Technical Bases in Life Insurance

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This article describes briefly the nature of the bases in common use in **life insurance** and, in particular, the different terminologies that have evolved through different approaches to regulation.

A *basis* is a set of assumptions that may be used to project future cash flows under a life insurance contract and to discount them to the present. The projection and discounting operations may be performed by any numerical method available, from the traditional use of expected present values through to explicit cash flow projection models.

The simplest basis must have assumptions about future rates of mortality and the rate of interest that will prevail in future, and the premiums that will be received in future. To these may be added, depending on the purpose for which the basis is required, assumptions about future expenses, lapse rates and surrender values (*see* **Surrenders and Alterations**), transfers to paid-up **policies**, morbidity, rates of retirement, marriage, and so on. The purpose of the basis may be pricing (i.e. setting premium rates or tariffs), valuation for **solvency**, or valuation in order to measure and distribute surplus (*see* **Surplus in Life and Pension Insurance**).

If the same basis is used for pricing and valuation, a particularly simple mathematical model is obtained because, if the future works out exactly as expected according to the elements of the basis, the retrospective accumulation of assets at any time is exactly what is needed in order then to reserve prospectively for future liabilities (technically, retrospective and prospective policy values are equal). If, in addition, this basis allows implicitly rather than explicitly for future expenses and contingencies, by making conservative or 'safe-side' assumptions about future interest and mortality, we have the classical net premium system. Until recently, this was used widely in most of Europe, and local regulations often enforced its use and even specified the elements of the basis. In such a system, the common pricing and valuation basis is often called the *first-order basis*. In other countries, including the United Kingdom, actuaries were free to choose pricing and valuation bases.

The use of 'safe-side' bases for pricing should result in the emergence of surpluses during the term of the policy. Note that, what is 'safe-side' in respect of mortality depends on the nature of the risk being insured; for assurance contracts, paying a benefit on death, it is prudent to assume that future mortality will be on the high side, but for **annuities** or pure endowment contracts, paying benefits upon survival, the opposite is the case. The measurement of emerging surplus depends on a valuation basis, because the terms of life insurance policies are so long that no retrospective system of accounting for profit is adequate. We can see how this works with a simple example, of a whole of life insurance with sum assured \$1 and level premiums payable continuously, taken out by a person age x. The prospective policy value at age $x+t$ is denoted ${}_t\overline{V}_x$, computed on the valuation basis, if the insured person is still alive then, or zero otherwise. Thiele's differential equation is then:

$$\frac{\mathrm{d}\,{}_t\overline{V}_x}{\mathrm{d}t} = \delta\,{}_t\overline{V}_x + \pi - \mu_{x+t}(1 - {}_t\overline{V}_x), \qquad (1)$$

where δ is the force of interest and μ_{x+t} is the force of mortality at age $x+t$ under the valuation or first-order basis, and π is the level annual premium, payable continuously, calculated on the same basis. Thiele's equation is interpreted intuitively as follows: the rate of change of prospective reserve is just the rate at which interest is earned on the current reserve, *plus* the rate at which premiums are received, *minus* the expected payments due to death (the sum assured is \$1, but the reserve is available to offset this amount, so the sum at risk is $(1 - {}_t\overline{V}_x)$).

To every assumption in the valuation basis, there corresponds the actual outcome in reality, variously called the experience basis or second-order basis, and the difference between the two is the contribution to surplus from that source. For example, if the force of interest actually earned on the assets is δ', and the rate at which deaths actually occur is represented by the force of mortality μ'_{x+t}, and the rate at which premiums net of expenses are actually received is π' we have

$$\frac{\mathrm{d}\,{}_t\overline{V}_x}{\mathrm{d}t} + S_t = \delta'\,{}_t\overline{V}_x + \pi' - \mu'_{x+t}(1 - {}_t\overline{V}_x), \qquad (2)$$

in which S_t is the rate at which surplus is emerging. Subtracting equation (1) from this, we get:

$$S_t = (\delta' - \delta)\ {}_t\overline{V}_x + (\pi' - \pi) + (\mu_{x+t} - \mu'_{x+t}) \times (1 - {}_t\overline{V}_x), \tag{3}$$

which analyzes the emerging surplus into its interest, loading (or expense), and mortality components. It is simplest to demonstrate the analysis of surplus in this somewhat idealized continuous-time setting, in which the key rôle of Thiele's equation is evident, but in practice emerging surplus may be measured and analyzed over some extended time period, often the insurance company's accounting period or the period between bonus declarations (*see* **Participating Business**), and then a discrete-time setting is appropriate. The analog of Thiele's equation is that there is a recursive relationship between the prospective reserves at successive durations; see [1, 3]. The analysis is then complicated by second-order contributions to the surplus, for example, interest surplus on mortality surplus; see [2], so from a pedagogical point of view, the continuous-time model has advantages.

Note that the experience basis or second-order basis may contain elements that are not included in the valuation or first-order basis, such as lapses. The valuation basis may be simplified by omitting such elements, but the experience basis is given by what actually happens. In the analysis of surplus, the 'missing' elements of the valuation basis are treated as if they were included but with null values, for example, lapse rates of zero at every duration.

Modern developments have introduced probabilistic models into all the elements of the traditional basis, and computer-intensive numerical methods to accompany them, such as simulation (*see* **Stochastic Simulation**). The technical basis as described above can be recognized, in modern parlance, as the parameterization of a particularly simple model of life insurance liabilities. In current practice, its place is taken by the parameters of the various models being used.

References

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(*See also* **Life Insurance Mathematics**; **Surplus in Life and Pension Insurance**; **Valuation of Life Insurance Liabilities**)

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