

# **Life Insurance**

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# Life Insurance

This article describes the main features and types of life insurance, meaning insurance against the event of death. Life insurers also transact **annuity** business.

## Long-term Insurance

The distinguishing feature of life insurance is that it provides insurance against the possibility of death, often on guaranteed terms, for very long terms. For that reason it is sometimes referred to as ‘long-term business’. Of course, it is also possible to insure risks of death over very short terms, or to have frequent reviews of premium rates, as is usual under **non-life insurance**. In the context of life insurance, this system is called *assessmentism*. But because the probability of dying during a short period, such as a year, increases rapidly with age (*see Mortality Laws*) and these probabilities would naturally be the basis for charging premiums, life insurance written along the lines of assessmentism becomes just as rapidly less affordable as the insured person ages. Moreover, deteriorating health might result in cover being withdrawn some years before death. The modern system of life insurance was initiated by the foundation of the Equitable Life Assurance Society in London in 1762, when **James Dodson** described how a level premium could be charged, that would permit the insurance cover to be maintained for life [15] (*see Early Mortality Tables; History of Actuarial Science*). Long familiarity makes it easy to forget just how ingenious an invention this was.

The chief drawback of assessmentism – that human mortality increased rapidly with age – in fact was the key to the practicability of Dodson’s system. The level premium was calculated to be higher than the premium under assessmentism for some years after the insurance was purchased. Starting with a group of insured people, this excess provided an accumulating fund, of premiums received less claims paid, which could be invested to earn interest. As the group aged, however, the level premium eventually would fall below that charged under assessmentism, but then the fund could be drawn upon to make good the difference as the aged survivors died. The level premium could, in theory, be calculated so that the fund would be exhausted just as the last

survivor might be expected to die. See Figure 1 for an example, based on the Swedish M64 (Males) life table. It is easy to see why a similar system of level premiums might not be a sound method of insuring a long-term risk that decreased with age, as the insured person would have little incentive to keep the insurance in force at the same rate of premium in the later years.

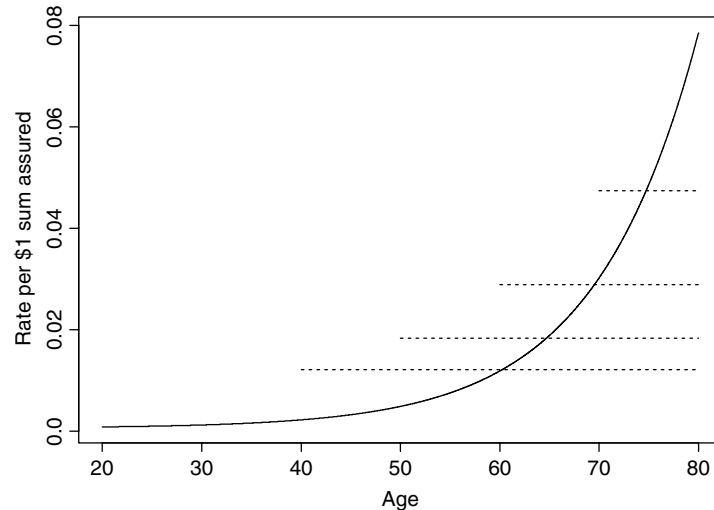
The level premium system, while solving one problem, introduced others, whose evolving solutions became the province of the actuary, and as it became the dominant method of transacting life insurance business, eventually, it gave rise to the actuarial profession. We mention three of the problems that have shaped the profession:

- A **life table** was needed in order to calculate the level premium; thus actuaries adopted as their own the scientific investigation of mortality. (A life table was also needed to calculate premiums under assessmentism, but its short-term nature did not lock the insurer into the consequences of choosing any particular table.)
- An understanding of the accumulation of invested funds was necessary because of the very long terms involved; thus actuaries were early proponents of what we would now call financial mathematics.
- It was necessary to check periodically that the assets were sufficient to meet the liabilities, leading to methods of valuing long-term liabilities (*see Valuation of Life Insurance Liabilities*).

A form of assessmentism remains in common use for **group life insurance** in which an employer (typically) buys life insurance cover for a workforce. A large enough group of persons, of varying ages, with young people joining and older people leaving, represents a reasonably stable risk in aggregate, unlike the rapidly increasing risk of death of an individual person, so this business is closer to non-life insurance than to long-term individual life insurance.

## Conventional Life Insurance Contracts

Here we describe the basic features of conventional life insurance benefits. We use ‘conventional’ to distinguish the forms of life insurance that have been transacted for a very long time, sometimes since the very invention of the level premium system, from



**Figure 1** An illustration of the level premium principle, showing the premiums payable per \$1 of life cover at the ages shown along the bottom. The increasing line is the force of mortality according to the Swedish M64 (Males) life table; this represents the premium that would be charged under assessmentism, regardless of the age at which life insurance was purchased, if the M64 table was used to calculate premiums. The level lines show the level premiums that would be payable by a person age 40, 50, 60, or 70, buying life cover until age 80

those that are distinctly modern, and could not be transacted without computers.

- The *policy term* is the period during which the insurance will be in force, and the insurer at risk.
- The *sum assured* is the amount that will be paid on, or shortly after, death. It is often a level amount throughout the **policy** term, but for some purposes, an increasing or decreasing sum assured may be chosen; for example, the insurance may be covering the outstanding amount under a loan, which decreases as the loan is paid off.

We list the basic forms of life insurance benefit here. Most actuarial calculations involving these benefits make use of their expected present values (EPVs) and these are used so frequently that they often have a symbol in the standard **international actuarial notation**; for convenience we list these as well. Note that the form of the benefit does not define the scheme of premium payment; level premiums as outlined above, and single premiums paid at inception, are both common, but other patterns of increasing or decreasing premiums can be found.

- A *whole of life insurance* pays the sum assured on death at any future age. It is also sometimes called

a *permanent assurance*. The EPV of a whole of life insurance with sum assured \$1, for a person who is now age  $x$ , is denoted  $\bar{A}_x$  if the sum assured is payable immediately on death, or  $A_x$  if it is paid at the end of the year of death (most contracts guarantee the former, subject only to the time needed to administer a claim, but the latter slightly odd-looking assumption facilitates calculations using life tables that tabulate survival probabilities (or, equivalently, the function  $l_x$ ) at integer ages).

- A *term insurance*, also called a *temporary insurance*, has a fixed policy term shorter than the whole of life. The EPV of a term insurance with sum assured \$1, for a person who is now age  $x$ , with remaining term  $n$  years, is denoted  $\bar{A}_{x:\overline{n}|}$  if the sum assured is payable immediately on death, or  $A_{x:\overline{n}|}^1$  if it is paid at the end of the year of death (if death occurs within  $n$  years).
- A *pure endowment* also has a fixed policy term shorter than the whole of life, but the sum assured is paid out at the end of that term provided the insured person is then alive; nothing is paid if death occurs during the policy term. The EPV of a pure endowment with sum assured \$1, for a person who is now age  $x$ , with remaining term  $n$  years, is denoted  $A_{x:\overline{n}|}$ , or alternatively  ${}_nE_x$ .

- An *endowment insurance* is a combination of a term insurance and a pure endowment. It is a savings contract that delivers the sum assured at the end of the policy term, but also pays a sum assured (often but not always the same) on earlier death. The EPV of an endowment insurance with sum assured \$1, for a person who is now age  $x$ , with remaining term  $n$  years, is denoted  $\overline{A}_{x:\overline{n}}$  if the sum assured is payable immediately on death, or  $A_{x:\overline{n}}$  if it is paid at the end of the year of death (if death occurs within  $n$  years). Since expected values behave linearly, we clearly have

$$\overline{A}_{x:\overline{n}} = \overline{A}_{x:\overline{n}}^1 + A_{x:\overline{n}}^1 \quad \text{and} \quad A_{x:\overline{n}} = A_{x:\overline{n}}^1 + A_{x:\overline{n}}^1. \quad (1)$$

- A *joint life insurance* is written on more than one life. The type of benefit can be any of those described above, and it is only necessary to specify the precise event (other than the end of the policy term) that will cause the sum assured to be paid. Common types (taking whole of life insurance as examples) are:
  - *Joint life first death insurance*, payable when the first of several lives dies. The EPV of a joint life, first death, whole of life insurance with sum assured \$1, for two persons now age  $x$  and  $y$ , is denoted  $\overline{A}_{x:y}$  if the sum assured is payable immediately on the first death, or  $A_{x:y}$  if it is paid at the end of the year of the first death.
  - *Joint life last death insurance*, payable when the last of several lives dies. The EPV of a joint life, last death, whole of life insurance with sum assured \$1, for two persons now age  $x$  and  $y$ , is denoted  $\overline{A}_{\overline{x:y}}$  if the sum assured is payable immediately on the second death, or  $A_{\overline{x:y}}$  if it is paid at the end of the year of the second death.
- A *contingent insurance* has a sum assured that is payable when the insured person dies within the policy term, but only if some specified event (often the death of another person) has happened first, or has not happened first. For example, consider a policy with sum assured \$1, payable on the death of a person now age  $y$ , but contingent upon the death or survival of another person now age  $x$ , denoted  $(x)$ . The benefit may be payable provided  $(x)$  is still alive, and then its EPV is denoted  $\overline{A}_{x:y}^1$ . Or, it may be payable

provided  $(x)$  has already died, and then its EPV is denoted  $\overline{A}_{x:y}^2$ . The notation can be extended to any number of lives, and any combination of survivors and prior deaths that may be needed. Such problems usually arise in the context of complicated bequests or family trusts. We clearly have the relations

$$\overline{A}_{x:y}^1 + \overline{A}_{x:y}^2 = \overline{A}_y, \quad (2)$$

$$\overline{A}_{x:y}^1 + \overline{A}_{x:y}^2 = \overline{A}_{x:y}, \quad (3)$$

$$\overline{A}_{x:y}^2 + \overline{A}_{x:y}^2 = \overline{A}_{x:y}. \quad (4)$$

Other forms of life insurance benefit, mainly relating to increasing or decreasing sums assured, are also found; see [4, 11, 13], for detailed accounts.

### Premiums and Underwriting

A life insurance contract may be paid for by a single premium at outset, or by making regular payments while the insurance remains in force, or over an agreed shorter period. Regular premiums may be level or may vary.

The amount of the premiums will be determined by the actuarial basis that is used, namely the assumptions that are made about future interest rates, mortality, expenses, and possibly other factors such as the level of **lapses** or surrenders (*see Surrenders and Alterations*), and the cost of any capital that is needed to support new business.

The terms that will be offered to any applicant will be subject to financial and medical underwriting (*see Life Table*).

- Many familiar forms of insurance indemnify the insured against a given loss of property. Life insurance does not, and the quantum of insurance is largely chosen by the insured. The scope for **moral hazard** is clear, and financial underwriting is needed to ensure that the sum assured is not excessive in relation to the applicant's needs or resources.
- The applicant will have to answer questions about his or her health in the proposal form, and this basic information may be supplemented by a report from the applicant's doctor, or by asking the applicant to undergo a medical examination. These more expensive procedures tend to be used only when the answers in the proposal form

indicate that it would be prudent, or if the sum assured is high, or if the applicant is older; see [5, 12], for a detailed account.

### Participating and Nonparticipating Business

Any insurer makes a loss if they charge inadequate premiums, and a non-life insurer will constantly monitor the claims experience so that premiums may be kept up-to-date. The opportunity to do this is much reduced for a life insurer, because the experience takes so long to be worked out. Therefore it has always been evident that prudent margins should be included in the premiums so that the chance that they will turn out to be insufficient is reduced to a very low level. For basic life insurance, this means that premiums should be calculated assuming: (a) that mortality will be worse than is thought likely; (b) that the assets will earn less interest than is thought likely; and (c) that the expenses of management will turn out to be higher than is thought likely. The future course of these three elements – mortality, interest, and expenses – must be assumed in order to calculate a premium, and this set of assumptions is called the premium basis in some countries, or the first-order basis in others (see **Technical Bases in Life Insurance**).

A consequence of this cautious approach is that, with high probability, the premiums will be more than sufficient to pay claims, and over time a substantial surplus (see **Surplus in Life and Pension Insurance**) will build up. Provided it is not squandered incautiously, this surplus may be returned to the policyholders in some form, so that they are rewarded for the contribution that their heavily loaded premiums have made to the security of the collective. This is known as *participation in the profits of the company*. In some jurisdictions (e.g. Denmark and Germany) it has been the practice that all life insurance contracts of all types participate in profits; in others (e.g. the UK) not all contracts participate in profits, and two separate classes of business are transacted: **participating business** (also known as *with-profits business*) and *nonparticipating business* (also known as nonprofits or without-profits business). Typically, with-profits business has included mainly contracts whose purpose is savings (whole of life and endowment insurances) and whose premiums have included

a very substantial loading intended to generate investment surpluses, while nonprofit business has mainly included contracts whose purpose is pure insurance (term insurances). Note, however, that **unit-linked business** is nonparticipating.

Surpluses give rise to two important actuarial problems: how to measure how much surplus has been earned and may safely be distributed, and then how to distribute it? The first of these is a central theme of the valuation of life insurance liabilities. Surpluses may be returned to participating policyholders in many ways; in cash, by reducing their premiums, or by enhancing their benefits. Collectively, these may be called *bonus systems*, and aspects of these are described in **Participating Business and Surplus in Life and Pension Insurance**; see also [6, 7, 16].

### Valuation of Liabilities

The keystone of the actuarial control of life insurance business is the valuation of life insurance liabilities. Ever since the invention of participating business, the valuation has struggled to meet two conflicting aims: a conservative basis of valuation is appropriate in order to demonstrate **solvency**, but a realistic basis of valuation is needed in order to release surplus for distribution to participating policyholders and to shareholders in an equitable manner. In addition, there was persistent debate over the very methodology that should be used, especially in countries whose insurance law allowed some freedom in this regard. The so-called gross premium methods made explicit assumptions about future premiums, expenses, and bonuses, while net premium (or pure premium) methods made highly artificial assumptions [6, 7, 16, 17].

Perhaps surprisingly from an outside perspective, the net premium method predominated in many jurisdictions, even being mandatory under harmonized insurance regulation in the European Community. There were, however, good practical reasons for this. Under certain circumstances (including fairly stable interest rates, identical pricing and valuation bases, and a uniform reversionary bonus system) the net premium method did provide a coherent mathematical model of the business (see **Life Insurance Mathematics**). In many countries, **insurance regulation** tended to enforce these circumstances, for example, by constraining the investment of the assets, or

the form of the bonus system, or the relationship between the pricing and valuation bases. In others, particularly the United Kingdom, there were no such constraints, and developments such as the investment of with-profits funds in equities and the invention of terminal bonuses meant that the use of the net premium method, in some respects, obscured rather than revealed the true state of the business, at least to outside observers.

Both net and gross premium methods are simple to use with conventional business, lending themselves to the use of aggregated rather than individual policy data, and without computers this is important. It is difficult, however, even to formulate the equivalent of a net premium method of valuation for nonconventional business such as unit-linked.

Discussions of ways to strengthen solvency reporting began in the 1960s, with arguments for and against the introduction of explicit solvency margins, as opposed to those implicit in a conservative valuation basis (e.g. [1, 14, 18]), which resulted in the introduction of an explicit solvency margin in Europe, **risk-based capital** in the United States, and Minimum Continuing Capital and Surplus Requirements in Canada. These margins were simple percentages of certain measures of exposure to risk; for example, in Europe, the basic requirement was 4% of mathematical reserves and 0.3% of sums at risk; in the United States, many more risk factors were considered (the so-called C-1 to C-4 risks), similarly in Canada, but the principles were the same. These approaches may be said, broadly, to belong to the 1980s and, however approximately, they began a move towards separate reporting requirements for earnings and for solvency, that has gathered pace to this day.

## Modern Developments

Modern developments in life insurance have been intimately associated with the rise of cheap, widespread computing power. Indeed, this may be said to have opened the way to a fundamental change in the actuarial approach to life insurance. A glance back at the actuarial textbooks on life insurance mathematics up to the 1980s [11, 13], will show an emphasis on computations using life tables, while textbooks on life insurance practice would be concerned with issues such as approximate methods of valuation of a life insurer's liabilities. Much of the

technique of actuarial science used to be the reduction of complex calculations to manageable form, but the need for this has been greatly reduced by computing power. Indeed the authors of [4], a modern textbook, said in their introduction:

‘This text represents a first step in communicating the revolution in the actuarial profession that is taking place in this age of high-speed computers.’

This freedom from arithmetical shackles has allowed modern actuaries to focus on the underlying probabilistic risk models, a development that has been, and is being, propelled by risk measurement and **risk management** techniques based on models being adopted by regulators worldwide.

Computing power allowed new products to be developed, that simply could not have been managed using traditional actuarial techniques and manual clerical labor. Unit-linked insurance is an example. Briefly, it allows each policyholder to direct their premiums into one or more funds invested in specified types of assets, and the value of their investment in each fund then moves precisely in line with the prices of the underlying assets. This is achieved by dividing each fund into identical units, that the policyholder ‘buys’ and ‘sells’, so it is very like a unit trust (UK) or mutual fund (USA) with a life insurance ‘wrapper’. Charges are levied explicitly in order to meet the insurer's expenses and provide a profit margin. Explicit charges can also be used to pay for insurance benefits such as life cover. Unit-linked policies ought to be transparent, compared with traditional with-profits policies, although they support a rich variety of charging structures that policyholders tend to find unclear. Their very essence is that they may be tailored to the needs of each policyholder, whereas conventional policies such as endowments are strongly collective, with shared tariffs and bonus scales. Only computers made this individuality feasible.

Once such a variety of products are marketed, questions of consistency arise, from the viewpoints of equity and profitability. Under conventional business, tariffs and bonuses would be set in an effort to achieve fairness, however rough, for all policyholders, but how can different unit-linked policies be compared? New techniques are needed, and chief among these is the profit test (*see Profit Testing*). At its simplest, this combines the explicit projection of future net cash flows, either expected cash flows or under various scenarios, with a model of capital

represented by a risk discount rate. Again, only computers make this new technique accessible. One of its most interesting features is that the use of a risk discount rate makes a link between actuarial technique and financial economics, even if, when it was introduced in 1959 [2], most of financial economics had yet to be developed.

An obvious step, once explicit cash-flow modeling had been introduced for pricing, was to extend it to models of the entire entity, the so-called **model office**. This became the basis of the next step in strengthening supervision, dynamic financial analysis (see **Dynamic Financial Modeling of an Insurance Enterprise**), in which model office projections under a wide range of economic and demographic scenarios were carried out, to establish the circumstances in which statutory solvency would be threatened. Dynamic financial analysis was, broadly, the major development of the 1990s, and at the time of writing it is the most advanced tool in use in many jurisdictions.

Beginning in the 1970s, modern financial mathematics introduced coherence between the assets and liabilities of a financial institution, centered on the concepts of **hedging** and nonarbitrage (see **Arbitrage**). Hedging ideas had been foreshadowed by actuaries, for example, [10, 17], but the technical apparatus needed to construct a complete theory was then still very new mathematics. In principle, these methodologies sweep away the uncertainties associated with the traditional methods of life insurance liability valuation. Instead of discounting expected future cashflows at a rate of interest that is, at best, a guess at what the assets may earn in future, the actuary simply finds the portfolio of traded assets that matches future liability cashflows, and the value of the liability is just the value of that portfolio of assets. This underlies the idea of the *fair value* (see **Accounting; Valuation of Life Insurance Liabilities**) of an insurance liability, as the price at which it would be exchanged between willing buyers and sellers [3].

In practice, there are significant difficulties in applying nonarbitrage methods to life insurance liabilities. The theory comes closest to resembling reality when there are deep and liquid markets in financial instruments that can genuinely match a liability. This is rarely the case in life insurance because of the long term of the liabilities and because there are no significant markets trading life insurance-like liabilities in

both directions. Moreover, the scale of life insurance liabilities, taken together with annuity and **pension** liabilities, are such that hedging in existing financial markets might disrupt them.

Nevertheless, driven by the aim (of the International Accounting Standards Board) of making life insurers' accounts transparent and comparable with the accounts of other entities [8], the entire basis of their financial reporting is, at the time of writing, being transformed by the introduction of 'realistic' or 'fair value' or 'market consistent' balance sheets, with a separate regime of capital requirements for solvency. In principle, this is quite close to banking practice in which balance sheets are marked to market and solvency is assessed by stochastic measures such as **value-at-risk** or conditional tail expectations. The methodologies that will be needed to implement this regime are still the subject of discussion [9].

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