Outline

- Introduction and motivation
- Hedging with an index-based call-spread option contract
- Anatomy of a hedging calculation in 22 easy steps!
- Numerical example
- Discussion
Motivation

- Longevity risk ⇒ Capital Requirement
- Why use General Population Longevity Index based risk transfer instruments?
  → Capacity and Price
- Pros/cons
  - Transferred risk is efficiently priced
  - But hedger left with basis risk
- Thus we need
  - a clear and rigorous approach to quantify basis risk
  - hedger and regulator agreement on approach
  - to quantify properly the Capital Relief
Introduction

- Underlying problem:
  - Life insurer
  - Aim 1: measure mortality/longevity risk
  - Aim 2: manage mortality/longevity risk
  - e.g. to reduce regulatory capital
  - ⇐⇒ regulatory engagement/acceptance
  - e.g. to reduce economic capital
  - e.g. to increase economic value

- Further aim:
  to bridge the Academic/Practitioner gap
Solvency II options:

- Solvency Capital Requirement, $\text{SCR} =$ difference between
  Best estimate of annuity liabilities (BE) and
  Annuity liabilities following an immediate
  20% reduction in mortality

- or $\text{SCR} =$ extra capital required at time 0 to
  ensure solvency at time 1 with 99.5% probability

- or $\text{SCR} =$ extra capital at time 0 to ensure
  solvency at time $T$ with $x\%$ probability
Liability to be Hedged

- \( L = \) random PV at time 0 of liabilities

- \( L(0) = \) point estimate of \( L \) based on time 0 info

- \( L(T) = \) point estimate of \( L \) based on info at \( T \)
  = PV of actual cashflows up to \( T \)
  + PV of estimated cashflows after \( T \)
Hedging Options

What type of hedge to modify capital requirements and manage risk?

- Index-based hedge
  - Synthetic $\tilde{L}(T) \approx \text{true } L(T)$
  - Call spread derived from underlying $\tilde{L}(T)$

Payoff at $T$, per unit

$$H(T) = \begin{cases} 
0 & \text{if } \tilde{L}(T) < AP \text{ (Attachment Point)} \\
\tilde{L}(T) - AP & \text{if } AP \leq \tilde{L}(T) < EP \text{ (Exhaustion Point)} \\
EP - AP & \text{if } EP \leq \tilde{L}(T)
\end{cases}$$
The Synthetic $\tilde{L}(T)$

- $\tilde{L} = \text{random PV at time 0 of a portfolio of synthetic liabilities}$
- Synthetic mortality experience
  - based on general population mortality
  - adjusted using experience ratios

- $\tilde{L}(T) = \text{point estimate of } \tilde{L} \text{ based on info at } T$
  = PV of actual \textit{synthetic} cashflows up to $T$
  + PV of estimated \textit{synthetic} cashflows after $T$
Questions and Observations

- What impact $L(T) \rightarrow L(T) - H(T)$?
- Need a two population mortality model
- Practical reality: calculation is more complex than academic ‘ideal world’
- What are good choices of $AP$, $EP$, $T$?
Anatomy of a Hedging Calculation in 22 Easy Steps!

\[
\begin{align*}
\text{General (National) Population} & : \\
G(x, t) & , D_G(x, t) , m_G(x, t) \\
E_R(0) & , \mu(0) \\
\text{Specific (Hedger's) Population} & : \\
S(x, t) & , D_S(x, t) , m_S(x, t) \\
E_R(T) & , \mu(T) \\
\end{align*}
\]
Anatomy of a Hedging Calculation: Steps 1, 2

General (National) Population

Specific (Hedger’s) Population

Population G

Population S

$t \leq 0$

$t \geq 91$

$t \geq 90$

$0 \leq t \leq T$

$1 \leq t \leq T$

$t > T$

$L(T)$

$H(T)$

$\hat{\mu}(0)$

$\hat{\mu}(T)$

$\hat{\mu}(0) + \hat{\mu}(T)$

$L(T)$

$H(T)$

$E_G(x, t)$

$D_G(x, t)$

$m_G(x, t)$

$E_P(x, t)$

$D_P(x, t)$

$m_P(x, t)$

$LC(0)\hat{\mu}(0)$

$C(T)\hat{\mu}(T)$

$SIM(0)$

$ER(0)$

$M1M5X(0)$

$SIM(T)$

$ER(T)$

$\hat{\mu}(0) + \hat{\mu}(T)$

$L(T)$

$H(T)$
Anatomy of a Hedging Calculation: Steps 3-5

General (National) Population

Specific (Hedger’s) Population

$t \leq 0$

$1 \leq t \leq T$

$t > T$

Population G

$E_G(x, t)$

$D_G(x, t)$

$m_G^c(x, t)$

Population S

$E_P(x, t)$

$D_P(x, t)$

$m_P^c(x, t)$

$t \leq 0$

$t > T$

$L(T)$

$H(T)$

$C(0) \xrightarrow{\hat{\mu}(0)} C(T) \xrightarrow{\hat{\mu}(T)} C(T)$

$\hat{\mu}(0) + \text{SIM}(0)$

$\hat{\mu}(T) + \text{SIM}(T)$

$\text{ER}(0)$

$\text{ER}(T)$

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The Impact of Longevity Risk Hedging on Economic Capital 12 / 32
Anatomy of a Hedging Calculation: Steps 6, 7, 14, 15, 17

General (National) Population

Population G

\( E_G(x, t) \)
\( D_G(x, t) \)
\( m_G(x, t) \)

\( C(0) \)
\( \hat{\mu}(0) \)

Population S

\( E_P(x, t) \)
\( D_P(x, t) \)
\( m_P(x, t) \)

\( \hat{\mu}(0) \)

Specific (Hedger’s) Population

Time:

\( t \leq 0 \)
\( 1 \leq t \leq T \)
\( t > T \)

Economic Risk (ER)

\( \hat{\mu}(0) \)
\( M1M5X(0) \)

Simulation (SIM)

\( \hat{\mu}(0) + \)

\( \hat{\mu}(T) \)

\( \hat{\mu}(T) + \)

\( \hat{\mu}(T) \)

L(T)

H(T)

LC(0)

\( \hat{\mu}(0) \)

SIM(0)

ER(T)

\( \hat{\mu}(T) \)

\( M1M5X(T) \)

\( C(T) \)

\( m_G(x, t) \)

\( m_P(x, t) \)
Anatomy of a Hedging Calculation: Steps 8, 9, 12

General (National) Population

Specific (Hedger’s) Population

Population G

Population S

0

90

≥ 91

≥ 90

≥ 89

≤ 40

≤ 39

$E_G(x, t)$

$D_G(x, t)$

$m_G(x, t)$

$E_P(x, t)$

$D_P(x, t)$

$m_P(x, t)$

$C(0)$

$\hat{\mu}(0)$

$\hat{\mu}(0)$

$\hat{\mu}(0) +$ SIM(0)

$C(T)$

$\hat{\mu}(T)$

SIM(T)

ER(T)

$\tilde{L}(T)$

$H(T)$

$t \leq 0$

$1 \leq t \leq T$

$t > T$

$LC(0)$

$\hat{\mu}(0) +$ SIM(0)

$ER(T)$

$SIM(T)$

$ER(0)$

$L(T)$

$M1M5X(0)$
Anatomy of a Hedging Calculation: Steps 10, 11, 13, 14, 16, 18

General (National) Population

Population $G$

$E_G(x, t)$

$D_G(x, t)$

$m_G(x, t)$

$t \leq 0$

$\mu(0)$

$C(0)$

$\hat{\mu}(0)$

$\text{M1M5X}(0)$

$\text{SIM}(0)$

$\text{ER}(T)$

$\text{ER}(0)$

$\mu(0)$

$\text{SIM}(0)$

$\text{ER}(0)$

$t > T$

$\tilde{L}(T) \rightarrow H(T)$

Specific (Hedger's) Population

Population $S$

$E_P(x, t)$

$D_P(x, t)$

$m_P(x, t)$

$0 \leq t \leq T$

$\mu(T)$

$C(T)$

$\text{SIM}(T)$

$\text{ER}(T)$

$\mu(T)$

$\text{SIM}(T)$

$\text{ER}(T)$

$t > T$

$L(T)$

$t \leq 0$

$\geq 91$

$90$

$0$

$\geq 90$

$89$

$\geq 90$

$89$

$40$

$\leq 39$
Anatomy of a Hedging Calculation: Steps 19-22

Population G
\[ E_G(x, t) \]  
\[ D_G(x, t) \]  
\[ m_G(x, t) \]

Population S
\[ E_P(x, t) \]  
\[ D_P(x, t) \]  
\[ m_P(x, t) \]

General (National) Population

Specific (Hedger’s) Population

\[ 1 \leq t \leq T \]
\[ \tilde{L}(T) \rightarrow H(T) \]

\[ t > T \]

\[ \hat{m}_G(x, t) \]

\[ \hat{m}_P(x, t) \]
How many models do you need?

**Academic ‘ideal’: One model**

**In practice:**

- **Time 0:**
  - Liability valuation model (BE + SCR)
  - Simulation model (0 → T)

- **Time T:**
  - Hedge instrument valuation model
  - Liability valuation model

- ‘Models’ for extrapolating to high (and low) ages
Time 0 Models

- **Unhedged Liabilities:**
  Deterministic BE + 20% stress

- **Simulation:** (by way of example)
  - General population: (Lee-Carter/M1)
  \[
  \ln m_{\text{gen}}(x, t) = A(x) + B(x)K(t) \quad (\text{Lee-Carter/M1})
  \]
  - Hedger’s own population: (M1-M5X)
  \[
  \ln m_{\text{pop}}(x, t) = \ln m_{\text{gen}}(x, t) + a(x) + k_1(t) + k_2(t)(x - \bar{x})
  \]
Time $T$ models

- **Hedge instrument:**
  - Lee-Carter (M1) for general population
  - Recalibration: *on basis specified at time 0*

$$q_{pop}^H(x, t) = q_{gen}^H(x, t) \times ER(x, 0) \rightarrow \tilde{L}(T) \rightarrow H(T)$$

- **Liability: specific (hedger’s) population**
  - Lee-Carter (M1) for general population
  - Possible different calibration from the hedge instrument

$$q_{pop}^L(x, t) = q_{gen}^L(x, t) \times ER(x, T) \rightarrow L(T)$$
Hedging Example

- Data: Netherlands
  - CBS national data
  - CVS insurance data (Dutch aggregated industry experience data)

- Hedge instrument maturity: $T = 10$
- Attachment and exhaustion points at 60% and 95% quantiles of $\tilde{L}(T)$
- Key point: $EP "<<" 99.5\% quantile of $\tilde{L}(T)$
Hedging Example

- Portfolio of deferred and immediate annuities
- Current ages 40 to 89
- Weights (≡ pension amounts):
  \[ w_x = \begin{cases} 
  x - 25 & \text{for } 40 \leq x < 50 \\
  25 & \text{for } 50 \leq x < 65 \\
  90 - x & \text{for } x \geq 65 
  \end{cases} \]
- Deferred to age 65
- Before and after: Compare \( L(T) \) with \( L(T) - H(T) \)
- SCR = 99.5% quantile − mean
Hedging Example ($n = 10,000$ scenarios)

Simulated Annuity Portfolio Present Values

Correlation=0.978

$\text{S.D.}=350$

$\text{S.D.}=395$
Hedging Example: Unheded VaR = 11,649
Hedging Example: Hedged VaR = 11,199
### Numerical Example: AP, EP = 60% and 95% quantiles

<table>
<thead>
<tr>
<th></th>
<th>SCR\textsubscript{20%stress}</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( L(0) )</td>
<td></td>
<td>840</td>
</tr>
<tr>
<td>( \tilde{L}(T) )</td>
<td>( SCR_{10} )</td>
<td>840</td>
</tr>
<tr>
<td>( \tilde{L}(T) - H(T) )</td>
<td>( SCR_{11} )</td>
<td>478</td>
</tr>
<tr>
<td>( L(T) )</td>
<td>( SCR_{20} )</td>
<td>960</td>
</tr>
<tr>
<td>( L(T) - H(T) )</td>
<td>( SCR_{21} )</td>
<td>598</td>
</tr>
</tbody>
</table>

Table: SCR values in excess of the mean liability. For the hedging instrument \( AP = 10779 \) (60% quantile) and \( EP = 11228 \) (95% quantile). Pop 1: synthetic \( \tilde{L}(T) \). Pop 2: true \( L(T) \).

What impact of Population basis risk on hedge effectiveness?

Haircut \( HC = 1 - \frac{SCR_{20} - SCR_{21}}{SCR_{10} - SCR_{11}} = 0.000 \).
Haircut $\approx 0$: Interpretation

- Here $EP$ "$\ll$" 99.5% quantile
- Above the 99.5% quantile the call spread (almost) always pays off in full
- So population basis risk $\Rightarrow$ little impact
- Structural basis risk prevails

- More detailed analysis $\Rightarrow$
  Haircut is worst (highest) when EP is close to the 99.5% quantile.
Reduction in SCR: Dependence on AP and EP

Reduction in SCR with Hedge as a Percentage of SCR without Hedge

Attachment Point Quantile

Exhaustion Point Quantile

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Hedge Maturity, $T$

- e.g. $T = 20$

- % reduction in SCR is *slightly* higher
- Haircut is *slightly* worse
- Haircut is still $\approx 0$ for $EP \leq 99.5\%$ quantile

The longer the maturity:

- less liquid market
- less confidence in future reserving method
- more future capital relief (everything else held constant)
Summary

- Bridging the gap:
  Academics ↔ Insurance practitioners ↔ Regulators

- Academics:
  practice is more messy than you would like!

- Practitioners: insightful exercise, ultimately allows for flexible longevity risk management.
Thank You!

Questions?