Forecasting Socio-Economic Differences in the Mortality of Danish Males

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Joint work with D. Blake, K. Dowd, M. Kallestrup-Lamb, C. Rosenskjold

International Mortality and Longevity Symposium 2016

ARC Research Programme

Modelling, Measurement and Management of Longevity and Morbidity Risk

- New/improved models for modelling longevity
- Management of longevity risk
- Underlying drivers of mortality
- Modelling morbidity risk for critical illness insurance

Outline

- Introduction and motivation for multi-population modelling
- Constructing a new dataset
- Modelling Danish sub-population mortality
- Forecast correlations and basis risk
- Applications

1. Motivation: Stochastic Mortality

n lives, probability p of survival, N survivors

- Unsystematic mortality risk:
  $\Rightarrow N|p \sim$ Binomial(n, p)
  $\Rightarrow$ risk is diversifiable, $N/n \to p$ as $n \to \infty$
- Systematic mortality risk:
  $\Rightarrow p$ is uncertain
  $\Rightarrow$ risk associated with p is not diversifiable

Motivation: Longevity Risk

Interested in *longevity risk*:

The risk that in aggregate people live longer than anticipated.

⇒ pension plan has insufficient cash to pay promised pensions

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Multi-Population Challenges

- Data availability
- Data quality and depth
- Model complexity
  - single population models can be complex
  - 2-population versions are more complex
  - multi-pop ......
- Multi-population modelling requires
  - (fairly) simple single-population models
  - simple dependencies between populations

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2. A New Case Study and a New Model

- Sub-populations differ from national population
  - socio-economic factors
  - other factors
- Denmark
  - High quality data on ALL residents
  - 1981-2012 available
  - Can subdivide population using covariates on the database

Danish Data

- *What can we learn from Danish data that will inform us about other populations?*

- Key covariates (amongst others):
  - Wealth
  - Income

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Problem

- High income $\Rightarrow$ "affluent" *and low mortality* BUT
- Low income $\not\Rightarrow$ not affluent, high mortality
- High wealth $\Rightarrow$ "affluent" *and low mortality* BUT
- Low wealth $\not\Rightarrow$ not affluent, high mortality

Empirical solution: use a combination

- Affluence, $A =$ wealth $+K \times$ income
- $K = 15$ seems to work well *statistically* as a predictor
- Low affluence, A, predicts poor mortality

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Subdividing Data (after much experimentation!)

- Males resident in Denmark for the previous 12 months
- Divide population in year t
  - into 10 equal sized Groups (approx)
  - using *affluence*, A
- Individuals can change groups up to age 67
- Group allocations are locked down at age 67

(better than not locking down at age 67)

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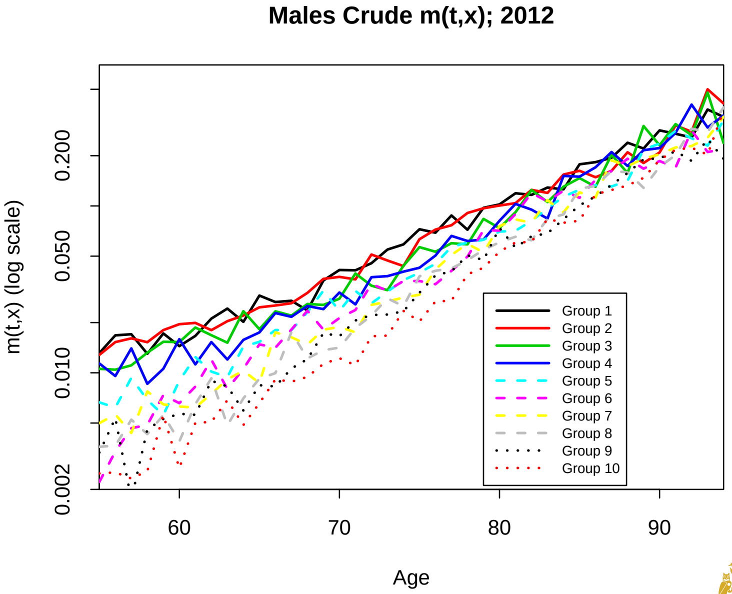
Subdivided Data

- Ages 55-94; Years 1985-2012
- Exposures $E^{(i)}(t, x)$ for groups $i = 1, \ldots, 10$
  range from over 4250 down to 13
- Deaths $D^{(i)}(t, x)$
  range from 151 down to 4
- Crude death rates
  $\hat{m}^{(i)}(t, x) = D^{(i)}(t, x)/E^{(i)}(t, x)$
- Small groups $\Rightarrow$ Poisson risk is important

Crude death rates 2012



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Modelling the underlying death rates, $m^{(k)}(t,x)$

$m^{(k)}(t,x) =$ pop. k death rate in year t at age x

Population k, year t, age x

$$\log m^{(k)}(t,x) = \beta^{(k)}(x) + \kappa_1^{(k)}(t) + \kappa_2^{(k)}(t)(x - \bar{x})$$

(Extended CBD with a non-parametric base table, $\beta^{(k)}(x)$)

- 10 groups, $k = 1, \ldots, 10$ (low to high affluence)
- 28 years, $t = 1985, \ldots, 2012$
- 40 ages, $x = 55, \ldots, 94$

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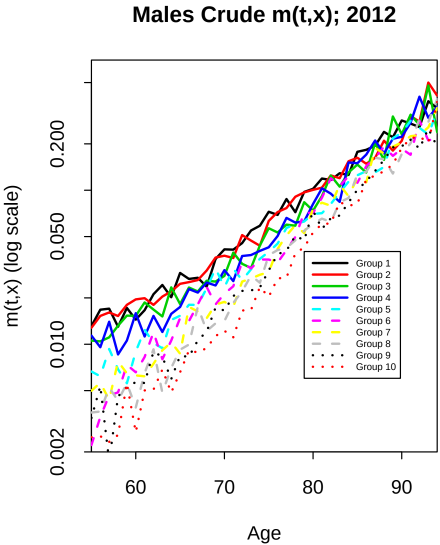
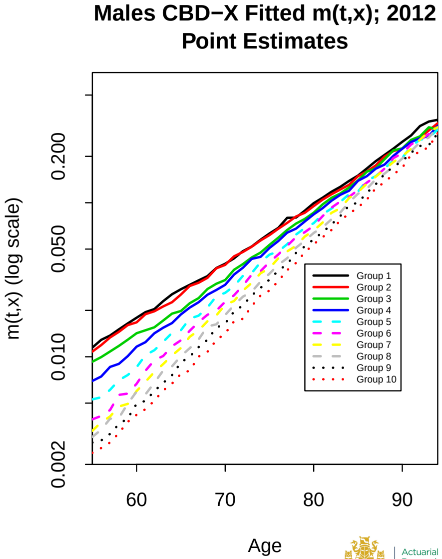
Model-Inferred Underlying Death Rates 2012





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$$\log m^{(k)}(t,x) = \beta^{(k)}(x) + \kappa_1^{(k)}(t) + \kappa_2^{(k)}(t)(x-\bar{x})$$

- Model fits the 10 groups well without a cohort effect
- Non-parametric $\beta^{(k)}(x)$ is essential to preserve group rankings
  - Rankings are evident in crude data
  - *"Bio-demographical reasonableness"*:

    more affluent $\Rightarrow$ healthier

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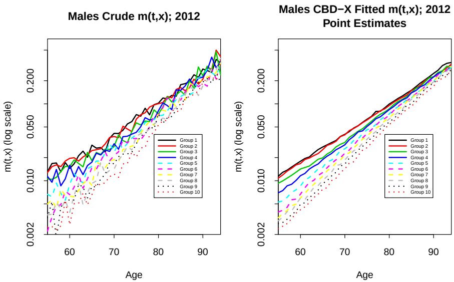
Model-Inferred Underlying Death Rates 2012



- Gap reduces from over $5\times$ to $1.3\times$
- Or $+14$ years difference for Group $1\rightarrow 10$, age 55; $+9$ at 67.
- Convergence $\Rightarrow$ way ahead for modelling very high ages???

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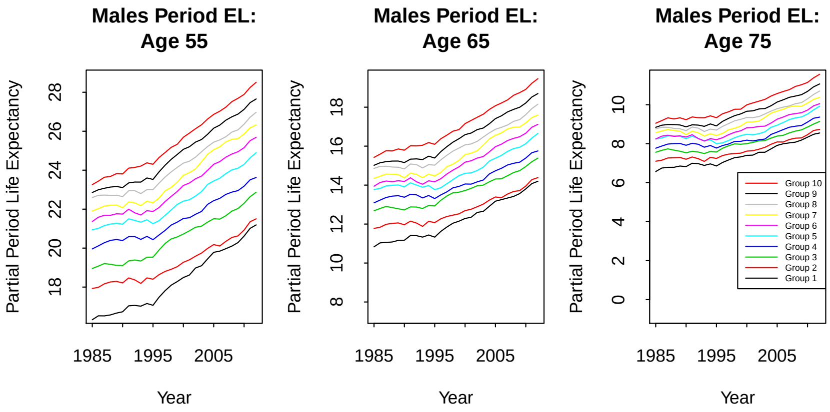
Partial Period Life Expectancy for Groups 1-10



Time series modelling

- $t \to t+1$: Allow for correlation
  - between $\kappa_1^{(k)}(t+1)$ and $\kappa_2^{(k)}(t+1)$
  - between groups $k = 1, \ldots, 10$

- Medium/long term, $t \to t+T$:
  group specific period effects *gravitate* towards the national trend (*coherence*)
  $\Rightarrow$ Bio-demographical reasonableness:
  groups should not diverge

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A specific model

$$\kappa_1^{(i)}(t) = \kappa_1^{(i)}(t-1) + \mu_1 + Z_{1i}(t) \quad \text{(random walk)}$$
$$-\psi\left(\kappa_1^{(i)}(t-1) - \bar{\kappa}_1(t-1)\right) \quad \text{(gravity between groups)}$$
$$\kappa_2^{(i)}(t) = \kappa_2^{(i)}(t-1) + \mu_2 + Z_{2i}(t)$$
$$-\psi\left(\kappa_2^{(i)}(t-1) - \bar{\kappa}_2(t-1)\right)$$

where

$$\bar{\kappa}_1(t) = \frac{1}{n}\sum_{i=1}^{n}\kappa_1^{(i)}(t) \quad \text{and} \quad \bar{\kappa}_2(t) = \frac{1}{n}\sum_{i=1}^{n}\kappa_2^{(i)}(t)$$

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A specific model

$$\kappa_1^{(i)}(t) = \kappa_1^{(i)}(t-1) + \mu_1 + Z_{1i}(t) - \psi\left(\kappa_1^{(i)}(t-1) - \bar{\kappa}_1(t-1)\right)$$

$$\kappa_2^{(i)}(t) = \kappa_2^{(i)}(t-1) + \mu_2 + Z_{2i}(t) - \psi\left(\kappa_2^{(i)}(t-1) - \bar{\kappa}_2(t-1)\right)$$

Model structure $\Rightarrow$

- $(\bar{\kappa}_1(t), \bar{\kappa}_2(t)) \sim$ bivariate random walk
- Each $\kappa_1^{(i)}(t) - \bar{\kappa}_1(t) \sim AR(1)$ reverting to 0
- Each $\kappa_2^{(i)}(t) - \bar{\kappa}_2(t) \sim AR(1)$ reverting to 0
- $\beta^{(i)}(x)$ vs $\beta^{(j)}(x) \Rightarrow$ intrinsic group differences

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Non-trivial correlation structure: between different ages and groups

$$\kappa_1^{(i)}(t) = \kappa_1^{(i)}(t-1) + \mu_1 + Z_{1i}(t) - \psi\left(\kappa_1^{(i)}(t-1) - \bar{\kappa}_1(t-1)\right)$$
$$\kappa_2^{(i)}(t) = \kappa_2^{(i)}(t-1) + \mu_2 + Z_{2i}(t) - \psi\left(\kappa_2^{(i)}(t-1) - \bar{\kappa}_2(t-1)\right)$$

The Z_{ki} are multivariate normal, mean 0 and

$$Cov(Z_{ki}, Z_{lj}) = \begin{cases} v_{kl} & \text{for } i = j \\ \rho v_{kl} & \text{for } i \neq j \end{cases}$$

$\rho =$ cond. correlation between $\kappa_1^{(i)}(t)$ and $\kappa_1^{(j)}(t)$ etc.

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Comments

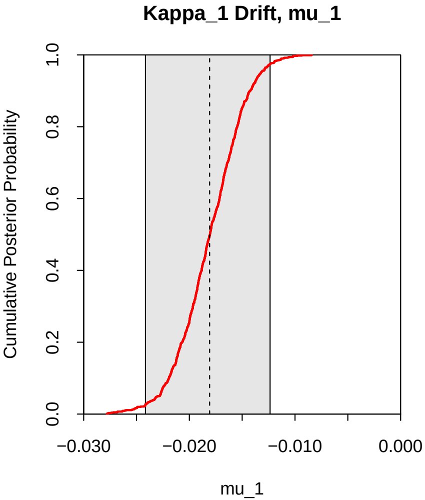
- Model is very simple
  - One gravity parameter, $0 < \psi < 1$
  - One between-group correlation parameter, $0 < \rho < 1$
- Many generalisations are possible
- But more parameters + more complex computing
- This simple model seems to fit quite well.
- Nevertheless $\Rightarrow$ potential for further work

Prior distributions

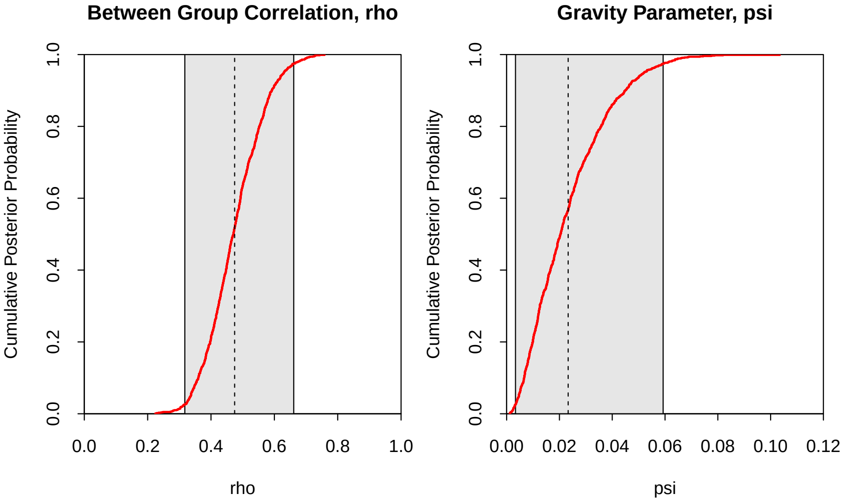
- As uninformative as possible
- $\mu_1,\ \mu_2 \sim$ improper uniform prior
- $\{v_{ij}\} \sim$ Inverse Wishart
- $\rho \sim \text{Beta}(2,2)$
- $\psi \sim \text{Beta}(2,2)$

State variables and process parameters estimated using MCMC (Gibbs + Metropolis-Hastings)

Posterior Distributions and 95% Credibility Intervals



Posterior Distributions and 95% Credibility Intervals



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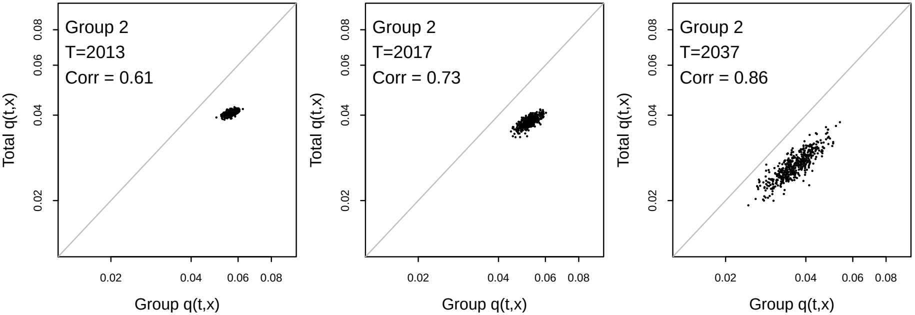
Simulated Group versus Population Mortality, $q(t, x)$



As T increases: $+1$ year; $+5$ years; $+25$years

- Scatterplots become more dispersed
- Shift down and to the left
- Correlation increasess

Forecast Correlations

Deciles are quite narrow subgroups

More diversified e.g.

- Blue collar pension plan

  $\Rightarrow$ equal proportions of groups 2, 3, 4
- White collar pension plan

  $\Rightarrow$ equal proportions of groups 8, 9, 10
- Mixed plan

  $\Rightarrow$ proportions $(0, 0, 1, 2, \ldots, 7, 8)/36$ (e.g. amounts)

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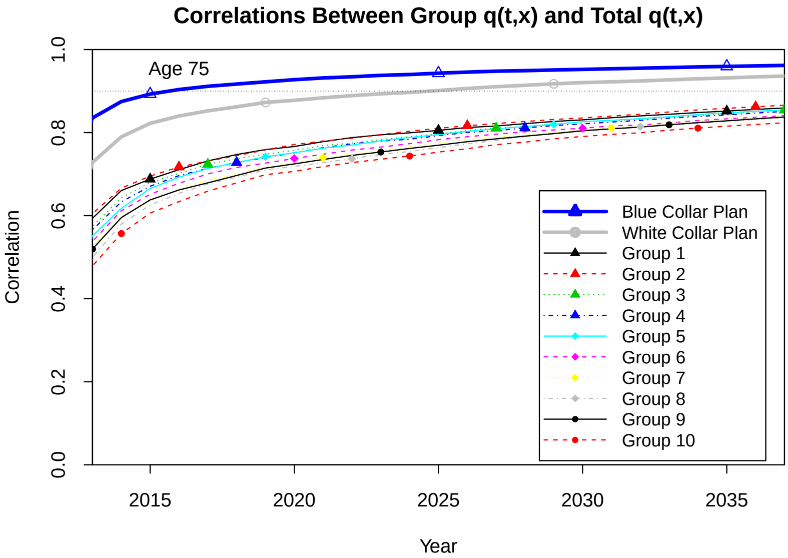
Forecast Correlations: Mortality Rates at Age 75



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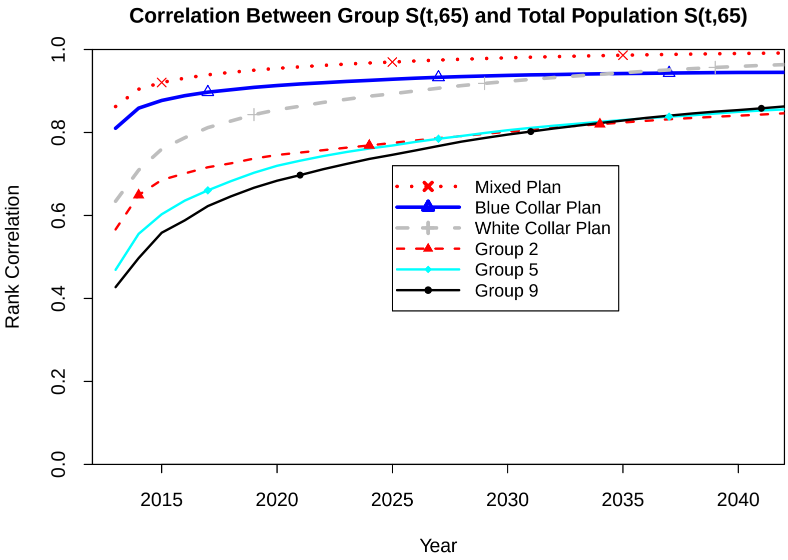
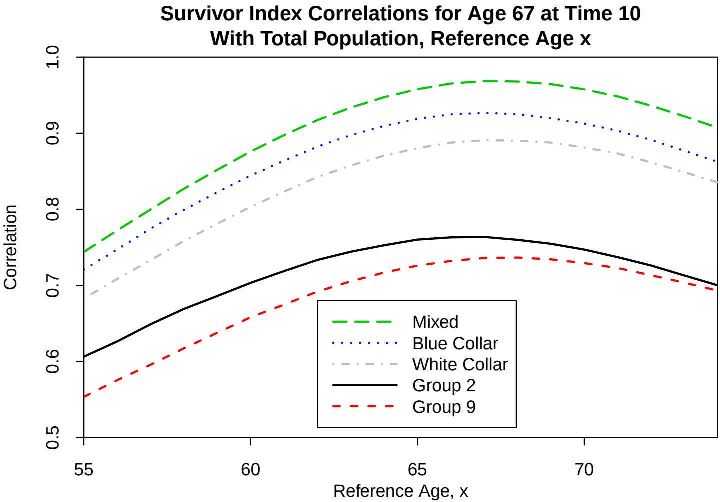
Forecast Correlations: Cohort Survivorship from 65



Forecast Correlations: Cohort Survivorship

Different reference ages: $cor\left(S_X(10, 67),\ S_{TOT}(10, x)\right)$



Modelling Conclusions

- Development of a new multi-population dataset for Denmark
  strong bio-demographically reasonable group rankings based on a new measure of affluence
- Unlike multi-country data
  a priori ranking of affluence-related groups
- Proposal for a simple new multi-population model
- Mortality rates converge at high ages
- Strong correlations over medium to long term
  even allowing for parameter uncertainty
- Correlations depend strongly on diversity of sub-population

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3. Postscript: Education as an Alternative Covariate

- Level of Highest Education also known to be a good predictor
  - Various US studies
  - Mackenbach et al. (2003) including Denmark: Std. Mortality Rates
  - Bronnum-Hansen and Baadsgaard (2012) Denmark: $LE(x = 30)$
- As close as possible on a *like for like* basis:

  Crude death rates; age 30+; matching years.

  Affluence $\Rightarrow$
  - Wider spread of SMR's than M. et al. (2003)
  - Wider spread of $LE(30)$ than BHB (2012)
- More to be done.

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Choices

- No hedging
- Hedge using own experience
- Hedge using standardised instrument: national mortality

Basis Risk

Two sources of basis risk considered here

- Population basis risk
- Sub-optimal choice of hedging instrument

  tradeoff: price *vs* basis risk

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Economic capital relief using longevity options

- Population 1: national population; reference for hedge

  notional portfolio of males aged 65: $A_1 =$P.V. pension payments

- Population 2: hedger's own population

  portfolio of males aged 65: $A_2 =$P.V. pension payments

Economic capital relief using longevity options

- Three choices:
  - No hedging of A_2
  - Hedge A_2 with population 1 longevity swap $A_1 - \hat{A}_1$
  - Hedge A_2 with out-of-the-money option on $A_1(T)$
    Payoff at $T = 20$; underlying $A_1(T)$ includes estimated $t > T$ cashflows

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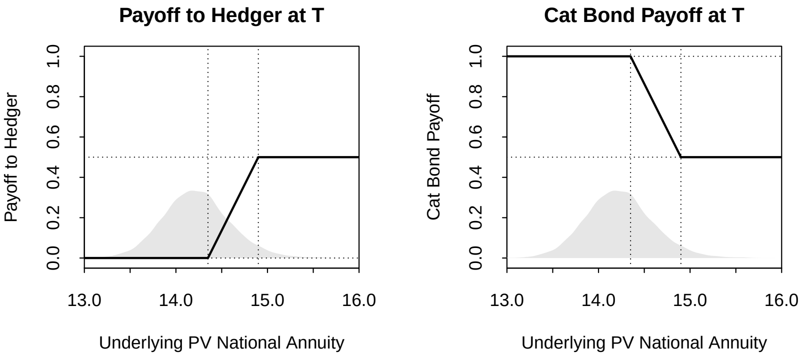
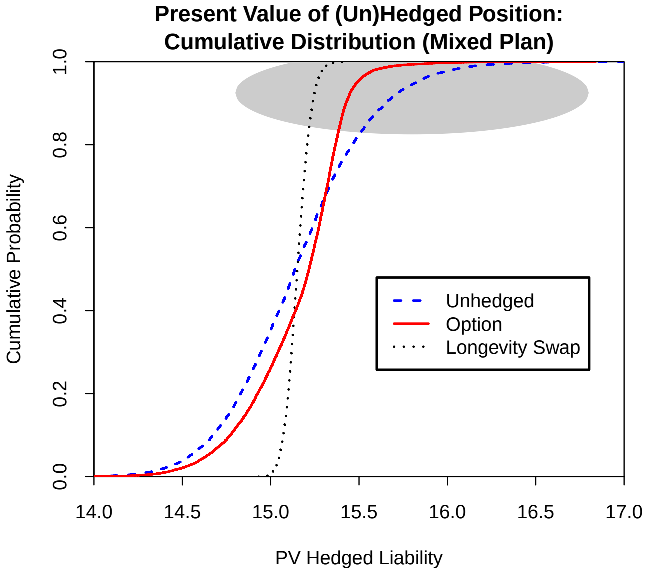
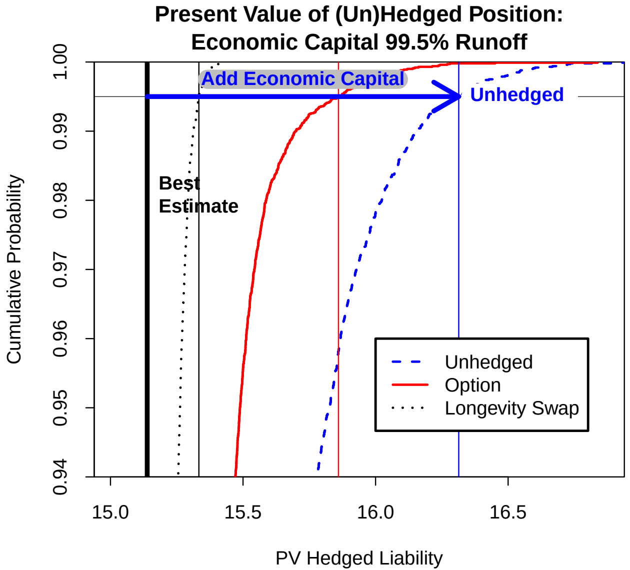


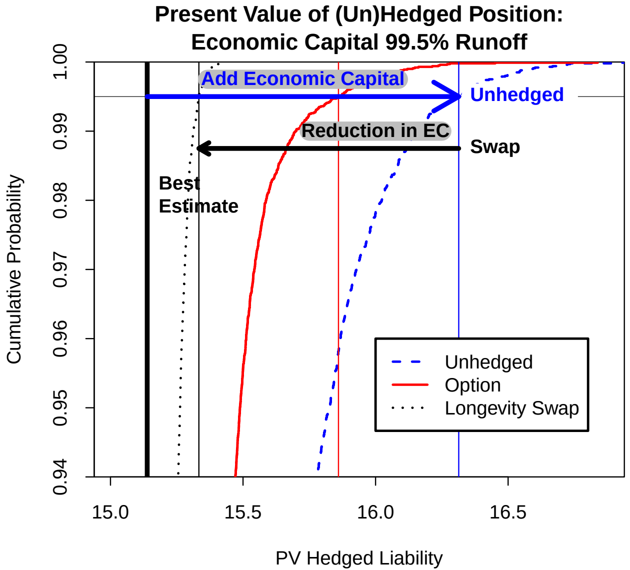
Attachment/Detachment at approx 60% / 95%

Impact of Hedging with $T = 20$ Option or Index Swap

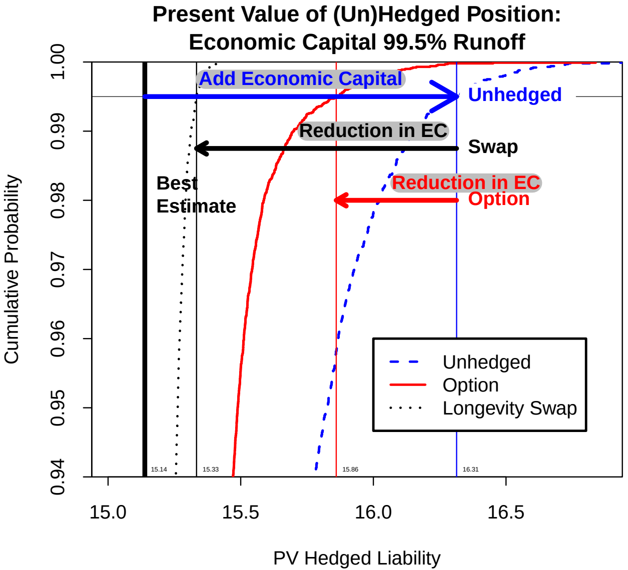


Impact of Hedging with $T = 20$ Option or Index Swap




Present Value of (Un)Hedged Position:
Economic Capital 99.5% Runoff
Cumulative Probability
0.94
0.95
0.96
0.97
0.98
0.99
1.00
15.0
15.5
16.0
16.5
PV Hedged Liability
Add Economic Capital
Unhedged
Reduction in EC
Swap
Best
Estimate
Unhedged
Option
Longevity Swap

Impact of Hedging with $T = 20$ Option or Index Swap



Challenges 1

- Simulation example assumes swaps and options priced at actuarially fair value
- But swap and option premiums might be more expensive
- Compare premium versus value of reduction in Economic Capital over multiple time periods

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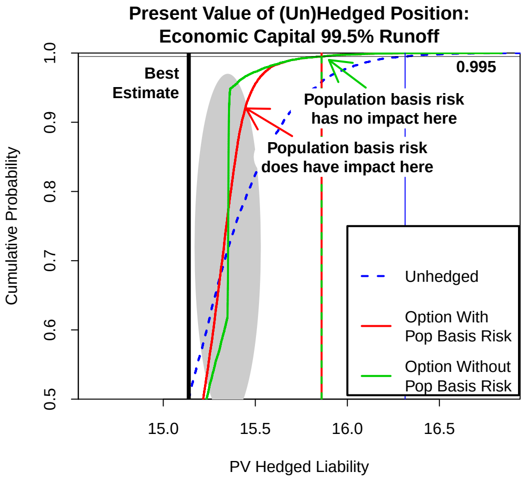
Challenges 2

- Bull spread option:
  - Choice of attachment/detachment points $K_1,\ K_2$
  - Maximum cat bond loss
  - Capital markets capacity $\leftrightarrow$ annuity liabilities
  - Risk premiums
  - Sub-optimal instrument basis risk

What is the impact of population basis risk?



Conclusion depends on $K_2 << 99.5\%$ quantile.

Basis risk from "sub-optimal" choice of hedging instrument.

5. Summary

- Danish data allows insight into relative mortality dynamics between socio-economic sub-populations
- Conclusions for other countries likely to be similar
- Economic capital example is one of many potential risk management applications

Working paper available on website.

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Thank You!

Questions?

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