MULTI-POPULATION MORTALITY MODELLING:

A Danish Case Study

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Plan

- Introduction and motivation for multi-population modelling
- Danish population data
- Modelling Danish sub-population mortality
- Applications

1. Motivation for multi-population modelling

A: Risk assessment

- Multi-country (e.g. consistent demographic projections)
- Males/Females (e.g. consistent demographic projections)
- Socio-economic subgroups (e.g. blue or white collar)
- Smokers/Non-smokers
- Annuities/Life insurance
- Limited data $\Rightarrow$ learn from other populations

$\rightarrow$ reserving calculations; diversification benefits

Motivation for multi-population modelling

B: Risk management for pension plans and insurers

- Retain systematic mortality risk; versus:
- 'Over-the-counter' deals (e.g. longevity swap)
- Standardised mortality-linked securities
  - linked to national mortality index
  - $< 100\%$ risk reduction: basis risk

Multi-Population Challenges

- Data availability
- Data quality and depth
- Model complexity
  - single population models can be complex
  - 2-population versions are more complex
  - multi-pop ......
- Multi-population modelling requires
  - (fairly) simple single-population models
  - simple dependencies between populations

2. A New Case Study and a New Model

- Sub-populations differ from national population
  - socio-economic factors
  - other factors
- Denmark
  - High quality data on ALL residents
  - 1981-2005 available (later data soon)
  - Can subdivide population using covariates on the database

Danish Data

- *What can we learn from Danish data that will inform us about other populations?*

- Key covariates
  - Wealth
  - Income

- *Affluence* $=$ Wealth$+15\times$Income

Problem

- High income $\Rightarrow$ "affluent" *and low mortality* BUT
- Low income $\not\Rightarrow$ not affluent, high mortality
- High wealth $\Rightarrow$ "affluent" *and low mortality* BUT
- Low wealth $\not\Rightarrow$ not affluent, high mortality

Empirical solution: use a combination

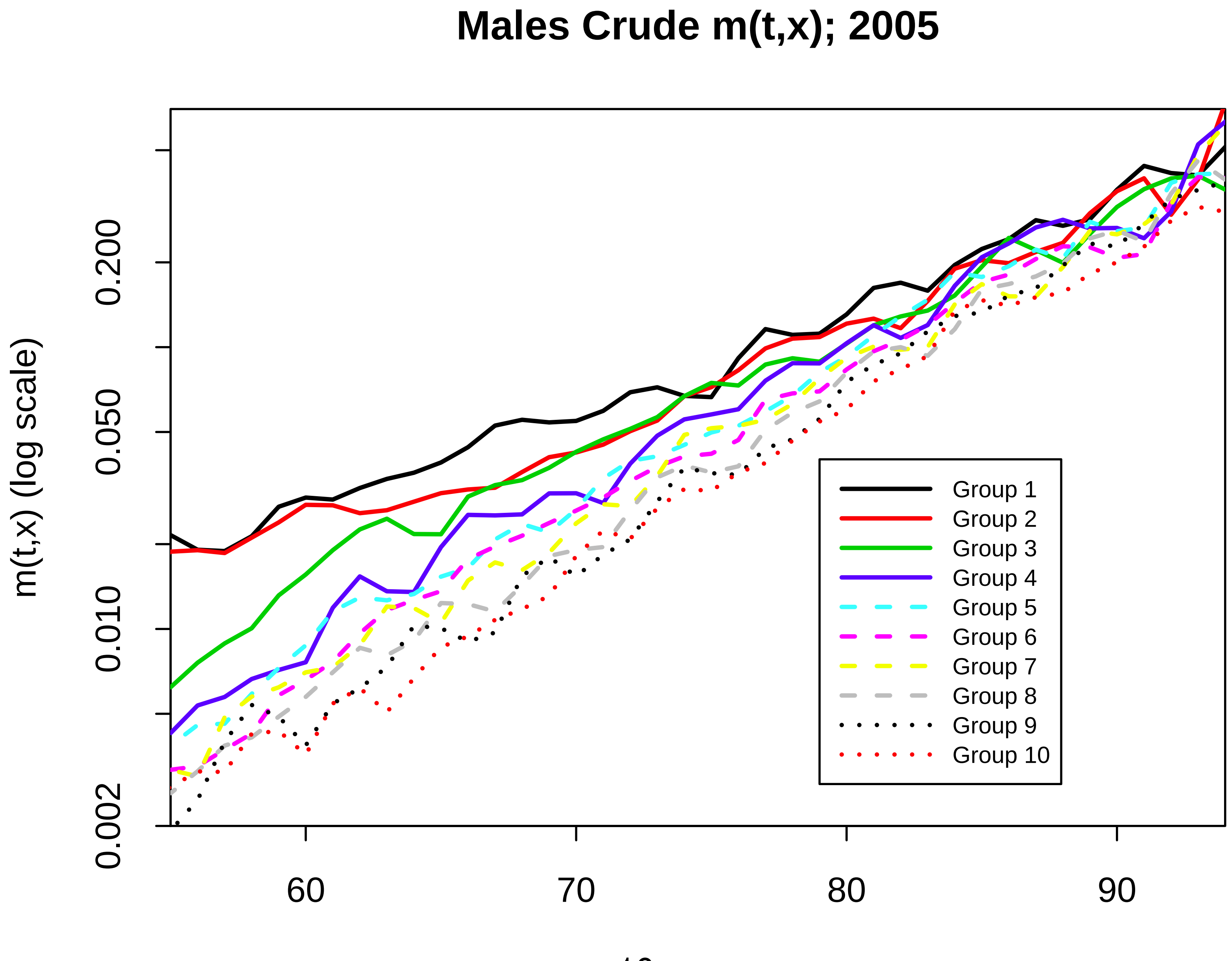
- Affluence, $A =$ wealth $+ K \times$ income
- $K = 15$ seems to work well *statistically* as a predictor
- Low affluence, A, predicts poor mortality

Subdividing Data (after much experimentation!)

- Males resident in Denmark for the previous 12 months
- Divide population in year t
  - into 10 equal sized Groups (approx)
  - using *affluence*, A
- Individuals can change groups up to age 67
- Group allocations are locked down at age 67

(better than not locking down at age 67)

Crude death rates 2005



Modelling the death rates, $m_k(t,x)$

$m^{(k)}(t,x) =$ pop. k death rate in year t at age x

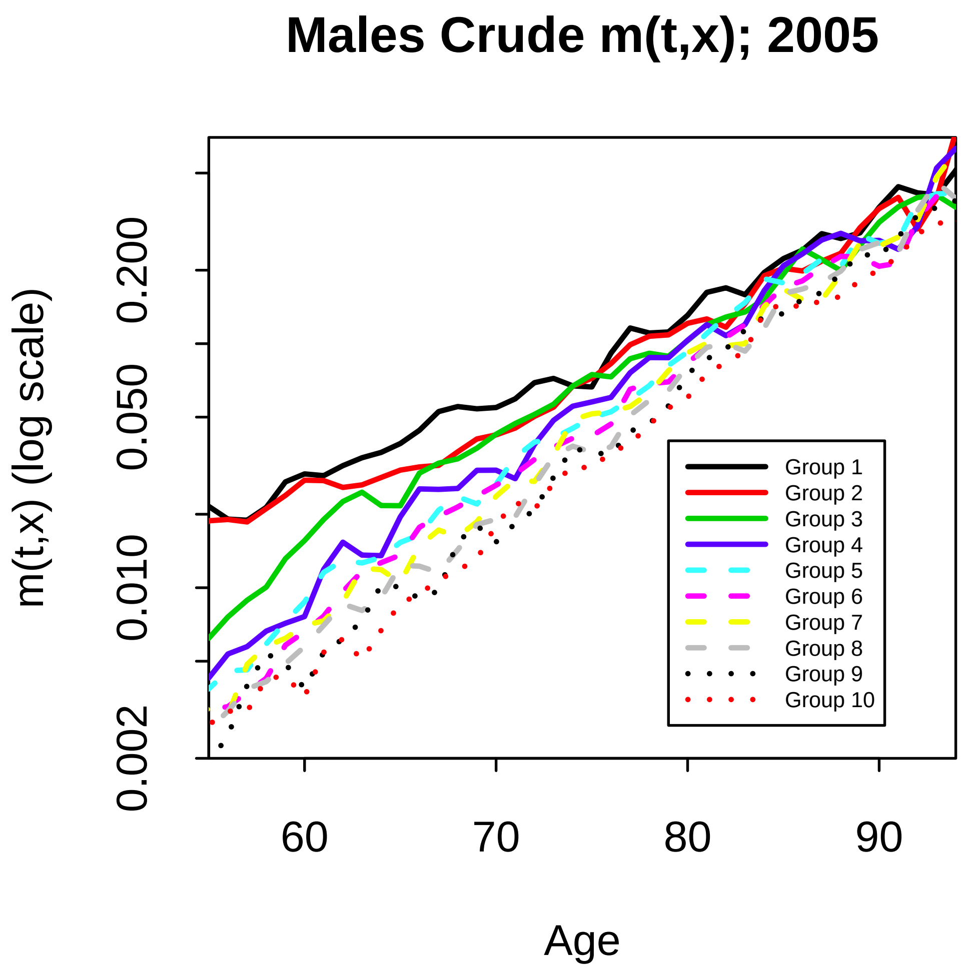
Population k, year t, age x

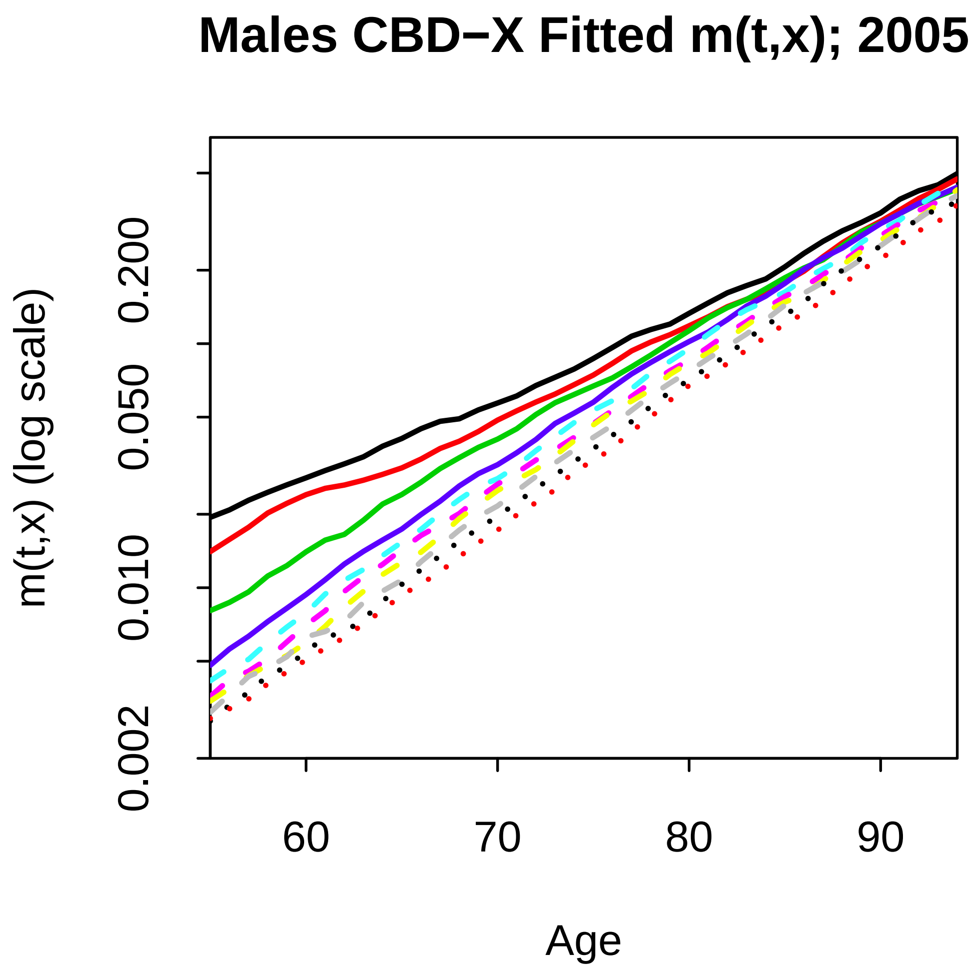
$$\log m^{(k)}(t,x) = \beta^{(k)}(x) + \kappa_1^{(k)}(t) + \kappa_2^{(k)}(t)(x-\bar{x})$$

(Extended CBD with a non-parametric base table, $\beta^{(k)}(x)$)

- 10 groups, $k = 1, \ldots, 10$ (low to high affluence)
- 21 years, $t = 1985, \ldots, 2005$
- 40 ages, $x = 55, \ldots, 94$

Model-Inferred Underlying Death Rates 2005





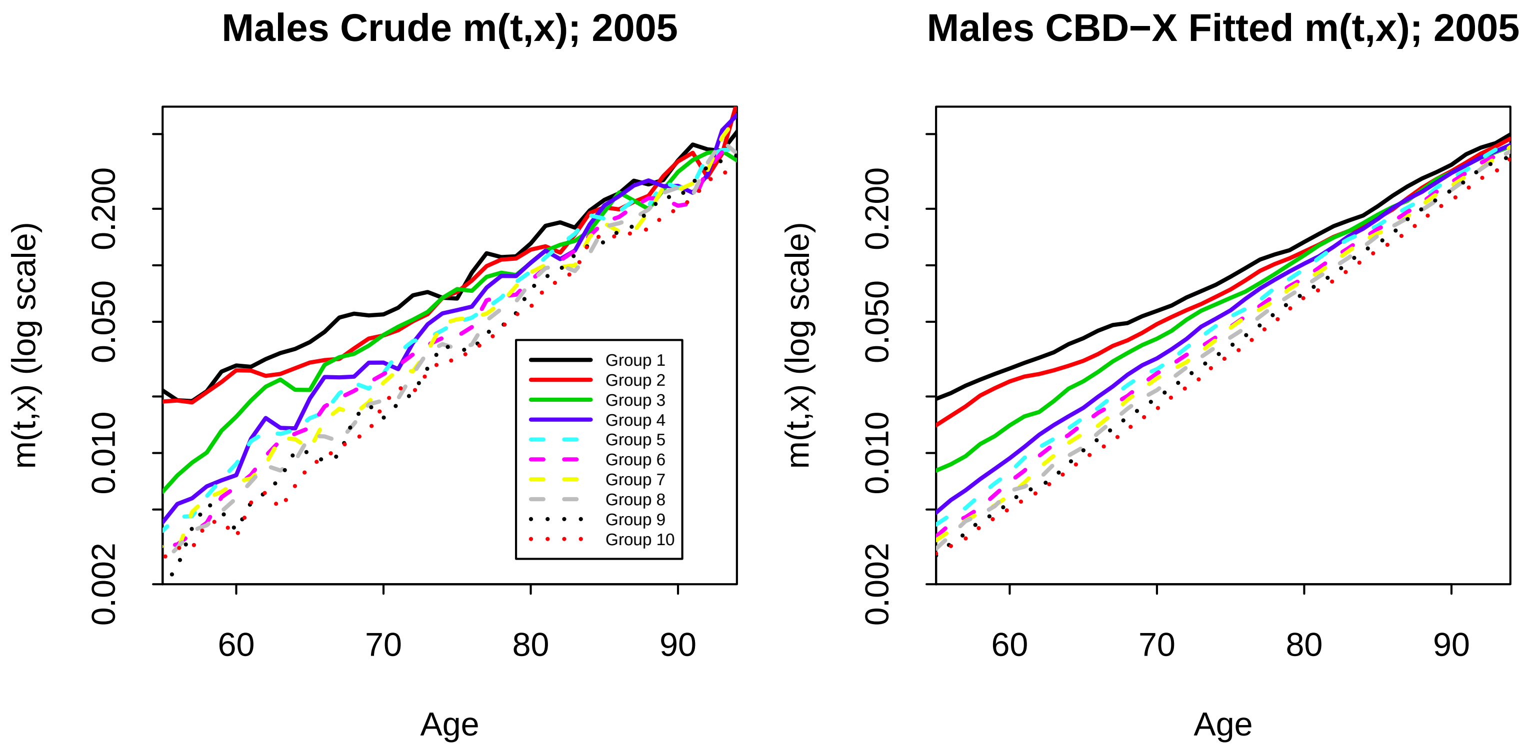
Modelling the death rates, $m_k(t, x)$

$$\log m^{(k)}(t, x) = \beta^{(k)}(x) + \kappa_1^{(k)}(t) + \kappa_2^{(k)}(t)(x - \bar{x})$$

- Model fits the 10 groups well without a cohort effect
- Non-parametric $\beta^{(k)}(x)$ is essential to preserve group rankings
  - Rankings are evident in crude data
  - *"Bio-demographical reasonableness"*:

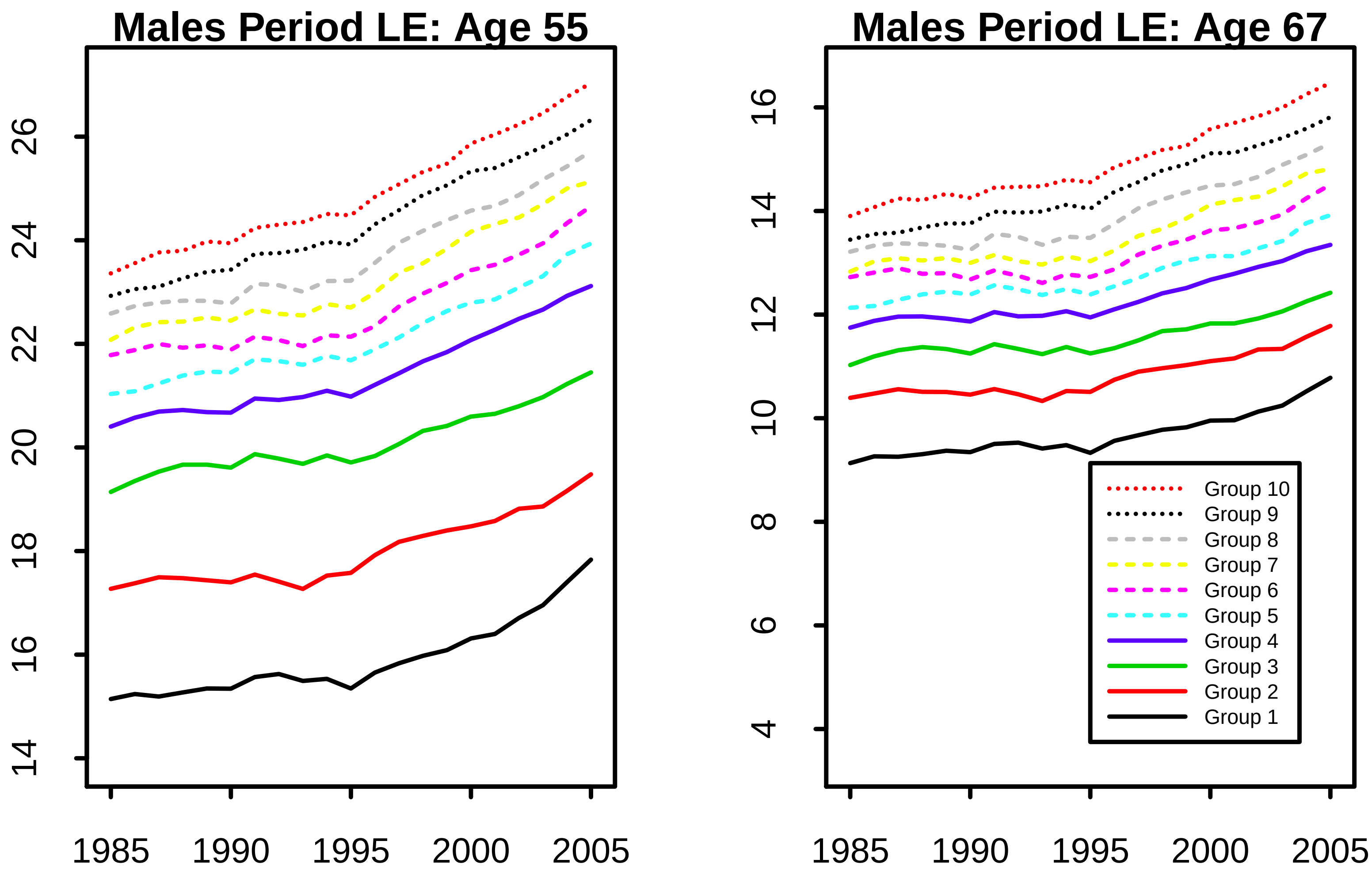
more affluent $\Rightarrow$ healthier

Model-Inferred Underlying Death Rates 2005



- Gap reduces from over $6\times$ to $1.5\times$
- Or $+17$ years difference for Group 1, age 55; $+11$ at 67.
- Convergence $\Rightarrow$ way ahead for modelling very high ages???

Life Expectancy for Groups 1 to 10



3. Applications

- Coherent forecasting
- Mortality
- Cohort survivorship
- Annuity risk measurement
- Hedging: customised *versus* index-linked hedges

Time series modelling

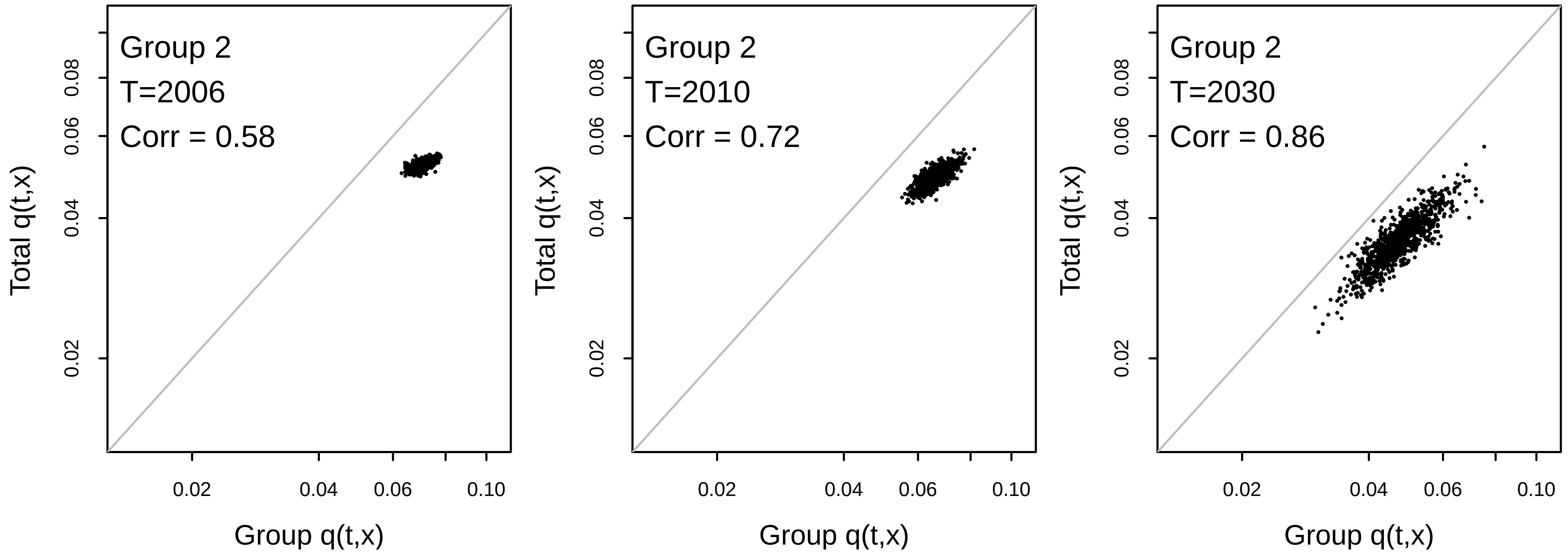
- $t \to t+1$: Allow for correlation
  - between $\kappa_1^{(k)}(t+1)$ and $\kappa_2^{(k)}(t+1)$
  - between groups $k = 1, \ldots, 10$
- Medium/long term:

  group specific period effects *gravitate* towards the national trend

  $\Rightarrow$ Bio-demographical reasonableness:

  groups should not diverge

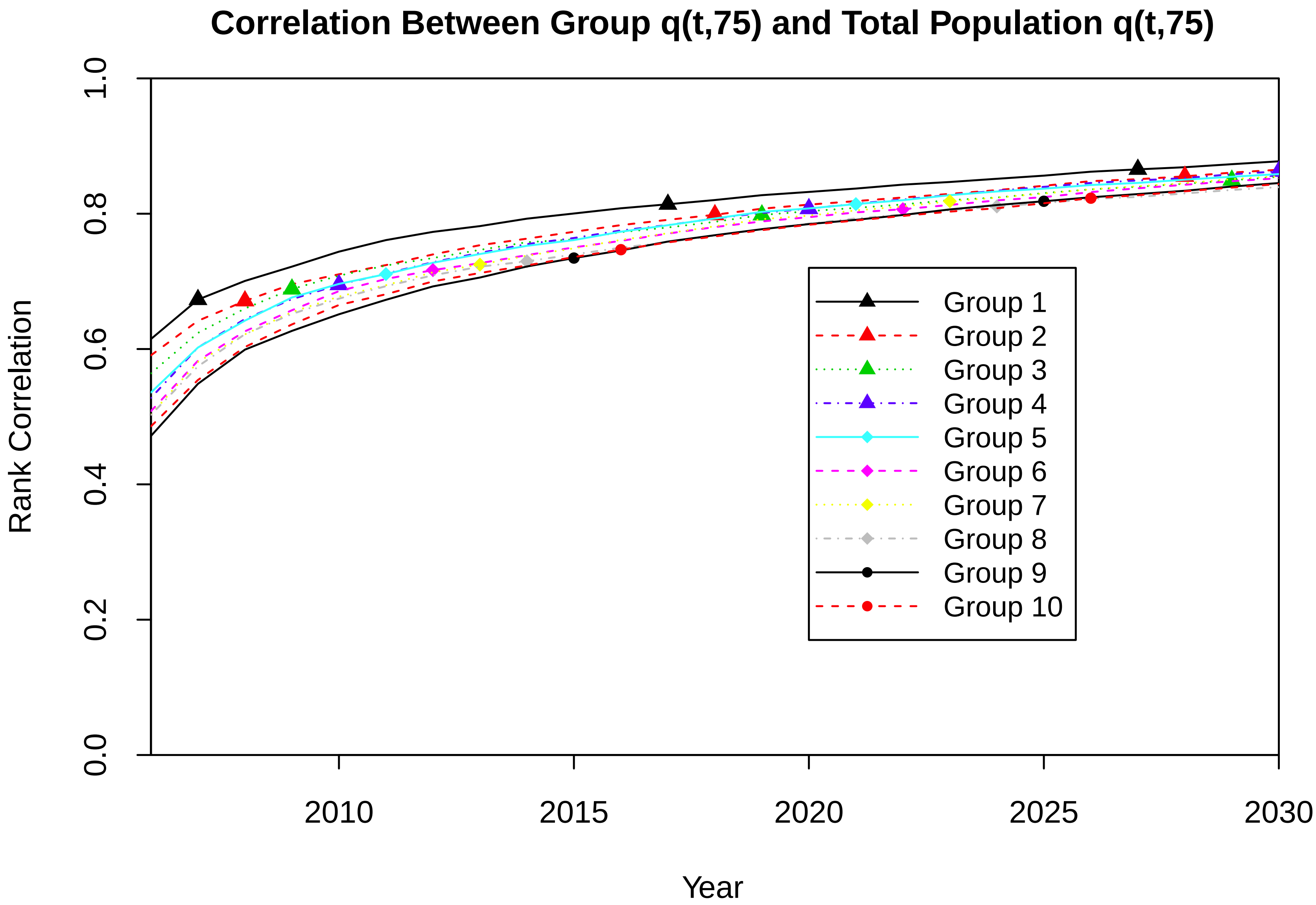
Simulated Group versus Population Mortality, $q(t,x)$



As T increases: $+1$ year; $+5$ years; $+25$years

- Scatterplots become more dispersed
- Shift down and to the left
- Correlation increasess

Forecast Correlations: Mortality Rates



Forecast Correlations

- Deciles are quite narrow subgroups

More diversified e.g.

- Blue collar pension plan

  $\Rightarrow$ equal proportions of groups 2, 3, 4

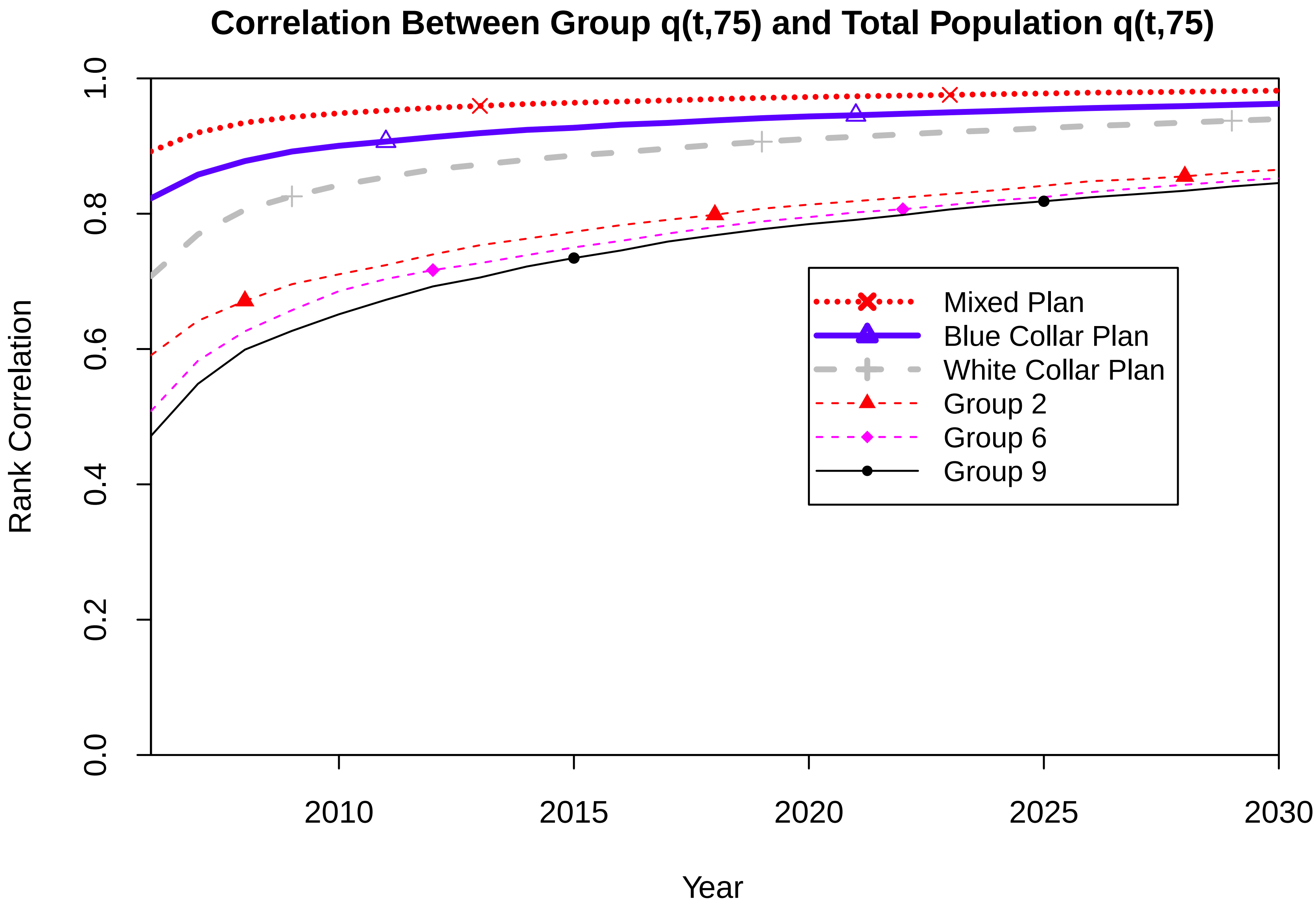
- White collar pension plan

  $\Rightarrow$ equal proportions of groups 8, 9, 10

- Mixed pension plan

  $\Rightarrow$ amounts proportional to $(0, 0, 1, 2, 3, 4, 5, 6, 7, 8)$

Forecast Correlations: Mortality Rates



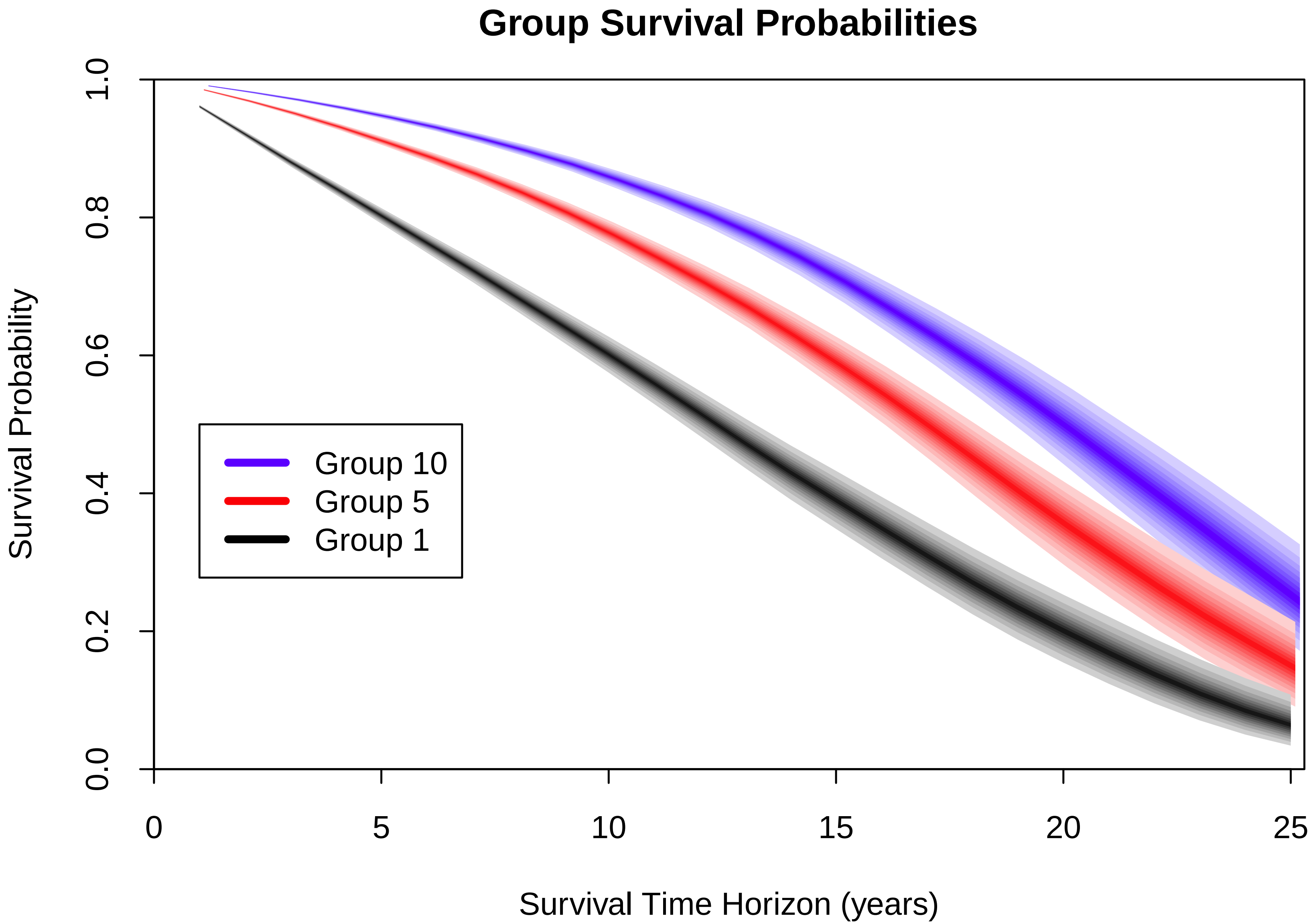
Cohort Survivorship

What proportion of a group survive from age 65 at time 0 to time t?

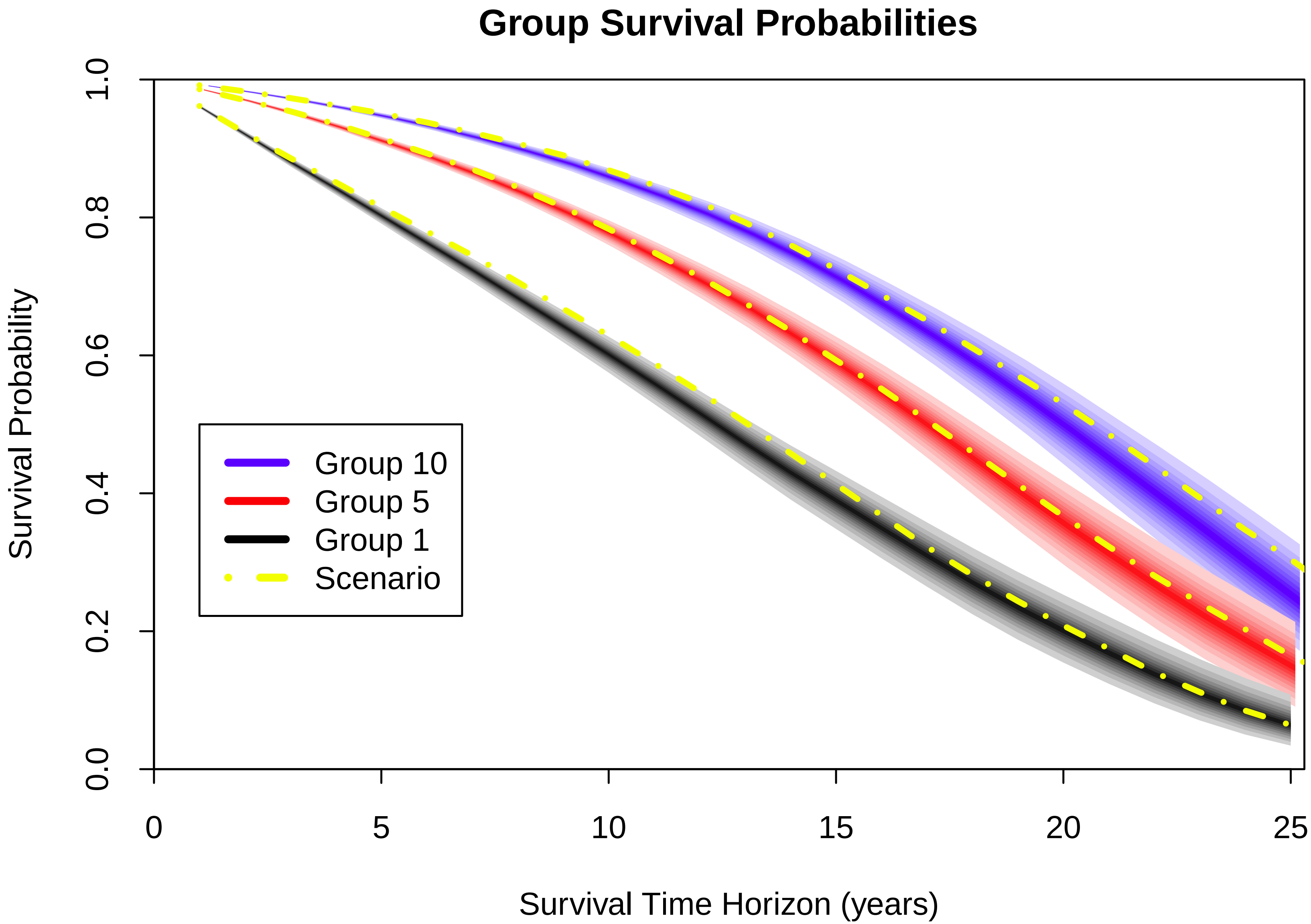
- $S_X(t, 65)$
- Groups 1 to 10 individually
- Blue collar plan
- White collar plan
- Mixed plan

Compare with the national population

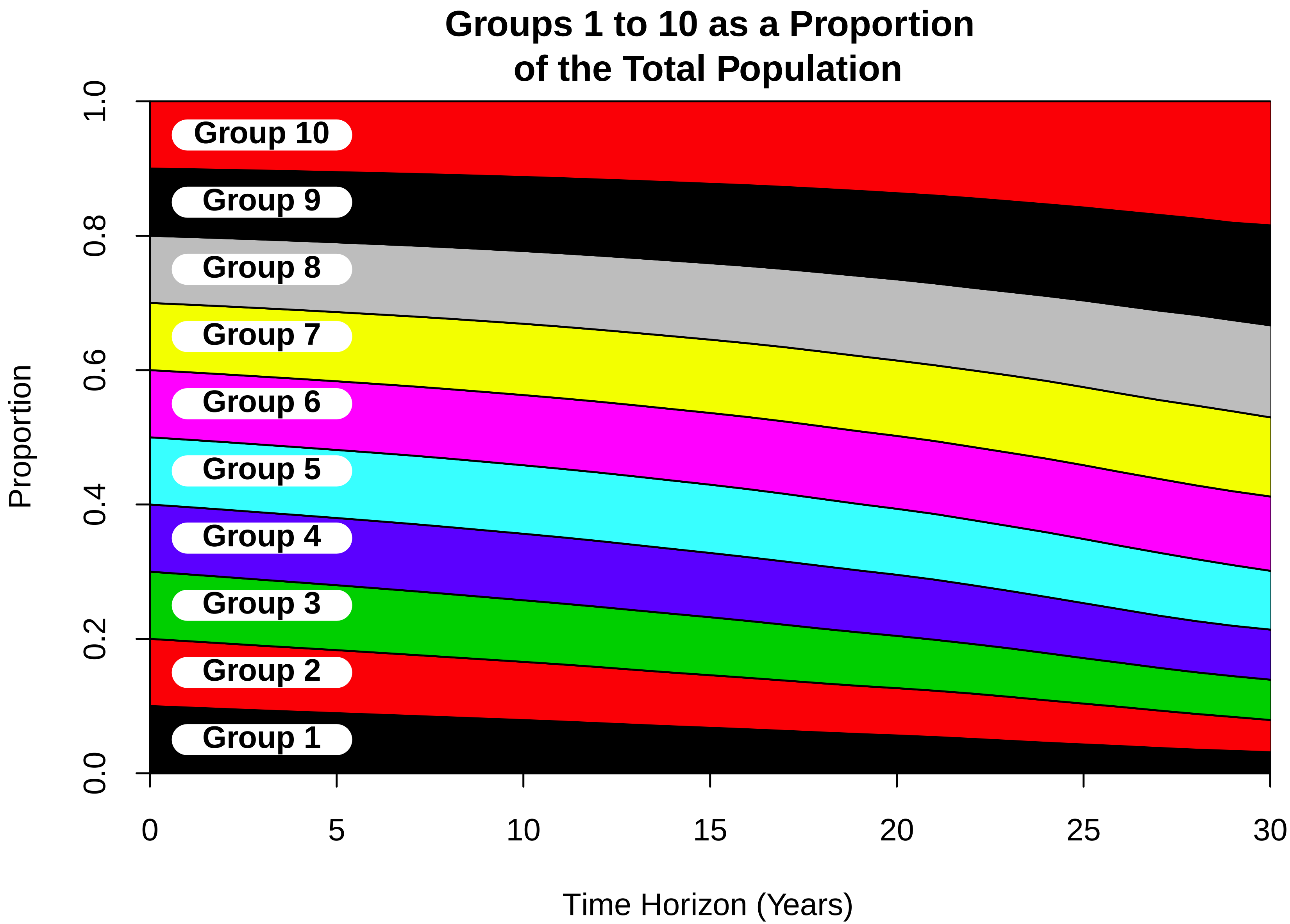
Cohort Survivorship: Fan Charts



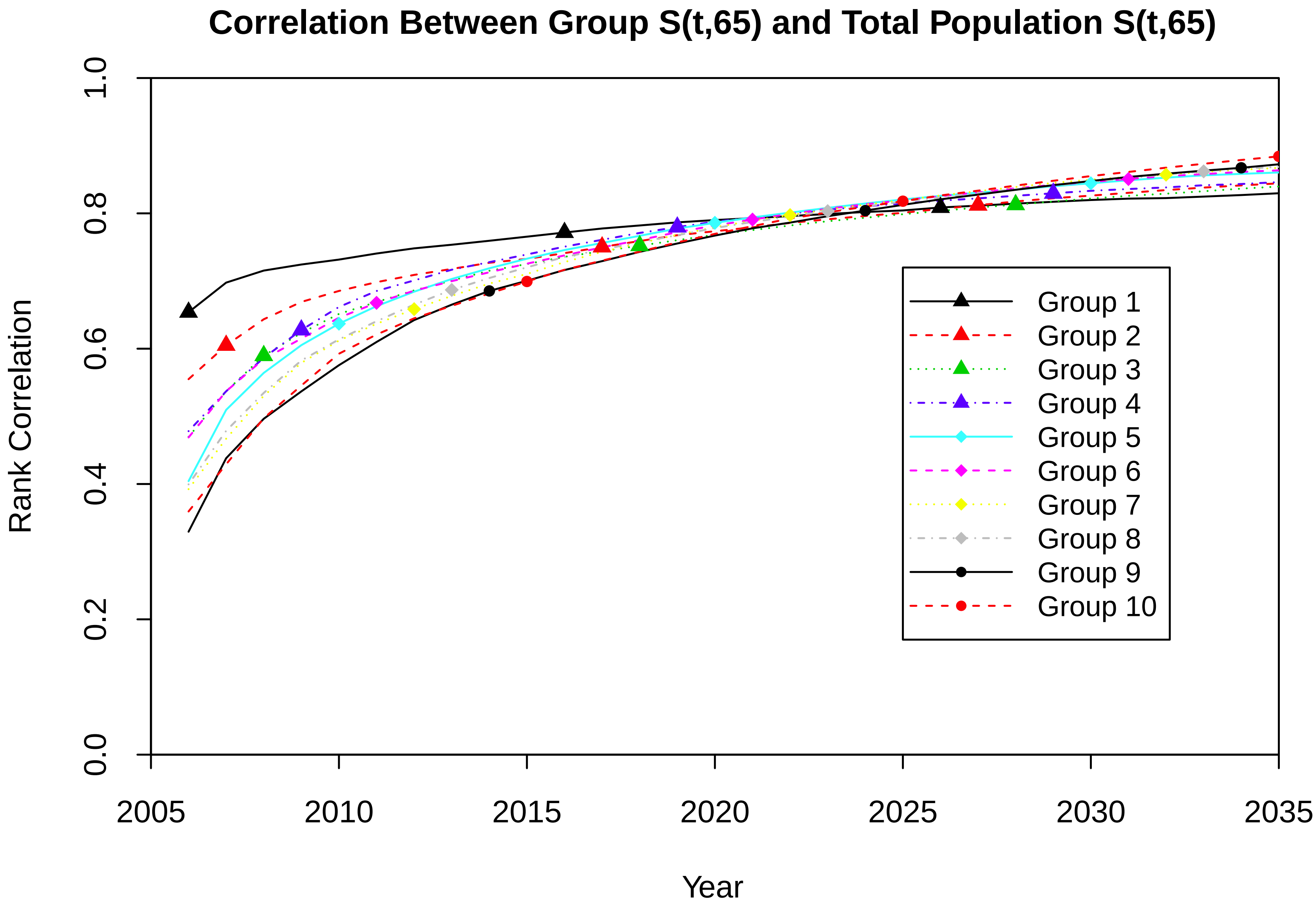
Cohort Survivorship: Individual Scenarios



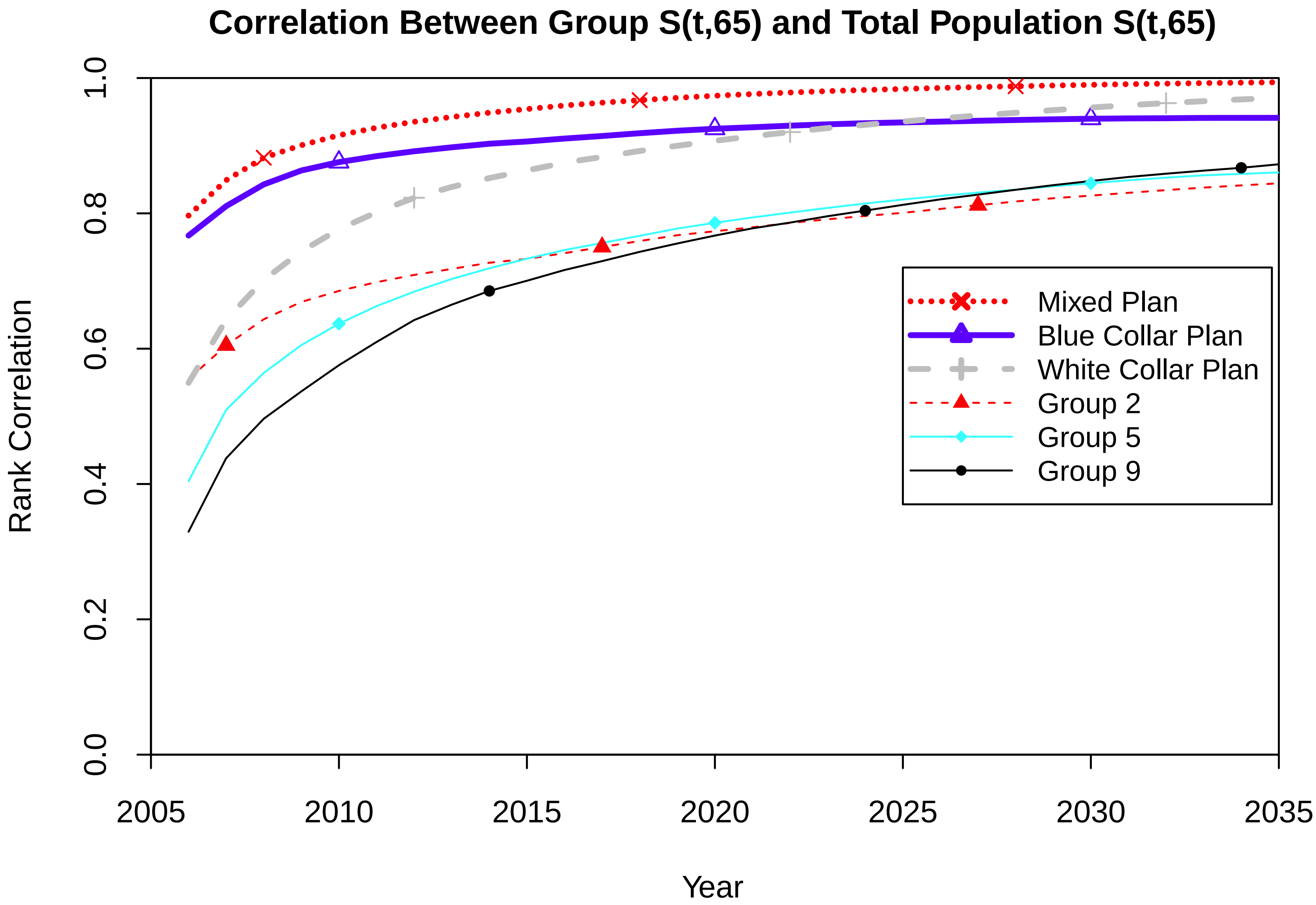
Cohort Survivorship: Changing Population Mix



Forecast Correlations: Cohort Survivorship



Forecast Correlations: Cohort Survivorship, 3 Plans



Comments

- Are the differences between groups shocking?
- Are the differences between groups surprising?
- `www.ubble.co.uk`
  - What is your probability of survival for the next 5 years?
  - Various health and lifestyle questions; sex and age
  - Output: what is your effective age?
  - e.g. “Typical” Research Actuary, male, aged 48
    5-year survival probability is:
    the same as an “average” male aged 33
  - Difference is consistent with Danish Males, Group 10 versus the average

Annuities from Age 65: Present Values (PV)

Group/Plan	Mean P.V.	Correlation with National Population
National	13.03	1.000
Group 1	10.34	0.805
Group 10	14.95	0.849
Blue Collar	11.95	0.938
White Collar	14.55	0.947
Mixed	14.06	0.985

Annuities from Age 65: Present Values (PV)

What is the relevance of annuity correlations?

- Risk management of longevity risk
- Customised *versus* Index-linked hedges
- $> 94\%$ correlation means a well designed index-linked hedge can be very effective.
- Choice depends on
  - Risk appetite (all schemes > 0!)
  - Scheme size: accessibility of customised transactions
  - Scheme size: small population risk

4. Summary

- Danish data allows insight into relative mortality dynamics between socio-economic sub-populations
- Conclusions for other countries likely to be similar
- Results allow us to explore many risk measurement and risk management applications

Working paper available soon.

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