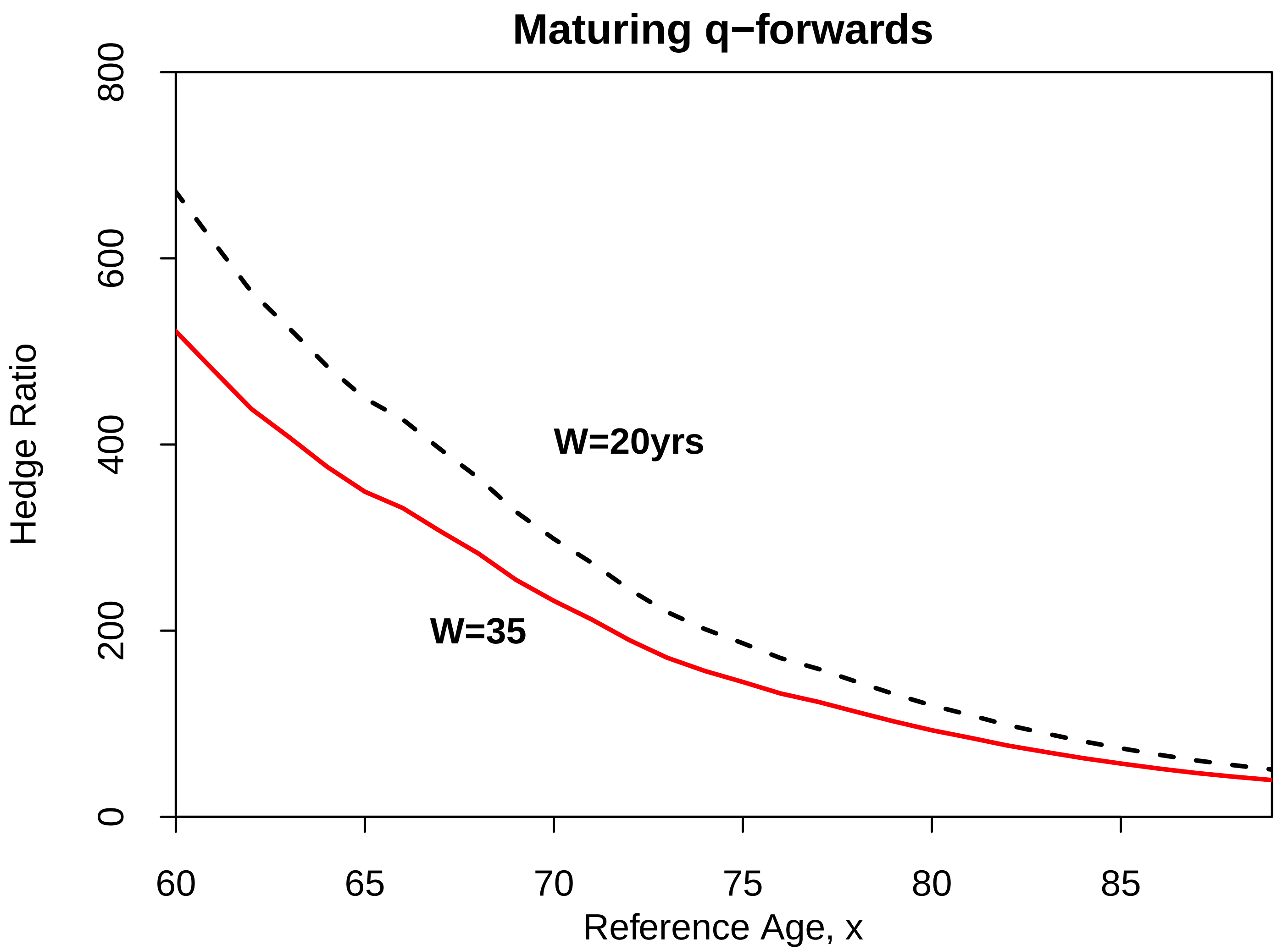
    $$H = q_1(T, x) - q_1^F(0, T, x)$$
- .... for a range of reference ages x

Robustness of Hedge Ratios



PC $\longrightarrow$ PC-R <u>not robust</u>; PC-R $\longrightarrow$ PU <u>robust</u>

Robustness relative to recalibration window, W



Robustness relative to recalibration window, W

q-forwards maturing at time 10 are not robust w.r.t. W

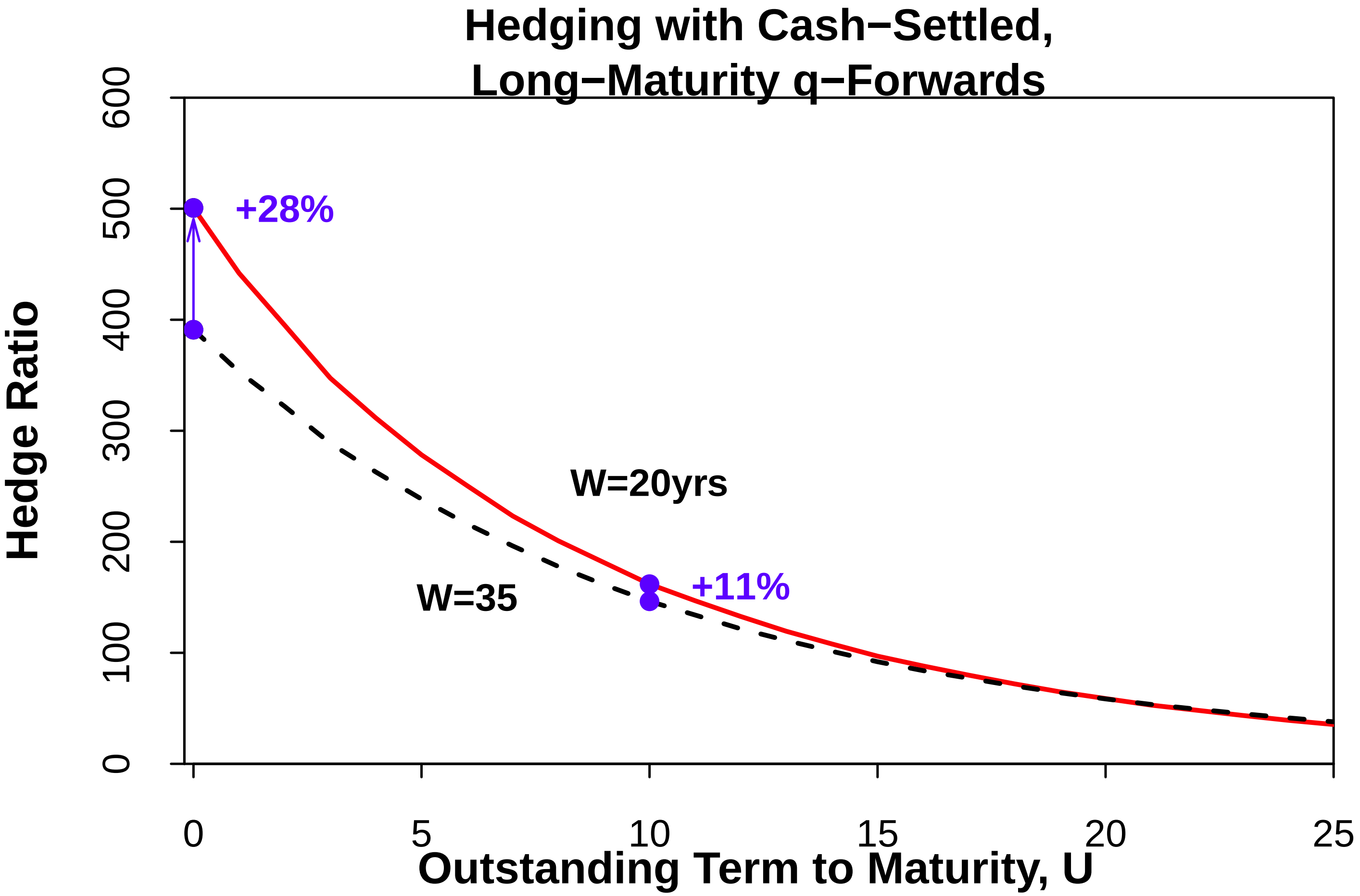
- Liability, L, depends on
  - $\kappa_T^{(2)}$ and ν_κ $(\kappa^{(1)}(T)$, $W \to \nu_\kappa)$
- Maturing q-Forward depends on $\kappa_T^{(1)}$ only

  $\Rightarrow$ not robust w.r.t. W
- Possible market solution:

  $(0, T + U, x)$ q-Forward, cash settled at T

  $\Rightarrow$ dependent on $\kappa_T^{(1)}$ and ν_κ

$x = 65$: Robustness relative to recalibration window, W



$T + U$ q-Forward is cash settled at time T

$\Rightarrow$ value depends on $\kappa_t^{(1)}$ and ν_κ

Robustness relative to recalibration window, W

- If we know W, then ν_κ linear in $\kappa_T^{(1)}$

  $\Rightarrow$ one hedging instrument sufficient

- If W is not known

  or, ν_κ determined by other methods

  $\Rightarrow$ two hedging instruments are required

  $\Rightarrow$ Delta and "Nuga" hedging

Delta and Nuga Hedging

Recall: $a_k(T,x) \approx f(\beta_{[x]}^{(k)}, \kappa_T^{(k)}, \gamma_{T-x+1}^{(k)}, \nu_\kappa)$

Liability: $L = a_2(T,x)$.

Hedge instruments:

$$H_1 = q_1(T,x_1) - q_1^{\text{fxd}}(0,T,x_1) \qquad \to h_1 \text{ units}$$

$$H_2 = q_1(T+U,x_2) - q_1^{\text{fxd}}(0,T+U,x_2) \qquad \to h_2 \text{ units}$$

(H_2 cash settled at T)

Delta and Nuga hedging $\Rightarrow$ require

$$\text{Deltas:} \quad \alpha \frac{\partial L}{\partial \kappa^{(2)}} = -h_1 \frac{\partial H_1}{\partial \kappa^{(1)}} - h_2 \frac{\partial H_2}{\partial \kappa^{(1)}}$$

$$\text{and Nugas:} \quad \frac{\partial L}{\partial \nu_\kappa} = -h_1 \frac{\partial H_1}{\partial \nu_\kappa} - h_2 \frac{\partial H_2}{\partial \nu_\kappa}$$

where $\alpha = Cov(\kappa_T^{(1)}, \kappa_T^{(2)})/Var(\kappa_T^{(1)})$.

Concept:

same idea as Vega hedging in equity derivatives ($\mathcal{V} = \partial V/\partial \sigma$)

– hedging against changes in a parameter that is supposed to be constant.

Numerical example: $L = a_2(T, 65), T = 10$

Four strategies:

A: No hedging

B: H_1 only; h_1 optimal for $W = 20$

C: H_1 only; h_1 optimal for $W = 35$

D: H_1 and H_2; Delta and Nuga hedging

Numerical example: $L = a_2(T, 65), T = 10$

	q-F$(T, 64)$	q-F$(T+T, 74)$			
Strategy	h_1	h_2	Var(Deficit)	Hedge Eff.	
$W = 20$					
A	0	0	0.3481	0	
B	500.7	0	0.03435	0.9013	(1)
C	389.0	0	0.04996	0.8565	(3)
D	-279.6	256.4	0.03797	*0.8909*	(2)
$W = 35$					
A	0	0	0.2233	0	
B	500.7	0	0.04953	0.7782	(3)
C	389.0	0	0.03392	0.8481	(1)
D	-279.6	256.4	0.03493	*0.8436*	(2)

Numerical example: discussion

- Nonlinearities $\Rightarrow D < B$ instead of $D = B$
- BUT
  - $W = 20 \Rightarrow$

    D is nearly optimal

    C is much worse
  - $W = 35 \Rightarrow$

    D is nearly optimal

    B is much worse

Robustness relative to other factors

Results are robust relative to:

- inclusion of parameter uncertainty in $\beta_x^{(k)}$, $\kappa_t^{(k)}$, $\gamma_c^{(k)}$
- pension plan's own small-population Poisson risk
- index population: EW-size Poisson risk, maybe smaller
- CMI data up to 2005 $+$ EW data up to 2005

  versus

  CMI data up to 2005 $+$ EW data up to 2008

Ongoing work

Economic capital relief using longevity options

- Option payoff at T based on
  - Pop 1 cashflows up to T
  - Estimated Pop 1 cashflows after T (commutation)
- Example: $BE =$ best estimate liability at time 0
- $EC =$ additional Economic Capital to cover 95% runoff
  - $EC_0 = EC$ without hedge
  - $EC_1 = EC$ with index-based option hedge

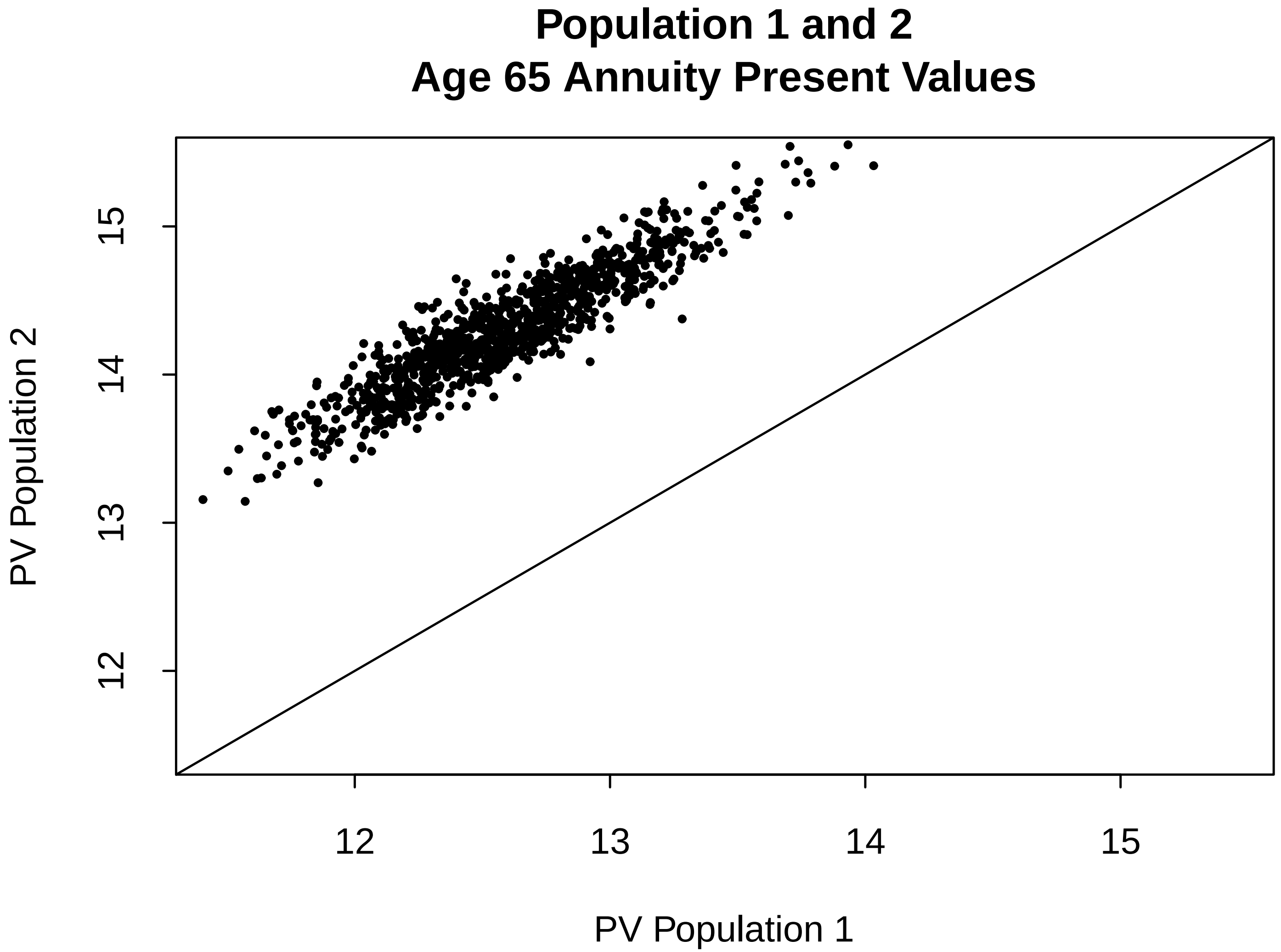
Practical issues

- Structure of the hedging instrument
- Price / risk premium payable by hedger
- Tradeoff:

Hedger	Counterparty
Customised	Index
Full term	Medium term
Uncapped payoff	Limited loss
Swap	Cat Bond format

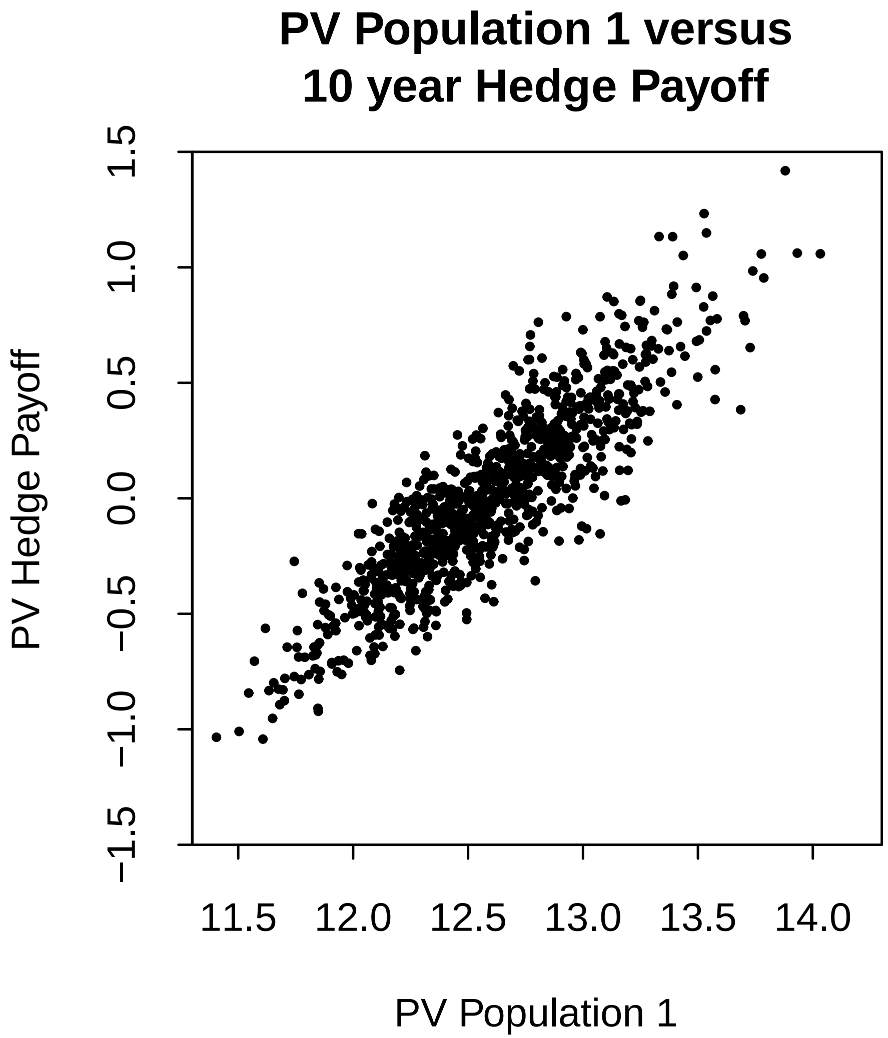
Left: PV of Uncertain Future Annuity Cashflows from Age 65

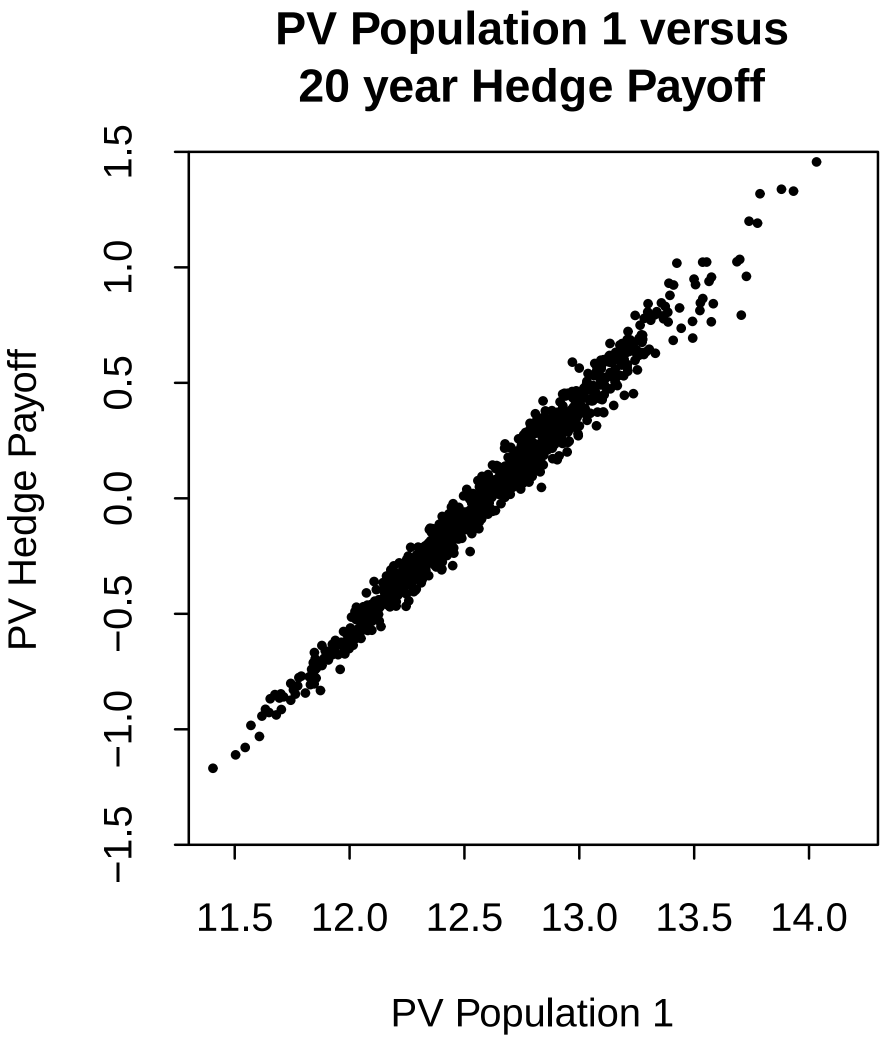
Right: Pop 1 PV versus PV 10-year Swap + Commutation



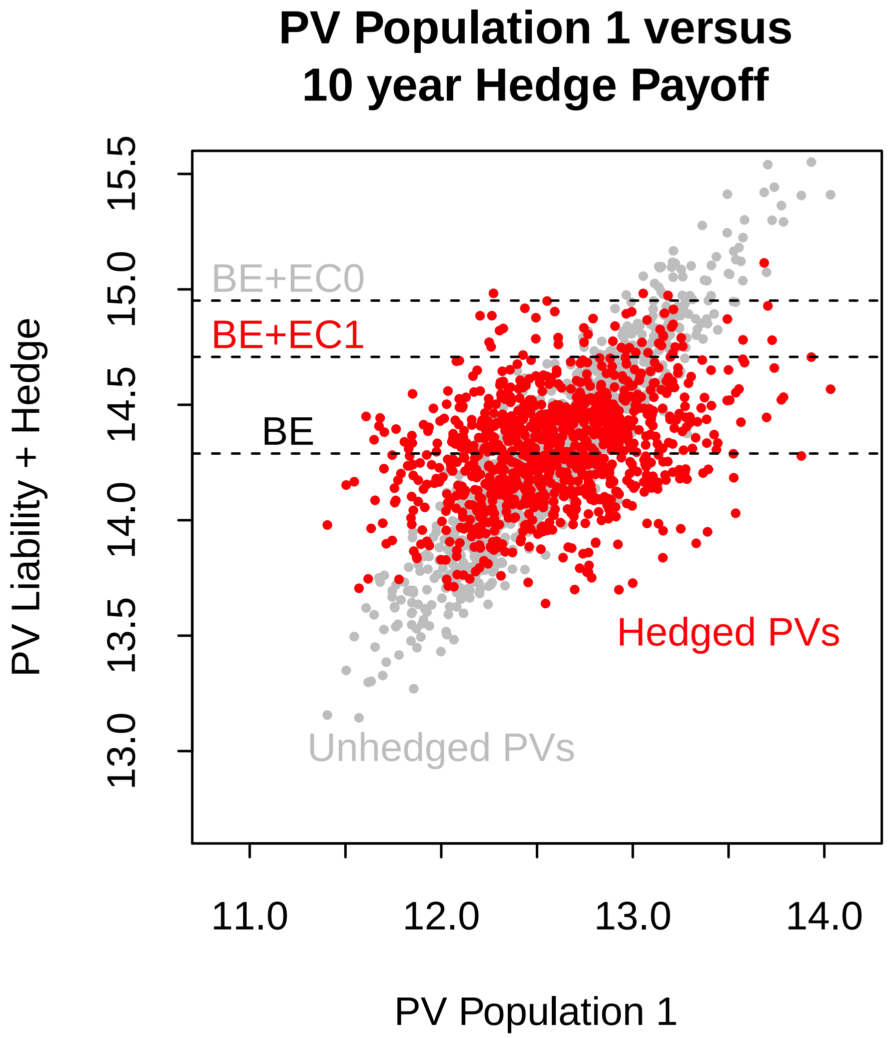
Pop 1 PV versus PV T-year Swap with Commutation

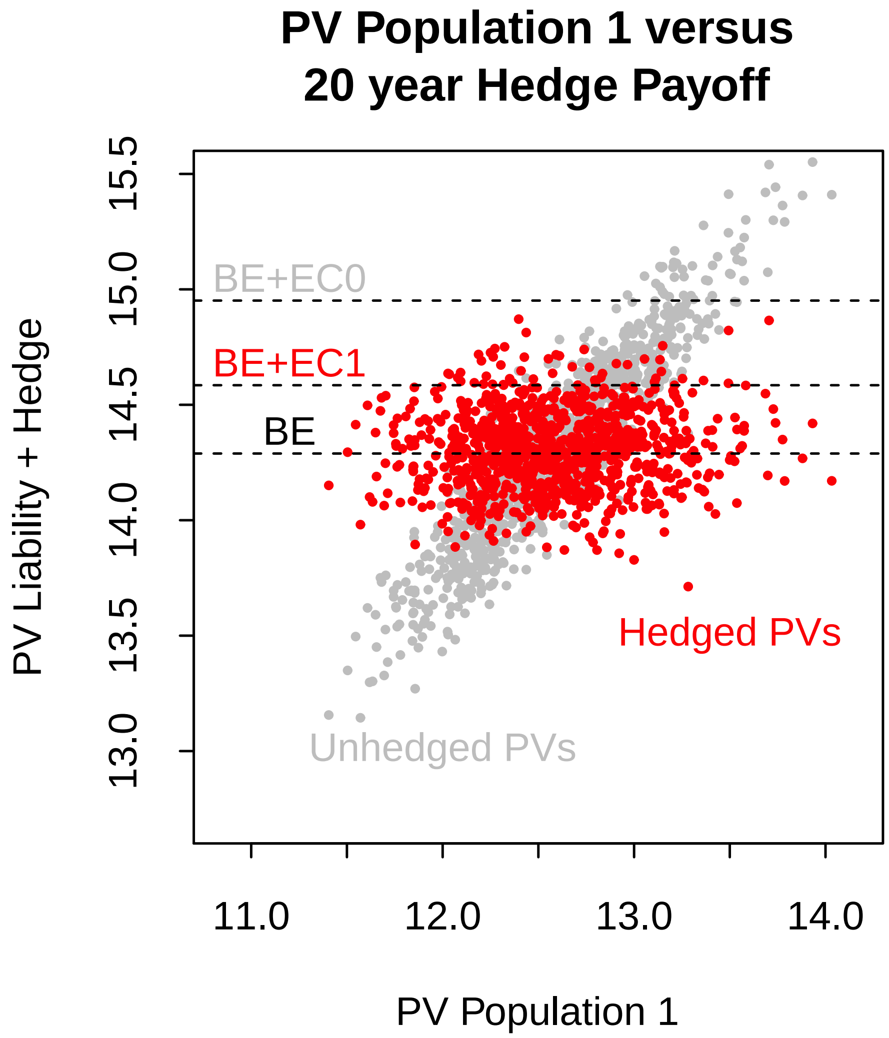
Survivor Swap with Commutation at $T = 10$ or $T = 20$





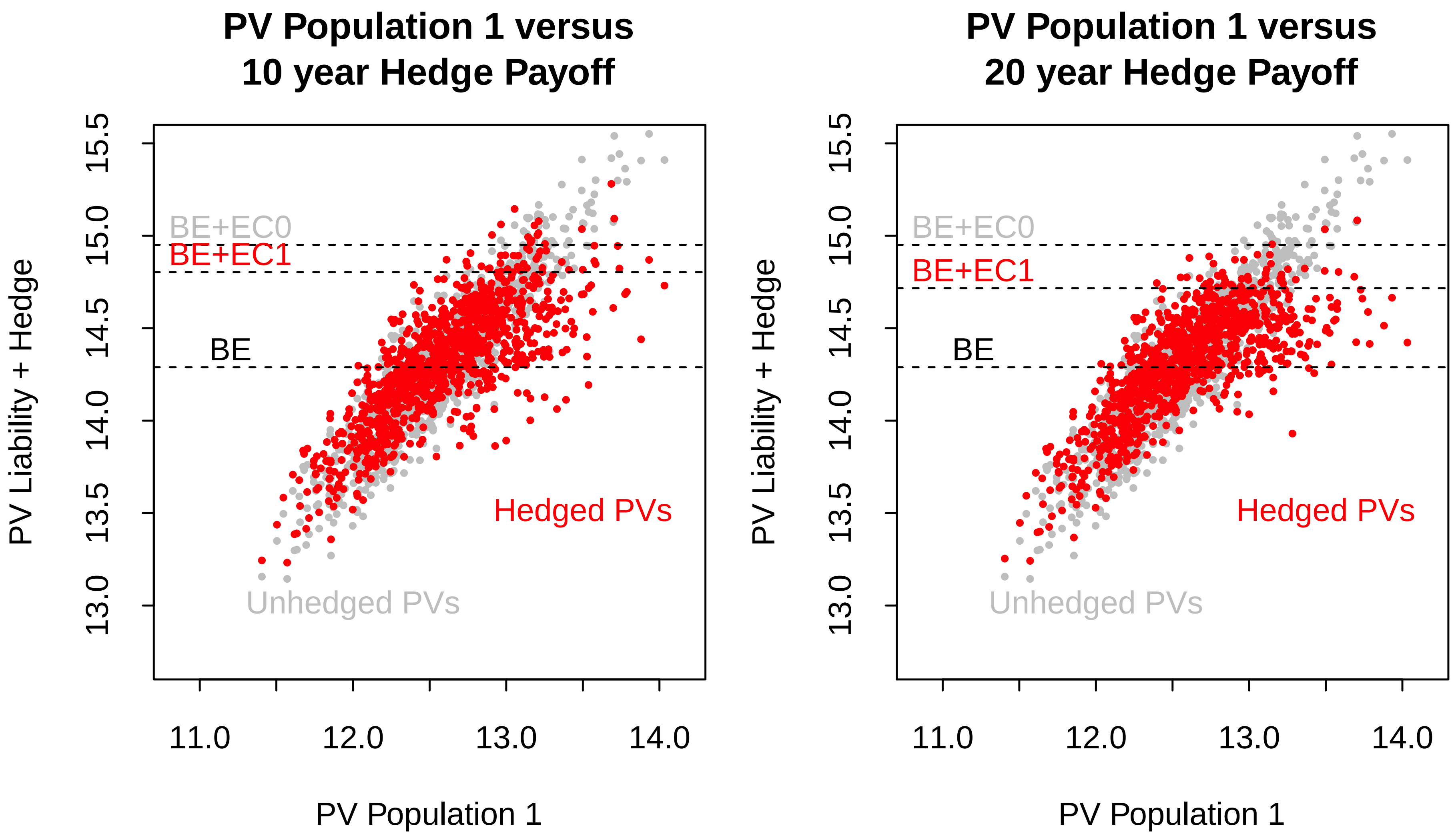
Impact of Swap on Economic Capital





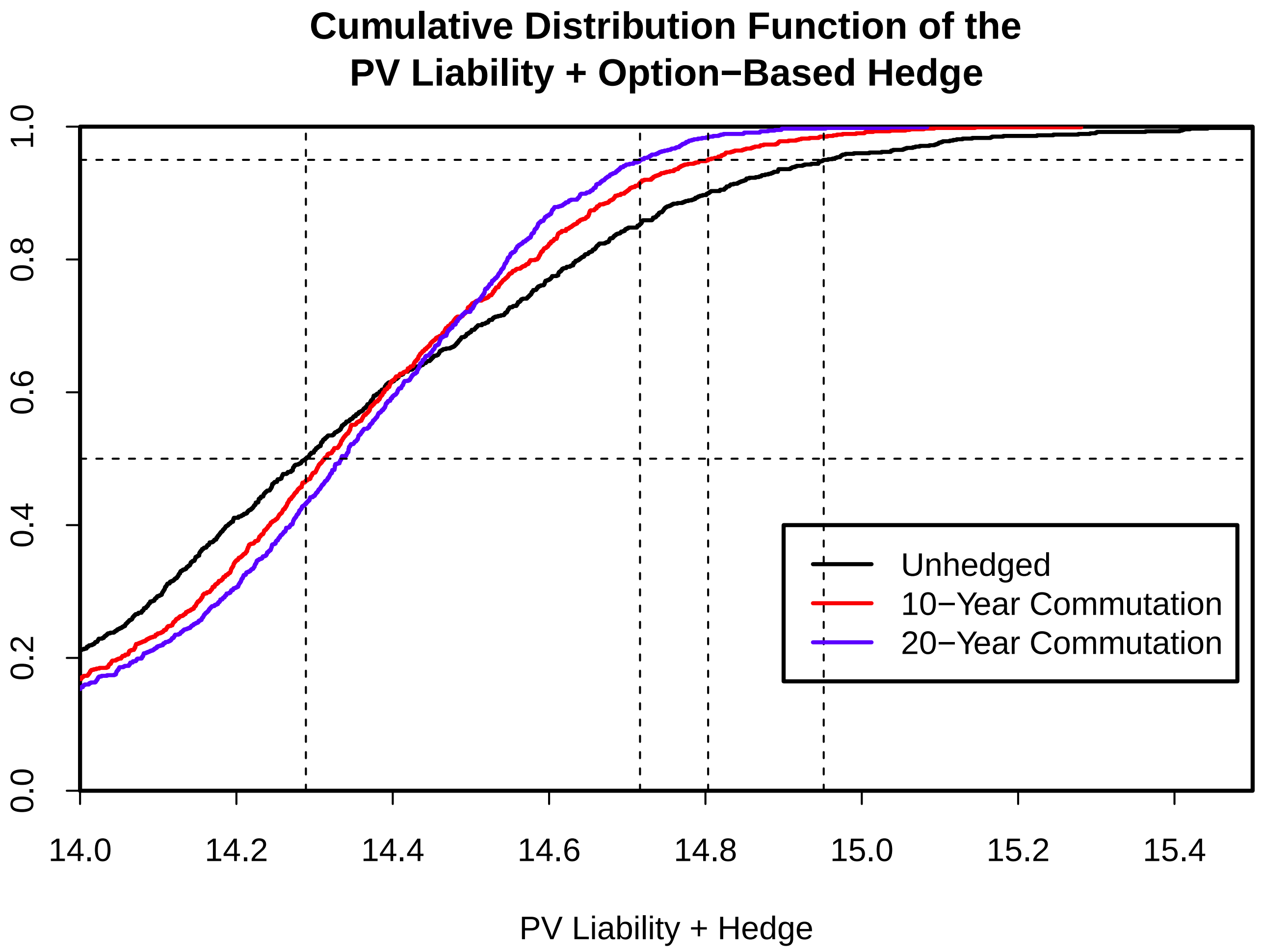
Impact of Option on Economic Capital

Option underlying: accumulated cashflows + commutation



Impact of Option on Economic Capital

Option underlying: accumulated cashflows $+$ commutation



Conclusions and the Future

Robust hedging requires *inclusion* of

- Recalibration risk (Nuga)
- Careful treatment of recalibration window
- Long-dated hedging instruments to handle *Nuga* risk

The future

- Cashflow hedging versus value hedging
- Hedging with different instruments
- Longevity risk is here to stay, but
- The problems might be different

Bonus slides

Value Hedging: basic idea

- $L =$ liability value
- $H =$ value of hedging instrument
- Objective: minimise $Var(\text{deficit}) = Var(L + hH)$

$$\Rightarrow \text{ optimal hedge ratio, } \hat{h} = -\frac{Cov(L,H)}{Var(H)} = -\rho \frac{S.D.(L)}{S.D.(H)}$$

$$\text{Hedge effectiveness} = 1 - \frac{Var(L + \hat{h}H)}{Var(L)} = \rho^2$$

More general: $\Rightarrow$ minimise $Var(L + h_1 H_1 + \ldots + h_n H_n)$

Simpler example: impact of recalibration on correlation

- $X_1 = \mu + Z_1, \quad X_2 = \mu + Z_2$
- Z_1, Z_2 independent
- μ known $\Rightarrow$ cor$(X_1, X_2) = 0$
- μ unknown and independent of Z_1, Z_2

  $\Rightarrow$ Var(X_1) and Var(X_2) both higher

  and cor$(X_1, X_2) > 0$