Modelling Socio-Economic Differences in the Mortality of Danish Males Using a New Affluence Index

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LoLitA Closing Conference, Paris, January 2018
Outline

- Introduction and motivation for multi-population modelling
- Constructing a new Danish dataset
- Modelling Danish sub-population mortality
- Forecast correlations and population basis risk
- Concluding remarks
1. Motivation: Multi-Population Stochastic Mortality

- Focus: pension plan or annuity provider
- Concerned about *longevity risk*: the risk that *in aggregate* people live longer than anticipated.
  \[\Rightarrow\text{ insufficient cash to pay promised pensions}\]
- Risk measurement $\rightarrow$ risk management
Multi-Population Questions and Challenges

Why model multiple populations?
- Different socio-economic groups
- Males and females
- Different countries
- etc.

We seek to understand and model the differences:
- How big are the differences?
- Are the differences changing over time?
Multi-Population Questions and Challenges (cont.)

Challenges:

- Data availability
- Data quality and depth
- Multi-population modelling requires
  - (fairly) simple single-population models
  - simple dependencies between populations
  - BUT also plausible correlation structure between groups, ages and through time
Multi-Population Questions and Challenges (cont.)

Applications:

- **Risk management e.g.**
  - Pension plan: reduce exposure to longevity risk
  - (Re)insurer: reduce exposure and concentration
    \[\Rightarrow \] release economic/regulatory capital
    \[\Rightarrow \] increased capacity

- **Compare risk management options**
  - Customised versus index-based hedges
  - Base table; central forecast; risk
  - Index-based \[\Rightarrow \] population basis risk
2. A New Case Study and a New Model

- Sub-populations differ from national population
  - socio-economic factors
  - other factors
- Denmark: National Register Database
  - High quality data on ALL residents
  - 1981-2012 available
  - Can subdivide population using covariates on the database
Danish Data

What can we learn from Danish data that will inform us about other populations?

Key covariates (amongst others):
- Wealth
- Income
Problem

- High income $\Rightarrow$ “affluent” \textit{and low mortality}
  BUT
- Low income $\not\Rightarrow$ not affluent, high mortality
- High wealth $\Rightarrow$ “affluent” \textit{and low mortality}
  BUT
- Low wealth $\not\Rightarrow$ not affluent, high mortality

Empirical solution: use a combination

- Affluence, $A = \text{wealth} + K \times \text{income}$
- $K = 15$ seems to work well statistically as a predictor
- Low affluence, $A$, predicts poor mortality
Subdividing Data (after much experimentation!)

- Males resident in Denmark for the previous 12 months
- Divide population in year $t$
  - into 10 equal sized Groups (approx)
  - using affluence, $A$
- Individuals can change groups up to age 67
- Group allocations are locked down at age 67
  (better than not locking down at age 67)
Subdivided Data

- Ages 55-94; Years 1985-2012
- Exposures $E^{(i)}(t, x)$ for groups $i = 1, \ldots, 10$
  range from over 4250 down to 13
- Deaths $D^{(i)}(t, x)$
  range from 151 down to 4
- Crude age-specific death rates
  $\hat{m}^{(i)}(t, x) = D^{(i)}(t, x)/E^{(i)}(t, x)$
  Population $i$, Year $t$, Age $x$
- Small groups $\Rightarrow$ Poisson risk is important
Modelling the underlying death rates, \( m^{(k)}(t, x) \)

\[
m^{(k)}(t, x) = \text{pop. } k \text{ death rate in year } t \text{ at age } x
\]

Population \( k \), year \( t \), age \( x \)

\[
\log m^{(k)}(t, x) = \beta^{(k)}(x) + \kappa_1^{(k)}(t) + \kappa_2^{(k)}(t)(x - \bar{x})
\]

(Extended CBD with a non-parametric base table, \( \beta^{(k)}(x) \))

- 10 groups, \( k = 1, \ldots, 10 \) (low to high affluence)
- 28 years, \( t = 1985, \ldots, 2012 \)
- 40 ages, \( x = 55, \ldots, 94 \)
Model-Inferred Underlying Death Rates 2012

Males Crude m(t,x); 2012

Males CBD–X Fitted m(t,x); 2012
Point Estimates

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Modelling the death rates, $m_k(t, x)$

$$\log m^{(k)}(t, x) = \beta^{(k)}(x) + \kappa_1^{(k)}(t) + \kappa_2^{(k)}(t)(x - \bar{x})$$

- Model fits the 10 groups well without a cohort effect
- Non-parametric $\beta^{(k)}(x)$ is essential to preserve group rankings
  - Rankings are evident in crude data
  - “Bio-demographic reasonableness”: more affluent $\Rightarrow$ healthier
• Gap reduces from over $5 \times$ to $1.3 \times$

• Or $+14$ years difference for Group 1→10, age 55; $+9$ at 67.

• Convergence $\Rightarrow$ way ahead for modelling very high ages???
Time series modelling

- $t \rightarrow t + 1$: Allow for correlation
  - between $\kappa_1^{(k)}(t + 1)$ and $\kappa_2^{(k)}(t + 1)$
  - between groups $k = 1, \ldots, 10$

- Medium/long term, $t \rightarrow t + T$:
  - *group specific* period effects *gravitate* towards the national trend (*coherence*)
  - $\Rightarrow$ Bio-demographical reasonableness:
    - groups should not diverge
A specific model

\[
\begin{align*}
\kappa_1^{(i)}(t) &= \kappa_1^{(i)}(t - 1) + \mu_1 + Z_{1i}(t) \\
&\quad - \psi \left( \kappa_1^{(i)}(t - 1) - \bar{\kappa}_1(t - 1) \right) \\
(\text{random walk})
\end{align*}
\]

\[
\begin{align*}
\kappa_2^{(i)}(t) &= \kappa_2^{(i)}(t - 1) + \mu_2 + Z_{2i}(t) \\
&\quad - \psi \left( \kappa_2^{(i)}(t - 1) - \bar{\kappa}_2(t - 1) \right) \\
(\text{gravity between groups})
\end{align*}
\]

where

\[
\bar{\kappa}_1(t) = \frac{1}{n} \sum_{i=1}^{n} \kappa_1^{(i)}(t) \quad \text{and} \quad \bar{\kappa}_2(t) = \frac{1}{n} \sum_{i=1}^{n} \kappa_2^{(i)}(t)
\]
A specific model

\[ \kappa_1^{(i)}(t) = \kappa_1^{(i)}(t - 1) + \mu_1 + Z_{1i}(t) - \psi \left( \kappa_1^{(i)}(t - 1) - \bar{\kappa}_1(t - 1) \right) \]

\[ \kappa_2^{(i)}(t) = \kappa_2^{(i)}(t - 1) + \mu_2 + Z_{2i}(t) - \psi \left( \kappa_2^{(i)}(t - 1) - \bar{\kappa}_2(t - 1) \right) \]

Model structure ⇒

- \((\bar{\kappa}_1(t), \bar{\kappa}_2(t)) \sim\) bivariate random walk
- Each \(\kappa_1^{(i)}(t) - \bar{\kappa}_1(t) \sim AR(1)\) reverting to 0
- Each \(\kappa_2^{(i)}(t) - \bar{\kappa}_2(t) \sim AR(1)\) reverting to 0
- \(\beta^{(i)}(x) \text{ vs } \beta^{(j)}(x) \Rightarrow\) intrinsic group differences
Non-trivial correlation structure: between different ages and groups

\[
\begin{align*}
\kappa_1^{(i)}(t) &= \kappa_1^{(i)}(t - 1) + \mu_1 + Z_{1i}(t) - \psi \left( \kappa_1^{(i)}(t - 1) - \bar{\kappa}_1(t - 1) \right) \\
\kappa_2^{(i)}(t) &= \kappa_2^{(i)}(t - 1) + \mu_2 + Z_{2i}(t) - \psi \left( \kappa_2^{(i)}(t - 1) - \bar{\kappa}_2(t - 1) \right)
\end{align*}
\]

The \( Z_{ki}(t) \) are iid multivariate normal, mean 0 and

\[
\text{Cov}(Z_{ki}, Z_{lj}) = \begin{cases} 
\nu_{kl} & \text{for } i = j \\
\rho \nu_{kl} & \text{for } i \neq j
\end{cases}
\]

\( \rho = \text{cond. correlation between } \kappa_1^{(i)}(t) \text{ and } \kappa_1^{(j)}(t) \) etc.
Comments

- Model is very simple
  - One gravity parameter, $0 < \psi < 1$
  - One between-group correlation parameter, $0 < \rho < 1$
- Many generalisations are possible
- But more parameters + more complex computing
- This simple model seems to fit quite well.
- Nevertheless $\Rightarrow$ potential for further work
Bayesian implementation: Prior distributions

- As uninformative as possible
- $\mu_1, \mu_2 \sim$ improper uniform prior
- $\{v_{ij}\} \sim$ Inverse Wishart
- $\rho \sim$ Beta$(2, 2)$
- $\psi \sim$ Beta$(2, 2)$

State variables and process parameters estimated using MCMC (Gibbs + Metropolis-Hastings)
Posterior Distributions and 95% Credibility Intervals

Kappa_1 Drift, mu_1

Kappa_2 Drift, mu_2

Cumulative Posterior Probability

Cumulative Posterior Probability

mu_1

mu_2
Posterior Distributions and 95% Credibility Intervals

**Between Group Correlation, \( \rho \)**

**Gravity Parameter, \( \psi \)**

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Danish Males Mortality
Simulated Group versus Population Mortality, $q(t, x)$

As $T$ increases: +1 year; +5 years; +25 years

- Scatterplots become more dispersed
- Shift down and to the left
- Correlation increases
Forecast Correlations

Deciles are quite narrow subgroups
More diversified e.g.

- **Blue collar pension plan**
  ⇒ equal proportions of groups 2, 3, 4

- **White collar pension plan**
  ⇒ equal proportions of groups 8, 9, 10

- **Mixed plan**
  ⇒ proportions \((0, 0, 1, 2, \ldots, 7, 8)/36\) (e.g. amounts)
Forecast Correlations: Mortality Rates at Age 75

Correlations Between Group $m(t,x)$ and Total $m(t,x)$

Year
Correlation

Blue Collar Plan
White Collar Plan
Group 1
Group 2
Group 3
Group 4
Group 5
Group 6
Group 7
Group 8
Group 9
Group 10
Forecast Correlations: Cohort Survivorship from 65

Correlation Between Group $S(t,65)$ and Total Population $S(t,65)$

- Mixed Plan
- Blue Collar Plan
- White Collar Plan
- Group 2
- Group 5
- Group 9

Year

Rank Correlation

0.0 0.2 0.4 0.6 0.8 1.0

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Forecast Correlations: Cohort Survivorship

Different reference ages: $\text{cor} \left( S_X(10, 67), S_{TOT}(10, x) \right)$

Survivor Index Correlations for Age 67 at Time 10
With Total Population, Reference Age $x$

Correlation

Reference Age, $x$

Mixed
Blue Collar
White Collar
Group 2
Group 9

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Modelling Conclusions

- Development of a new multi-population dataset for Denmark
  
  strong bio-demographically reasonable group rankings
  
  based on a new measure of affluence

- Unlike multi-country data
  
  a priori ranking of affluence-related groups

- Proposal for a simple new multi-population model

- Mortality rates converge at high ages

- Strong correlations over medium to long term
  
  even allowing for parameter uncertainty

- Correlations depend strongly on diversity of sub-population
3. Postscript 1: Education as an Alternative Covariate

- **Level of Educational Attainment** also known to be a good predictor
  - Various US studies
  - Mackenbach et al. (2003) including Denmark: Std. Mortality Rates
  - Brønnum-Hansen and Baadsgaard (2012) Denmark: $LE(x = 30)$

- As close as possible on a *like for like* basis:
  
  Crude death rates; age 30+; matching years.

  Affluence $\Rightarrow$
  - Wider spread of SMR’s than M. et al. (2003)
  - Wider spread of $LE(30)$ than BHB (2012)

- Issue: “grade inflation” distorts results

- More to be done.
3. Postscript: Education as an Alternative Covariate

Age Standardised Mortality Rates per 1000
Ages 45–54; European Standard Population (1976)

- Affluence Group 1
- Low Education
- High Education
- Affluence Group 10

Year
Age Standardised Mortality Rate (per 1000 person years)
3. Postscript: Education as an Alternative Covariate

Dig a bit deeper:
Affluence + Education: average ASMR’s over 5 years

Mortality Improvement Rates (%)
Period 1987–2009; Age Band 45–54
By Affluence and Education Group

<table>
<thead>
<tr>
<th>Education Level</th>
<th>Affluence Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>HIGH</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
</tr>
<tr>
<td>MED</td>
<td></td>
</tr>
<tr>
<td>LOW</td>
<td></td>
</tr>
</tbody>
</table>

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4. Summary

- Danish data allows insight into relative mortality dynamics between socio-economic sub-populations
- Applications
  - Improved risk measurement
  - Assess effectiveness of longevity hedges
- Education as an alternative covariate
- Health inequalities
- Work in progress:
  - Canada, USA, England
  - Hedging: Regulatory and economic capital relief

Working paper available on website.

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Thank You!

Questions?

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Postscript 2: Cause of Death Data – Health Inequalities

- Deaths subdivided into 29 CoD groups
- Compare affluence groups
- Biggest differences at younger age groups e.g. 51-55
- Causes of death linked to lifestyle
  \[ \Rightarrow \text{some CoD death rates are up to } 20 \times \text{ higher for low affluence groups} \]
- Growing gaps: liver diseases; diabetes
- *Almost all CoD groups have a strong statistically significant difference*
Denmark: Cause of Death Data – Health Inequalities

- 5 × 5 ages and years
- CoD4: Lung cancer and related cancers
- CoD9: Cancer of lymphatic or blood-forming tissues
Some causes of death have **no obvious link** to lifestyle/affluence/education

Possible explanations (a very non-expert view)

- *onset* is not dependent on lifestyle/affluence/education
- BUT less affluent/educated $\Rightarrow$
  - later diagnosis
  - engage less well with treatment process
  - lower quality housing