

1 **THE EVOLUTION OF HOST DEFENCE WHEN PARASITES IMPACT**
2 **REPRODUCTION**

3 ALEX BEST¹, ANDY WHITE², AND MIKE BOOTS³

4 ¹SCHOOL OF MATHEMATICS & STATISTICS, UNIVERSITY OF SHEFFIELD, SHEFFIELD, S10 2TN.

5 ²DEPARTMENT OF MATHEMATICS & THE MAXWELL INSTITUTE FOR MATHEMATICAL SCIENCES, HERIOT-
6 WATT UNIVERSITY, EDINBURGH, EH14 4AS

7 ³DEPARTMENT OF INTEGRATIVE BIOLOGY, UNIVERSITY OF CALIFORNIA BERKELEY, BERKELEY, CA 94720-
8 3140

9 ABSTRACT

10 **Question:** How does the evolution of host defences to parasitism depend on the level of
11 disease-induced sterility?

12 **Mathematical Methods:** Evolutionary invasion analysis (adaptive dynamics) applied to
13 susceptible-infected host-parasite model.

14 **Key assumptions:** Hosts can evolve defence through avoidance (lower transmission), clear-
15 ance (higher recovery) or tolerance (lower virulence), in isolation or simultaneously, at a cost
16 to their reproductive rate. Separation of ecological and evolutionary timescales and mutations
17 of small phenotypic effect.

18 **Conclusions:** Avoidance and clearance are maximised when sterility is high, but tolerance
19 is greatest when sterility is low. However when clearance and tolerance co-evolve there is
20 greater tolerance at high sterility as this boosts the effectiveness of clearance. Patterns of
21 investment along other environmental gradients can change as the level of sterility changes.
22 Evolutionary branching to coexistence in avoidance and clearance is most likely when sterility
23 is high.

24 **Keywords:** Host-Parasite, Evolution, Adaptive Dynamics, Sterility

25

1. INTRODUCTION

26 Understanding the evolution of host defences to parasitism, and specifically the ecological
27 feedbacks that drive the patterns of selection, is a major area of theoretical inquiry (van
28 Baalen, 1998; Boots and Haraguchi, 1999; Boots and Bowers, 1999, 2004; Roy and Kirchner,
29 2000; Restif and Koella, 2003, 2004; Miller et al, 2005, 2007; Best et al, 2008, 2009; Toor and
30 Best, 2014). For the most part these studies assume that the main fitness impact of parasitism
31 is through disease-induced mortality, generally defined to be ‘virulence’. However, it is also
32 well understood that parasitism can cause sub-lethal effects, specifically through sterilising
33 effects that reduce the reproduction rate of infected hosts. Indeed, many parasites are known
34 to cause complete castration in their hosts, such as *Pasteuria ramosa* in *Daphnia* (Little et al,
35 2002; Ebert et al, 2004), anther smut infections in *Silene* (Antonovics et al, 1996) and insect
36 baculoviruses (Boots and Begon, 1993; Boots and Meador, 2007). Many more parasites cause
37 reduced fecundity in their infected hosts, for example in red grouse (Dobson and Hudson,
38 1992; Hudson et al, 1992) and vole (Deter et al, 2007; Feore et al, 1997) populations due to
39 parasitic nematodes.

40 A few theoretical studies have considered the evolution of defence traits that combat
41 parasite-induced sterility directly, that is, sterility itself is being selected on (Restif and Koella,
42 2004; Bonds, 2006; Best et al, 2008, 2009). These have shown that tolerance mechanisms that
43 reduce the degree of sterility may be a more effective defence strategy for the host than resis-
44 tance (Best et al, 2009), and that investment in such a tolerance mechanism will be greatest
45 against parasites with a high growth rate (Restif and Koella, 2004). More generally, the-
46 oretical studies focus on defences that combat epidemiological processes of transmission or
47 virulence, and this will also be our focus here. Commonly, these studies consider the invest-
48 ment in these defences along environmental gradients. Yet, the majority of these studies have
49 not allowed sterility to vary but instead assumed that sterility is either complete (such that
50 infected hosts do not reproduce at all) or absent (such that infected hosts reproduce at the
51 same rate as susceptible hosts). It is important for us to assess how these assumptions might
52 impact the predicted outcomes of host evolution.

53 Boots and Haraguchi (1999) showed that resistance (through ‘avoidance’) is maximised
54 against the least virulent parasites, while van Baalen (1998) showed that resistance (through
55 ‘clearance’) is highest when virulence is intermediate. However, these two studies made

56 very different assumptions about disease-induced sterility, with either complete (Boots and
57 Haraguchi, 1999) or no sterilising effects (van Baalen, 1998). Donnelly et al (2015) recently
58 offered some insight in to this difference by showing that the main driver of selection alters
59 between disease exposure (castrators) and disease prevalence (non-castrators) depending on
60 whether sterility is complete or absent. A few studies of host defence evolution have explored
61 investment in host defences along a gradient of sterility. For example, while exploring the
62 effects of host plasticity, McLeod and Day (2015) showed that avoidance increased as the
63 level of sterility is increased, a result that Toor and Best (2014) showed remains true when
64 a (dynamic) predator is also present. In a separate study McLeod and Day (2014) also
65 considered the impact of sterility on host reproductive strategy as a defence against sexually-
66 transmitted disease, while we showed that investment in immune priming in invertebrates
67 would be highest when parasites are sterilising (Best et al, 2012). Thus, while we have some
68 knowledge of how parasite-driven sterility impacts host evolution in specific circumstances,
69 we still lack a general overview of its role in the evolution of host defences.

70 A key finding has been the distinction between resistance and tolerance mechanisms due
71 to their ecological feedbacks. Resistance mechanisms directly reduce parasite fitness (i.e.
72 through lowered transmission or increased clearance) creating negative frequency-dependence.
73 Therefore when these mechanisms are costly to the host there can be evolutionary branching
74 to coexistence of host strains (Boots and Haraguchi, 1999; Boots and Bowers, 1999). In con-
75 trast, tolerance mechanisms generally increase parasite fitness (i.e. through reduced virulence,
76 and therefore increased infectious period) creating positive frequency-dependence (Roy and
77 Kirchner, 2000; Miller et al, 2005). When tolerance is to the sterilising rather than mortality
78 effects of parasitism, then parasite fitness is not impacted directly, and depending on where
79 the costs of sterility tolerance are incurred branching can arise (Best et al, 2008, 2009). In
80 another study, Ashby and Gupta (2014) showed that castration may be crucial to the mainte-
81 nance of temporal diversity through co-evolutionary cycles in gene frequencies between hosts
82 and parasites, but it remains an open question as to how static diversity (coexistence due to
83 evolutionary branching) in resistance depends on the degree of sterility.

84 In this study we consider the evolution of three forms of host defence: avoidance, clearance
85 and mortality tolerance, in a similar manner to the study of Miller et al (2007) on the effects
86 of host lifespan. Our focus here is on how investment in each type of defence varies with the

sterilising effects of the parasite, as well as on how this interacts with variation along other environmental gradients. We also consider the effects of sterility on the potential for diversity through evolutionary branching and on the outcome when two of the defence mechanisms coevolve together.

91

2. MODEL

For consistency with many earlier studies, and in particular Boots and Haraguchi (1999), we use a standard SIS epidemiological model with emergent density dependence on births, with the population dynamics of susceptible (S) and infected (I) hosts given by,

$$\frac{dS}{dt} = (a - q(S + I))(S + fI) - bS - \beta SI + \gamma I \quad (1)$$

$$\frac{dI}{dt} = \beta SI - (\alpha + b + \gamma)I. \quad (2)$$

All hosts reproduce at rate a which is reduced due to crowding, q . All hosts die at natural death rate b , with infected hosts suffering additional mortality (virulence) at rate α . Transmission is a mass-action term with parameter β . Again for consistency with earlier studies, notably Boots and Haraguchi (1999) and van Baalen (1998), we assume that transmission is density-dependent. We note, however, that since many sexually-transmitted infections induce infertility in their hosts (Lockhart et al, 1996) that frequency-dependent transmission would be an equally relevant assumption, but we leave this for future work. Infected hosts may recover back to susceptibility at rate γ . Our key parameter is that infection may reduce the reproduction rate of infected hosts by a fecundity factor f . We note that a low value of f indicates high sterility (with $f = 0$ representing full castration), and high f low sterility (with $f = 1$ meaning infected hosts reproduce at the same rate as susceptibles).

We model evolution of host defence through an evolutionary invasion (adaptive dynamics) framework (Geritz et al, 1998). As such we assume that a rare mutant type attempts to invade a resident population at its dynamic attractor (\hat{S}, \hat{I}) with $\hat{N} = \hat{S} + \hat{I}$. We will consider three forms of host defences; (i) avoidance (lowered susceptibility to infection, β), (ii) clearance (increased recovery, γ), and (iii) tolerance (lowered virulence, α).

In each case we will assume that there is a cost to higher defence through reduced reproduction, as is commonly assumed in the theoretical literature (e.g Boots and Haraguchi, 1999;

111

112

113 Boots and Bowers, 1999; Restif and Koella, 2003; Miller et al, 2007) and has empirical support
 114 (Boots and Begon, 1993). For example, let us assume that defence is through avoidance (the
 115 other cases can be expressed similarly), where a mutant host has strategy, $(\beta_m, a(\beta_m))$. We
 116 emphasise that this implies a reduced likelihood of a susceptible host becoming infected, not
 117 a reduced rate of infection by infected hosts. For the first part of the study we use a generic
 118 trade-off function of the form,

$$a(\beta) = a_{\max} - (a_{\max} - a_{\min}) \left(1 - \frac{\beta - \beta_{\min}}{\beta_{\max} - \beta_{\min}} \right) / \left(1 + p \frac{\beta - \beta_{\min}}{\beta_{\max} - \beta_{\min}} \right) \quad (3)$$

119 which links maximum and minimum birth and avoidance (defence) values through a smooth
 120 function the shape of which is controlled by parameter p (the trade-off is concave for $p > 0$ and
 121 convex for $-1 < p < 0$). We note that for the recovery trade-off the values of a_{\max} and a_{\min}
 122 must be swapped for the trade-off to be decreasing rather than increasing. We emphasise
 123 that in all cases, including the evolution of recovery and tolerance, the costs of reduced
 124 reproduction are paid by both susceptible and infected hosts. An alternative approach might
 125 be to assume that induced defences are plastic, and are only ‘switched on’ once a host is
 126 infected, and therefore only infected hosts would pay the costs. Here we assume that all
 127 defence mechanisms are constitutive and always present.

128 By considering the transversal stability of the resident equilibrium, specifically the deter-
 129 minant of the mutant’s Jacobian, it can be found that host fitness is given by (see appendix
 130 A1 for the derivation),

$$s = [a(\beta_m) - q\hat{N} - b - \beta_m\hat{I}][\alpha + b + \gamma] + \beta_m\hat{I}[\gamma + f(a(\beta_m) - q\hat{N})]. \quad (4)$$

131 If $s > 0$ then the mutant can invade to replace or coexist with the resident, whereas if $s < 0$
 132 the mutant will die out. Through a mutation-substitution sequence the population will evolve
 133 in the direction of the local selection gradient (e.g. $\partial s / \partial \beta_m |_{\beta_m = \beta}$ for avoidance) until this
 134 gradient is zero and an evolutionary ‘singular point’ has been reached. Here, the evolutionary
 135 outcome depends on two second-order derivatives: evolutionary stability (ES), $\partial^2 s / \partial \beta_m^2 |_{\beta_m = \beta}$
 136 (is the point evolutionarily invadible?), and convergence stability (CS), $\partial^2 s / \partial \beta_m^2 |_{\beta_m = \beta} +$
 137 $\partial^2 s / \partial \beta_m \partial \beta |_{\beta_m = \beta}$ (is the point evolutionarily attracting?). If both expressions are negative
 138 then the point is said to be a ‘continuously stable strategy’ (CSS), a long-term attractor of
 139 evolution. As our main focus here is on how sterility impacts quantitative investment in each

140 defence mechanism we shall concentrate on examining the location of CSSs. We shall also
 141 look at ‘evolutionary branching points’ where a dimorphic population emerges at a singular
 142 point that is CS (second expression is negative) but not ES (first expression is positive).

143

3. DIRECT EFFECTS

144 We first consider the direct effects of varying the fecundity of infected hosts, f , to the
 145 evolutionary outcome, focussing on whether investment at an attracting singular point (a CSS)
 146 will increase or decrease. We plot numerical examples for each case in figure 1. For consistency
 147 we plot $B^* = \beta_{\max} - \beta^*$ for avoidance and $A^* = \alpha_{\max} - \alpha^*$ for tolerance, such that in all
 148 cases high values indicate high defence and low values low defence. These show the general
 149 patterns that avoidance decreases with increasing fecundity (figure 1a), clearance decreases
 150 with increasing fecundity (figure 1b), and tolerance increases with increasing fecundity (figure
 151 1c). Therefore we see that resistance (avoidance or clearance) is highest when fecundity is
 152 low (sterility high), while tolerance is highest when fecundity is high (sterility low). In the
 153 appendix (A.2) we demonstrate analytically that this pattern is always true for clearance.
 154 Numerical exploration suggests that the patterns are also always true for the other two cases,
 155 but we cannot prove this analytically.

156 The pattern for resistance is to be expected. As the level of sterility is increased the fitness
 157 contribution from infected hosts is reduced. Thus selection for resistance mechanisms, which
 158 act to keep or move more of the population in to the more fecund susceptible state, will
 159 increase. The pattern for tolerance, in contrast, may not have initially been expected but
 160 in fact follows similar reasoning. At high rates of sterility, there is almost no contribution
 161 to fitness by infected hosts (due to both the direct effect of reduced reproduction and the
 162 indirect effect of a reduced infected density), meaning that there is very little reason to invest
 163 in tolerance mechanisms. However, when infected hosts do reproduce, infected hosts can still
 164 make a significant contribution to fitness, and this contribution can be increased by investing
 165 in tolerance and thus lengthening the infectious period.

166 It is clear from the fitness expression given above that there is a special case where $f = 0$.
 167 In this case a part of the fitness term disappears. This has an important simplifying effect
 168 when calculating the fitness gradient of each case. For example in the case of avoidance, the
 169 full fitness gradient is,

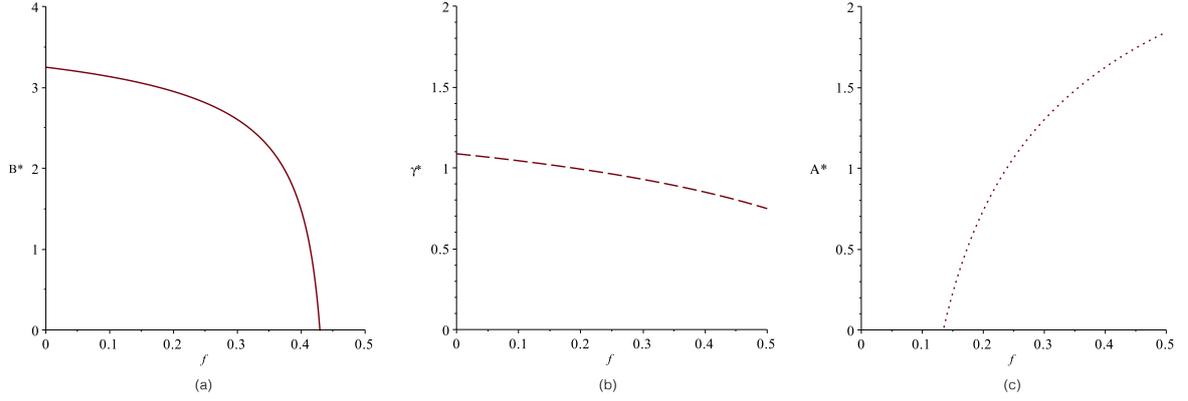


FIGURE 1. Direct effects of varying infected fecundity, f on host investment. (a) Avoidance (for consistency we plot $B^* = \beta_{\max} - \beta^*$); (b) Clearance (γ^*); (c) Tolerance (for consistency we plot $A^* = \alpha_{\max} - \alpha^*$). Parameter values: $\beta = 2$, $b = 1$, $q = 0.1$, $\alpha = 1$, $\gamma = 1$. Trade-offs: (a) $a(\beta) = 2.76 - 1.18(1.25 - 0.25\beta)/(0.84 + 0.16\beta)$, (b) $a(\gamma) = 1.70 + 0.50(1 - 0.5\gamma)/(1 - 0.17\gamma)$, (c) $a(\alpha) = 2.14 - 0.54(1 - 0.5\alpha)/(1 + 0.91\alpha)$.

$$\left. \frac{\partial s}{\partial \beta_m} \right|_{\beta_m = \beta} = [a'(\beta) - \hat{I}][\alpha + b + \gamma] + \hat{I}[\gamma + f(a(\beta) - q\hat{N})] + f a'(\beta) \beta \hat{I}. \quad (5)$$

170 If $f = 0$ this expression simplifies to,

$$\left. \frac{\partial s}{\partial \beta_m} \right|_{\beta_m = \beta} = [a'(\beta) - \hat{I}][\alpha + b + \gamma] + \hat{I}\gamma, \quad (6)$$

171 such that the total population size, \hat{N} , no longer appears in the fitness gradient and thus does
 172 not impact the location of the singular point. In this case, \hat{I} , that is the density of infecteds,
 173 is the key driver of evolution. However, in the more general case then both \hat{I} and \hat{N} impact
 174 the location of the singular point. (The same argument can be applied to the other two cases,
 175 see Appendix - section A.3).

176

4. COMBINED EFFECTS

177 In this section we will look at how investment across other environmental gradients is
 178 affected by altering the amount of sterility. We limit our study to the range $f \in [0, 0.5]$,
 179 noting that further investigation found no further qualitative changes of behaviour occur
 180 once $f > 0.5$.

181 **4.1. Parasite characteristics.** We first focus on how fecundity interacts with virulence
 182 (note that we cannot consider the case of tolerance here as this selects on virulence. The
 183 respective plots will be absent when considering recovery (clearance) and transmission (avoid-
 184 ance) also). In figure 2a we plot the CSS level of transmission as α (horizontal axis) and f
 185 (vertical axis) vary. Blue colours indicate high defence (low β) and yellow colours low de-
 186 fence (high β). Here we see that for very low f defence is highest when virulence is lowest.
 187 However, as f increases the pattern shifts so that defence is maximised against parasites with
 188 intermediate virulence. Once f is reasonably high, the pattern has completely reversed and
 189 defence is highest against the most virulent parasites. The changes in behaviour can be seen
 190 as line plots for the cases of $f = 0$ and $f = 0.5$ in figure A.1 in the appendix.

191 We now look at the relationship, α and f , when defence is through clearance. In figure 2b
 192 again yellow colours denote low defence (low γ) and blue colours high defence. Here we see a
 193 similar pattern as with avoidance. Defence is maximised against parasites with low virulence
 194 when there is low fecundity (high sterility), but against parasites with intermediate virulence
 195 when fecundity is greater. Again, the behaviour at the extremes of $f = 0$ and $f = 0.5$ can be
 196 seen in figure A.1 in the appendix.

197 Next we look at varying recovery. For avoidance (figure 2c) we see that investment always
 198 decreases with increasing clearance, but that the strength of this effect lessens at higher
 199 fecundity. For tolerance (figure 2d) we again see that the level of fecundity alters the pattern
 200 of investment. When fecundity is low (sterility high), tolerance is greatest at high clearance
 201 rates. However, as fecundity increases tolerance is instead maximised at intermediate and
 202 then low rates of recovery. Finally we show the patterns for varying transmission for defence
 203 in clearance and tolerance in figure 2e,f). In both cases, fecundity has no qualitative effect
 204 on the relationship, with investment increasing with transmission.

205 **4.2. Host characteristics.** We now consider the variation in investment against different
 206 host characteristics, starting with lifespan. In general, figure 3 (top-row) shows investment
 207 is highest (blue colours) in all three defence mechanisms for high lifespans (low death rates).
 208 However, at intermediate rates of fecundity we see that for the resistance mechanisms of
 209 avoidance (3a) and clearance (3b) investment may be maximised at intermediate lifespans.
 210 (We note that the black region in 3a represents a repelling singular point that is neither ES
 211 nor CS instead of an attractor. In this case evolution will lead to the host either maximising

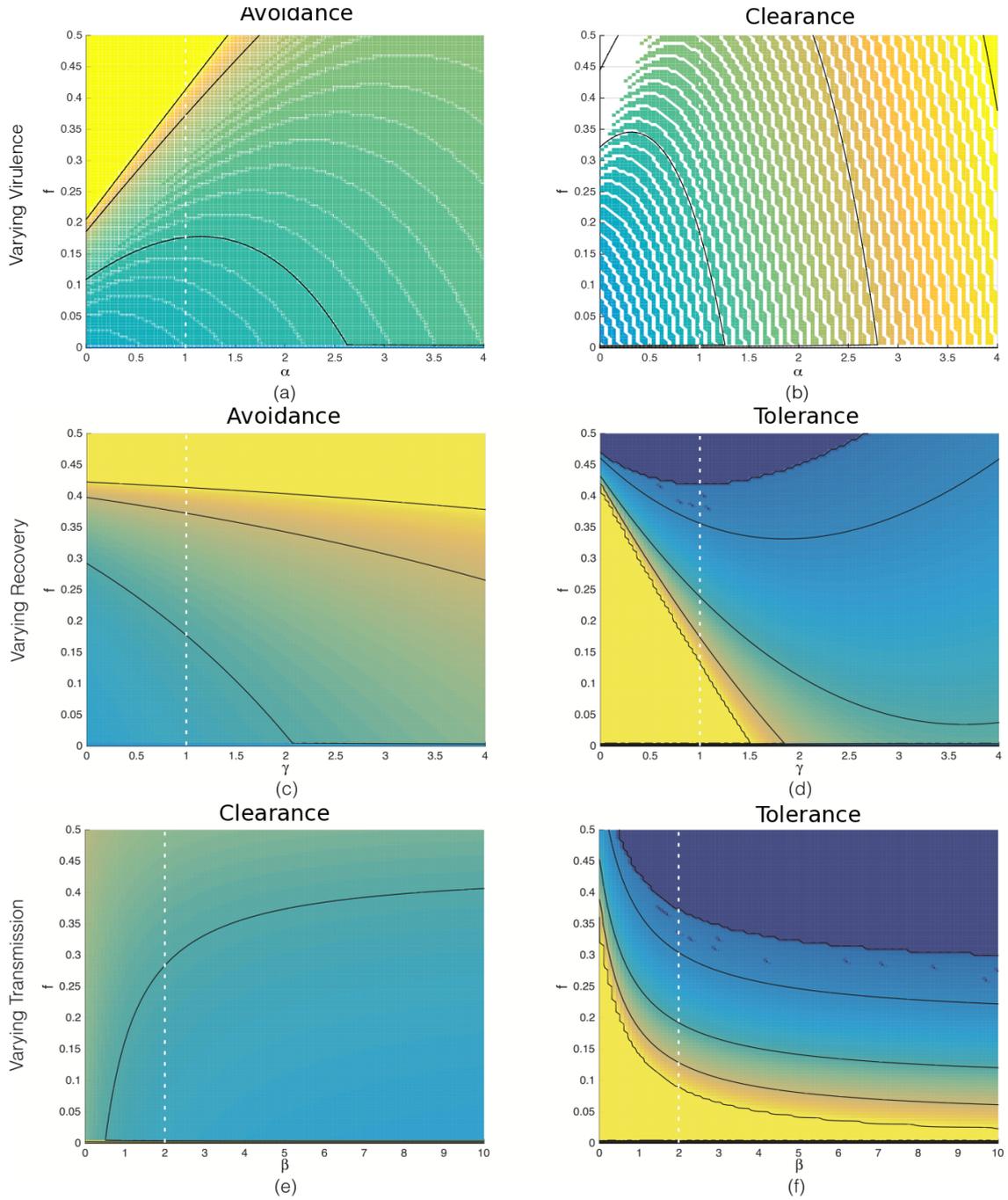


FIGURE 2. Combined effects of varying fecundity, f (y-axis) and (top-row) virulence, α , (middle-row) recovery, γ , (bottom-row) transmission, β . Plots are of investment in (a and c) avoidance, (b and e) clearance, and (d and f) tolerance. In each case blue colours indicate high defence, and yellow colours low defence. Contours are added for clarity. The dashed white line marks the gradient along which the single variable plots from figure 1 are taken. Parameter values are as of figure 1

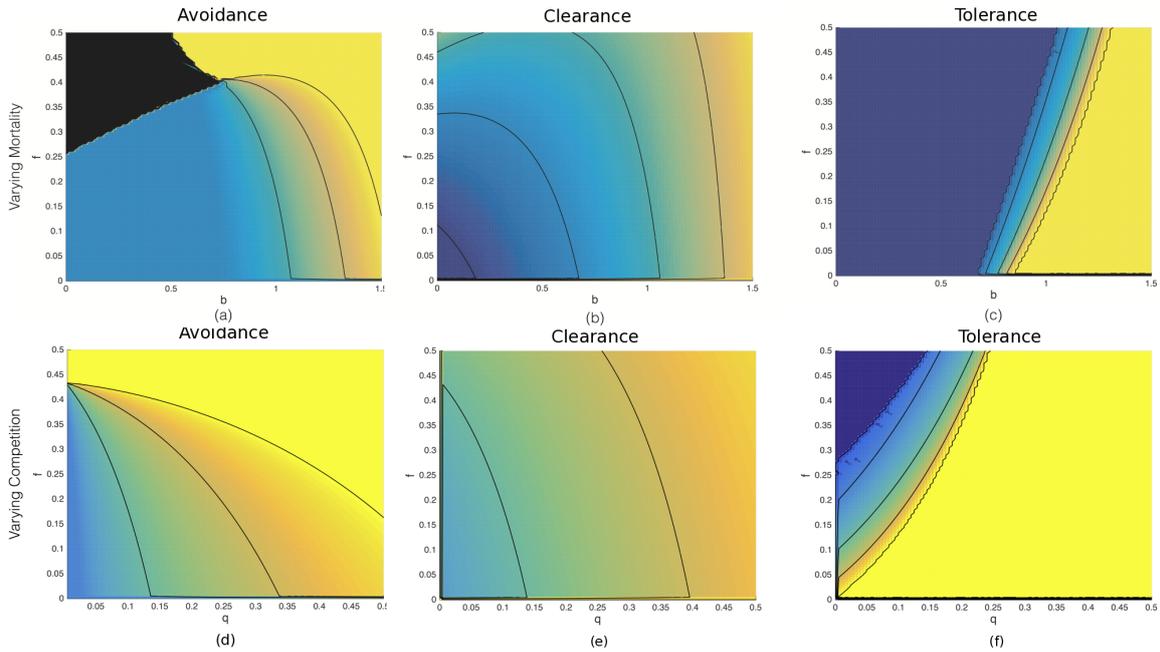


FIGURE 3. Combined effects of varying fecundity, f (y-axis) and (top-row) host mortality, b , and (bottom-row) competition, q . Plots a and d are for avoidance, b and e for clearance and c and f for tolerance. Again, blue colour indicate high defence and yellow colours low defence. Note that the black region of the top-left plot in fact denotes an evolutionary repeller. Parameter values are as of figure 1

212 or minimising avoidance depending on the initial conditions). We also consider investment
 213 as the competition coefficient q is varied. Figure 3 (bottom-row) shows that for all three de-
 214 fence mechanisms investment is highest when competition is low and therefore the population
 215 density is high.

216

5. STABILITY AND EVOLUTIONARY BRANCHING

217 We now examine how fecundity impacts the potential for dimorphism through evolutionary
 218 branching, in particular through the evolutionary and convergence stability of a fixed singular
 219 point. At such a point, the population is attracted to the singular point (it is CS), but once
 220 there find s it is a local fitness minimum (it is not ES). This results in disruptive selection
 221 and the emergence of two coexisting strains either side of the singular point (Geritz et al,
 222 1998). We now fix the singular point to be at a particular level of defence (and related
 223 cost). We no longer choose a fixed trade-off function, but the existence of the singular point

224 at the chosen values requires us to fix the gradient of the trade-off. We then consider the
225 change in behaviour at the singular point as we vary the curvature (second derivative) of the
226 trade-off at the singular point (Bowers et al, 2005; de Mazancourt and Dieckmann, 2004).
227 Specifically we plot the boundaries of evolutionary stability (ES) and convergence stability
228 (CS) at a fixed singular point in terms of the trade-off curvature (y-axis) at that point, as a
229 function of sterility (x-axis) in figure 4. Figure 4a shows the relationship for avoidance, 4b
230 for clearance and 4c for tolerance. In each case, curvatures below the solid line are ES and
231 those below the dashed line are CS. As is known to be generally true from earlier work (Hoyle
232 et al, 2008), trade-offs with strongly negative ("accelerating") curvatures tend to produce
233 CSS points (both ES and CS), those with strongly positive ("decelerating") curvatures tend
234 to produce repelling points (neither ES nor CS), with branching and 'Garden of Eden' points
235 (ES but not CS) generally occurring for near-linear trade-offs.

236 In the first two cases, avoidance and clearance, we see that when fecundity is not too
237 high (sterility not too low), there are a range of trade-off curvatures for which the singular
238 point is CS but not ES, and therefore an evolutionary branching point. However, for both
239 cases we see that the potential for branching decreases with increasing f , with no branching
240 predicted for this parameter set when reproduction is unaffected by infection. As we show
241 in the appendix (section A.5, figures A.2, A.3), we explored a range of parameter values and
242 found that this qualitative pattern, of a decreasing range of trade-offs that allow branching
243 as f increases, is generally preserved. For the final case, tolerance, we see that there is never
244 any evolutionary branching. It is well known that branching of tolerance mechanisms is not
245 possible in standard models as the derivatives are such that a singular point can never be
246 simultaneously CS but not ES (Roy and Kirchner, 2000; Miller et al, 2005). After branching
247 has occurred, for most standard trade-off forms (such as that used earlier in this study) the
248 two strains would evolve to the maxima/minima of evolution leaving two coexisting extreme
249 strains (though we note more complex trade-offs may lead to extinction of one of the strains
250 (Best et al, 2015)).

251

6. COEVOLUTION OF DEFENCES

252 **6.1. Direct trade-offs.** We now consider the outcome when two of the defence mechanisms
253 co-evolve together. We first assume that the two defences are directly traded-off against one

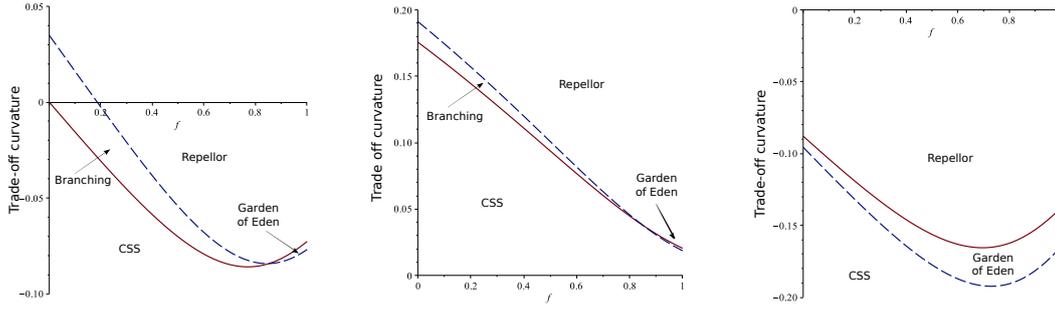


FIGURE 4. Boundaries of evolutionary stability and convergence stability at a fixed singular point when the host evolves (a) Avoidance, (b) Clearance and (c) Tolerance. Below the solid line the singular point is evolutionarily stable (ES), below the dashed line the singular point is convergence stable (CS). Evolutionary branching occurs for points that are CS but not ES. Parameters are as of figure 1 with respective singular points chosen at $\beta^* = 2$, $\gamma^* = 1$, $\alpha^* = 1$ and $a^* = 2$ for all three cases.

254 another, for example where increased avoidance is costly to clearance, $\gamma = \gamma(\beta)$, with no
 255 further life-history costs. Let us consider the selection gradient for example in the case for
 256 $\gamma(\beta)$,

$$\left. \frac{\partial s}{\partial \beta_m} \right|_{\beta_m = \beta} = -\hat{I}[\alpha + b + f(a - q\hat{N})] + \gamma'(\beta)[a - q\hat{N} - b]. \quad (7)$$

257 In this specific case, the level of fecundity has no impact on the evolutionary singular point,
 258 and thus allocation between the two defence mechanisms of avoidance and recovery will remain
 259 the same whatever the degree of sterility. This can be shown to be the case by noting that
 260 at equilibrium,

$$\frac{dN}{dt} = (a - q\hat{N})(\hat{S} + f\hat{I}) - b\hat{N} - \alpha\hat{I} = 0 \quad (8)$$

$$\implies \hat{S} = \frac{\hat{I}[\alpha + b - f(a - q\hat{N})]}{a - q\hat{N} - b}, \quad (9)$$

261 yet we also know that $\hat{S} = (\alpha + b + \gamma)/\beta$. By comparing this to equation (7) we see that
 262 the first term of (7) can be re-written without f appearing explicitly. We can then factor
 263 out the term $[a - q\hat{N} - b]$, noting that this term must be positive at the endemic equilibrium
 264 (as $\hat{S} < (a - b)/q$, the disease-free equilibrium), meaning that the solution to equation (7) is
 265 independent of f .

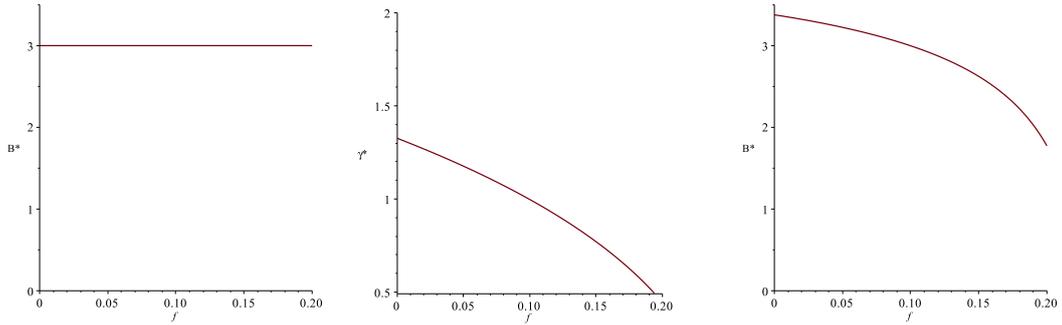


FIGURE 5. Singular levels of investment for varying fecundity when defence mechanisms are traded-off against one another. Parameter values are as of figure 1, with (a) $\gamma = 4 - 4.8(5/4 - \beta/4)/(0.8 + 0.2\beta)$, (b) $\alpha = 3.29 - 3.46(1 - \gamma/2)/(1 - 0.244\gamma)$, (c) $\alpha = -0.26 - 3.30(5/4 - \beta/4)/(3.97 - 2.97\beta)$

266 For the other two possibilities this is not the case. When clearance and tolerance are linked
 267 (e.g. $\alpha = \alpha(\gamma)$) then we find that hosts will favour recovery at low levels of fecundity but
 268 tolerance at high levels (figure 5b). When avoidance and tolerance are linked (e.g. $\alpha = \alpha(\beta)$)
 269 we find that hosts favour avoidance (low transmission, β^*) at low levels of fecundity but
 270 tolerance at higher levels (figure 5c). Thus, in each case, tolerance is favoured more strongly
 271 relative to resistance at higher rates of fecundity, as we might have predicted from the initial
 272 results in figure 1. However if the two resistance mechanisms are traded-off there is no impact
 273 of sterility.

274 **6.2. Simultaneous evolution.** We now assume that the two defences are not directly traded-
 275 off against one another, but instead evolve together simultaneously (i.e. coevolution). We
 276 assume both defences incur costs to the birth rate, as above, with the resulting birth rate
 277 being a linear combination of the two cost structures (similarly to the approach by Restif and
 278 Koella (2004); see the legend of figure 6 for the trade-off functions used).

279 We plot the results of the three cases in figure 6. As when the two resistance mechanisms
 280 were directly traded-off, we find that investment in avoidance and recovery remains relatively
 281 constant for varying levels of sterility when they simultaneously evolve, suggesting that the
 282 balance of investment in resistance mechanisms is largely independent of sterility. We find
 283 that when tolerance and clearance coevolve in this way, both defence mechanisms are favoured
 284 at low rates of fecundity (we are again plotting here $A^* = \alpha_{max} - \alpha^*$ such that high A^* means
 285 high tolerance). This is interesting since it reverses the result from the first part of this

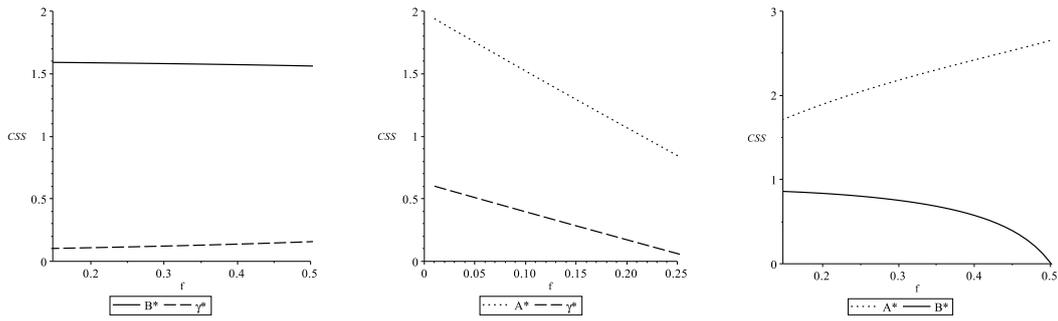


FIGURE 6. Singular levels of investment for varying fecundity when defence mechanisms evolve simultaneously. As before we in fact plot $B^* = \beta_{max} - \beta^*$ and $A^* = \alpha_{max} - \alpha^*$. Parameter values are as of figure 1, with (a) $a = 0.5(1.5 + 0.5(1 - 0.5\gamma)/(1 - 0.25\gamma)) + 0.5(2.8 - 1.15(1.25 - 0.25\beta)/(0.84 + 0.25\beta))$, (b) $a = 0.5(1.7 + 0.5(1 - 0.5\gamma)/(1 - 0.05\gamma)) + 0.5(2.14 - 0.54(1 - 0.5\alpha)/(1 + 0.9\alpha))$, (c) $a = 0.5(2.14 - 1.5(1 - 0.5\alpha)/(1 + 4\alpha)) + 0.5(2.76 - 1.5(1.25 - 0.25\beta)/(0.5 + 0.75\beta))$.

286 study for tolerance (i.e. fig 1c). When tolerance evolved in isolation there was little benefit
 287 of defence when sterility is high, whereas now increased tolerance boosts the effectiveness of
 288 increased recovery. Finally when avoidance and tolerance coevolve we find that both defences
 289 retain the patterns of investment as when they evolved in isolation, with high avoidance at
 290 low fecundity and high tolerance at high fecundity. In particular, we note that the pattern of
 291 tolerance differs markedly from its coevolution with clearance due to the differing feedbacks
 292 of the two resistance mechanisms.

293

7. DISCUSSION

294 While the sterilising effects of disease are known to be important to host-parasite interac-
 295 tions, few studies have specifically studied the impact this will have on host evolution (Best
 296 et al, 2012; McLeod and Day, 2014; Toor and Best, 2014; Donnelly et al, 2015; McLeod and
 297 Day, 2015). We have shown here that higher sterility selects for higher resistance (reduced
 298 transmission (see also McLeod and Day, 2015) or increased clearance), but lower tolerance
 299 (increased virulence). For resistance, this is because when sterility is high selection drives
 300 individuals to maximise their time in the susceptible state as this becomes the only source of
 301 reproduction. However, understanding the pattern for tolerance needs more consideration. As
 302 sterility increases, infected hosts contribute less to fitness and there is therefore less selection
 303 to extend the infectious period through tolerance. We find that these results largely hold even

304 when two of the defence mechanisms simultaneously evolve, except that high clearance leads
305 the host to invest in high tolerance to boost the effectiveness of increased recovery. Our study
306 therefore not only further highlights the importance of the distinction between resistance and
307 tolerance mechanisms due to their feedbacks to population densities, but stresses how these
308 feedbacks can influence the simultaneous evolution of each other.

309 An important result from previous studies is that host resistance may not be highest against
310 the most virulent parasites as might be intuitively expected, due to the ecological feedbacks.
311 Specifically, Boots and Haraguchi (1999) showed that avoidance is greatest against the least
312 virulent parasites, while van Baalen (1998) showed that clearance is maximised at intermedi-
313 ate rates of virulence. However, these studies focussed on specific cases of full and no sterility
314 respectively. Here we have explored in finer detail how these patterns depend on the degree of
315 sterility. In general, if sterility is very high, resistance through either mechanism will be great-
316 est against the least virulent parasites (c.f Boots and Haraguchi, 1999). This is because the
317 key effect of reduced virulence is increased exposure (since reduced disease-induced mortality
318 naturally leads to higher infected densities), which leads to increased selection for resistance.
319 However, as sterility decreases there is a shift to maximise resistance at intermediate levels of
320 virulence (c.f van Baalen, 1998) and then, for avoidance, at the highest virulence rates. Now,
321 infected hosts are able to make a significant contribution to fitness through reproduction. Not
322 only does reduced virulence lead to increased exposure, but also a greater contribution from
323 infected hosts. The balance of these two feedbacks is such that resistance is now greatest at
324 higher rates of virulence. These conclusions fit with the findings of McLeod and Day (2015)
325 and Donnelly et al (2015), the latter of who showed that for castrating diseases ($f = 0$) the
326 driver of selection is purely parasite exposure (i.e. the density of infected hosts) whereas for
327 non-castrators ($f = 1$) the driver is disease prevalence (i.e. the proportion of infected hosts).
328 Our work extends the findings of Donnelly et al (2015) by showing the range of behaviours
329 as sterility varies from one extreme to the other (see also McLeod and Day, 2015).

330 As well as virulence, we have seen similar shifts in behaviour when sterility is small or
331 large as other parameters are varied with the same reasoning applying. For example, we
332 saw such a relationship as host lifespan is varied to both avoidance and clearance evolution.
333 This pattern was again discussed by Donnelly et al (2015) in more detail for the specific
334 cases of $f = 0$ and $f = 1$. We also saw that as recovery is varied, tolerance is greatest at

335 high recovery rates when parasites are sterilising, but at intermediate or low recovery rates
336 when infected fecundity is higher. When $f = 0$ infected hosts contribute little to fitness. If
337 in addition recovery rates are high, however, increased tolerance gives infected hosts more
338 chance of recovering to susceptibility where they can contribute more. For larger f , when
339 there is low recovery tolerance may seem beneficial in order to extend the time producing
340 offspring, but in this case the costs of reduced reproduction are not worth paying. For high f
341 at high recovery rates, hosts are likely to return to susceptibility quickly, making reproduction
342 more important than tolerance. We therefore emphasise how important the degree of sterility
343 is to the evolution of host defences due to the feedbacks to population dynamics.

344 It is well-known that resistance mechanisms can create negative frequency-dependence lead-
345 ing to evolutionary branching and coexistence (Boots and Bowers, 1999) but that tolerance
346 mechanisms cannot (Roy and Kirchner, 2000; Miller et al, 2005), and we have recovered those
347 patterns here. However, we have also shown that branching in resistance is more likely (that
348 is, possible for a wider range of trade-offs) when sterility is high. High rates of sterility act
349 to increase the dichotomy between extreme strategies of a slow but long-lasting reproductive
350 strategy (high resistance) and a fast but short reproductive strategy (low resistance), which
351 makes diversity more likely. We found these results held across different parameter sets, and
352 we have previously found that branching in invertebrate immune priming is also most likely
353 for high levels of sterility (Best et al, 2012). Interestingly, Ashby and Gupta (2014) also
354 showed that the maintenance of temporal diversity (co-evolutionary cycles) similarly required
355 a high degree of sterility. It therefore appears that both static and temporal forms of diversity
356 are far more likely to evolve in host-parasite systems where the disease is sterilising.

357 We found that if two of the defences coevolved there was little change to our general
358 predictions. There were however some key differences. Firstly, when tolerance coevolves with
359 clearance, increased tolerance boosts the effectiveness of evolving higher clearance. Secondly,
360 we found that when the two resistance mechanisms coevolve, whether directly or by evolving
361 simultaneously, the relative investment in each mechanism stays constant no matter the level
362 of sterility. Here we have focussed purely on the role sterility plays in the evolution of host
363 defences. It is, of course, very likely that parasites would co-evolve with their hosts to combat
364 these defences. Previous theory has shown that if the parasite can target its negative impacts
365 towards either host mortality or reproduction it will always evolve to completely sterilise its

366 host, since fecundity has no impact on the parasite's R_0 but mortality does (Jaenike, 1996;
367 O'Keefe and Antonovics, 2002). It is for this reason that in a previous study we argued that
368 tolerance to sterility is likely to be a better defence strategy for the host than resistance, since
369 resistance mechanisms cannot prevent a parasite co-evolving to sterilise its host but tolerance
370 can (Best et al, 2009). McLeod and Day (2015) showed that for avoidance resistance the
371 pattern of investment along a sterility gradient found here is qualitatively the same when the
372 parasite coevolves, but further study is needed to explore this question more generally.

ACKNOWLEDGEMENTS

373
374 AB was supported by a Leverhulme Early Career Fellowship.

REFERENCES

- 375
376 Antonovics J, Stratton D, Thrall PH, Jarosz AM (1996) An anther-smut disease (*Ustilago*
377 *violacea*) of fire-pink (*Silene virginica*): its biology and relationship to the anther-smut
378 disease of white campion (*Silene alba*). American Midland Naturalist 135:130–143
- 379 Ashby B, Gupta S (2014) Parasitic castration promotes coevolutionary cycling but also im-
380 poses a cost on sex. Evolution 68:2234–2244
- 381 van Baalen M (1998) Coevolution of recovery ability and virulence. Proceedings of the Royal
382 Society of London B: Biological Sciences 265(1393):317–325
- 383 Best A, White A, Boots M (2008) Maintenance of host variation in tolerance to pathogens
384 and parasites. PNAS 105:20,786–20,791
- 385 Best A, White A, Boots M (2009) Resistance is futile but tolerance can explain why parasites
386 do not always castrate their hosts. Evolution 64:348–357
- 387 Best A, Tidbury H, White A, Boots M (2012) The evolution of within-generation immune
388 priming in invertebrate hosts. Journal of the Royal Society Interface 10:2012,887, unpub-
389 lished
- 390 Best A, Bowers RG, White A (2015) Evolution, the loss of diversity and the role of trade-offs.
391 Mathematical Biosciences 264:86–93
- 392 Bonds MH (2006) Host life-history strategy explains pathogen-induced sterility. The American
393 Naturalist 168:281–293

- 394 Boots M, Begon M (1993) Trade-offs with resistance to a granulosis virus in the Indian meal
395 moth, examined by a laboratory evolution experiment. *Functional Ecology* 7:528–534
- 396 Boots M, Bowers RG (1999) Three mechanisms of host resistance to microparasites - avoid-
397 ance, recovery and tolerance - show different evolutionary dynamics. *Journal of Theoretical*
398 *Biology* 201:13–23
- 399 Boots M, Bowers RG (2004) The evolution of resistance through costly acquired immunity.
400 *Proceedings of the Royal Society of London B: Biological Sciences* 271:715–723
- 401 Boots M, Haraguchi Y (1999) The evolution of costly resistance in host-parasite systems. *The*
402 *American Naturalist* 153:359–370
- 403 Boots M, Meador M (2007) Local interactions select for lower pathogen infectivity. *Science*
404 315:1284–1286
- 405 Bowers RG, Hoyle A, White A, Boots M (2005) The geometric theory of adaptive evolution:
406 trade-off and invasion plots. *Journal of Theoretical Biology* 233:363–377
- 407 Deter J, Cosson JF, Chaval Y, Charbonnel N, Morand S (2007) The intestinal nematode
408 *Trichus arvicolae* affects the fecundity of its host, the common vole *Microtus arvalis*. *Par-*
409 *asitology Research* 101:1161–1164
- 410 Dobson AP, Hudson PJ (1992) Regulation and stability of a free-living host-parasite system:
411 *Trichostrongylus tenuis* in Red Grouse. ii. population models. *Journal of Animal Ecology*
412 61:487–498
- 413 Donnelly R, White A, Boots M (2015) The epidemiological feedbacks critical to the evolution
414 of host immunity. *Journal of Evolutionary Biology* p In Press
- 415 Ebert D, Carius HJ, Little T, Decaestecker E (2004) The evolution of virulence when parasites
416 cause host castration and gigantism. *The American Naturalist* 164(5):S19–S32
- 417 Feore SM, Bennett M, Chantrey J, Jones T, Baxby D, Begon M (1997) The effect of cowpox
418 virus infection on fecundity in bank voles and wood mice. *Proceedings of the Royal Society*
419 *of London B: Biological Sciences* 264:1457–1461
- 420 Geritz SAH, Kisdi E, Meszéna G, Metz JAJ (1998) Evolutionarily singular strategies and the
421 adaptive growth and branching of the evolutionary tree. *Evolutionary Ecology* 12:35–57
- 422 Hoyle A, Bowers RG, White A, Boots M (2008) The influence of trade-off shape on evolution-
423 ary behaviour in classical ecological scenarios. *Journal of Theoretical Biology* 250:498–511

- 424 Hudson PJ, Newborn D, Dobson AP (1992) Regulation and stability of a free-living host-
425 parasite system: *Trichostrongylus tenuis* in Red Grouse. i. monitoring and parasite reduc-
426 tion experiments. *Journal of Animal Ecology* 61:477–486
- 427 Hurford A, Cownden D, Day T (2010) Next-generation tools for evolutionary invasion anal-
428 yses. *Journal of the Royal Society Interface* 7:561–571
- 429 Jaenike J (1996) Suboptimal virulence of an insect-parasitic nematode. *Evolution* 50(6):2241–
430 2247
- 431 Little T, Carius HJ, Sakwinska O, Ebert D (2002) Competitiveness and life-history char-
432 acteristics of *Daphnia* with respect to susceptibility to a bacterial pathogen. *Journal of*
433 *Evolutionary Biology* 15:796–802
- 434 Lockhart A, Thrall PH, Antonovics J (1996) Sexually transmitted diseases in animals: eco-
435 logical and evolutionary implications. *Biology Review* 71:415–471
- 436 de Mazancourt C, Dieckmann U (2004) Trade-off geometries and frequency-dependent selec-
437 tion. *The American Naturalist* 164(6):765–778
- 438 McLeod D, Day T (2014) Sexually transmitted infection and the evolution of serial monogamy.
439 *Proceedings of the Royal Society of London B: Biological Sciences* 281:20141,726
- 440 McLeod D, Day T (2015) Pathogen evolution under host avoidance plasticity. *Proceedings of*
441 *the Royal Society of London B: Biological Sciences* 282:20151,656
- 442 Miller MR, White A, Boots M (2005) The evolution of host resistance: tolerance and control
443 as distinct strategies. *Journal of Theoretical Biology* 236:198–207
- 444 Miller MR, White A, Boots M (2007) Host life span and the evolution of resistance charac-
445 teristics. *Evolution* 61:2–14
- 446 O’Keefe KJ, Antonovics J (2002) Playing by different rules: The evolution of virulence in
447 sterilizing pathogens. *The American Naturalist* 159:597–605
- 448 Restif O, Koella JC (2003) Shared control of epidemiological traits in a coevolutionary model
449 of host-parasite interactions. *The American Naturalist* 161(6):827–836
- 450 Restif O, Koella JC (2004) Concurrent evolution of resistance and tolerance to pathogens.
451 *The American Naturalist* 164(4):E90–E102
- 452 Roy BA, Kirchner JW (2000) Evolutionary dynamics of pathogen resistance and tolerance.
453 *Evolution* 54(1):51–63

454 Toor J, Best A (2014) The evolution of host resistance to disease in the presence of predators.
 455 Journal of Theoretical Biology 365:104–111

456 APPENDIX A. MATHEMATICAL DETAILS

457 **A.1. Derivation of fitness equation.** We show here how fitness is derived in the case of
 458 evolving avoidance. The other two cases can be found similarly. Consider the system of
 459 population dynamic equations where both the resident and mutant are present,

$$\frac{dS_r}{dt} = (a(\beta_r) - q(S_r + I_r + S_m + I_m))(S_r + fI_r) - bS_r - \beta_r S_r(I_r + I_m) + \gamma I_r \quad (\text{A.1})$$

$$\frac{dI_r}{dt} = \beta_r S_r(I_r + I_m) - (\alpha + b + \gamma)I_r \quad (\text{A.2})$$

$$\frac{dS_m}{dt} = (a(\beta_m) - q(S_r + I_r + S_m + I_m))(S_m + fI_m) - bS_m - \beta_m S_m(I_r + I_m) + \gamma I_m \quad (\text{A.3})$$

$$\frac{dI_m}{dt} = \beta_m S_m(I_r + I_m) - (\alpha + b + \gamma)I_m \quad (\text{A.4})$$

460 where a subscript r denotes a resident density or trait, and a subscript m a mutant density
 461 or trait. If we assume that the mutant is rare, then the resident can be assumed to have
 462 reached a stable equilibrium of the one-strain system given in equations (1)-(2). The mutant
 463 dynamics are now given by,

$$\frac{dS_m}{dt} = (a(\beta_m) - q(\hat{S} + \hat{I}))(S_m + fI_m) - bS_m - \beta_m S_m(\hat{I}) + \gamma I_m \quad (\text{A.5})$$

$$\frac{dI_m}{dt} = \beta_m S_m(\hat{I}) - (\alpha + b + \gamma)I_m \quad (\text{A.6})$$

464 where the resident equilibrium densities are given by \hat{S} and \hat{I} . The transversal stability of the
 465 two-strain system (e.g. (A1)-(A4)), where the resident is at equilibrium and the mutant at 0
 466 density, is governed by a 4x4 Jacobian matrix that can be separated in to four independent
 467 2x2 matrices. Since the top-left matrix is for the resident dynamics (which is assumed to be
 468 stable) and the bottom-left matrix is 0, the stability depends entirely on the Jacobian relating
 469 to the mutant dynamics, given by,

$$\mathbf{J} = \begin{pmatrix} a(\beta_m) - q(\hat{S} + \hat{I}) - b - \beta_m \hat{I} & f(a(\beta_m) - q(\hat{S} + \hat{I})) + \gamma \\ \beta_m \hat{I} & -(\alpha + b + \gamma) \end{pmatrix}. \quad (\text{A.7})$$

470 This system is then unstable, and thus the mutant can invade, whenever the determinant
 471 of the Jacobian is negative, giving,

$$s = [a(\beta_m) - q(\hat{S} + \hat{I}) - b - \beta_m \hat{I}][\alpha + b + \gamma] + \beta_m \hat{I}[\gamma + f(a(\beta_m) - q(\hat{S} + \hat{I}))] > 0 \quad (\text{A.8})$$

472 as in equation (4) of the main text.

473 **A.2. Proof of the direct effects of fecundity on clearance evolution.** We first assume
 474 that defence is through lowered transmission. In this case, the fitness gradient is given by,

$$\left. \frac{\partial s}{\partial \beta_m} \right|_{\beta_m = \beta} = a'(\beta)[\alpha + b + \gamma + f\beta I] + I[f(a - qN) - \alpha - b], \quad (\text{A.9})$$

475 If we take the derivative of this with respect to f we find how f affects the selection gradient
 476 at any point along the trade-off,

$$\frac{d\left(\left. \frac{\partial s}{\partial \beta_m} \right|_{\beta_m = \beta}\right)}{df} = a'(\beta)[\beta I(f) + f\beta I'(f)] + [fI'(f) + I(f)][a - qN] + I'(f)[- \alpha - b - fqI(f)] \quad (\text{A.10})$$

477 Here $I'(f) > 0$ and we assume $a'(\beta) > 0$, meaning that the first two terms are positive, and
 478 the third term is negative. We are unable to draw a general conclusion from this equation,
 479 but numerics suggest that the total is always positive, such that an increase in f leads to
 480 evolution to higher values of β , and therefore lower defence. See figure 1a.

481 We next assume that defence is through clearance. In this case the selection gradient is,

$$\left. \frac{\partial s}{\partial \gamma_m} \right|_{\gamma_m = \gamma} = a'(\gamma)[\alpha + b + \gamma + f\beta I] + [a - qN - b]. \quad (\text{A.11})$$

482 In this case the derivative of the selection gradient with respect to f is,

$$\frac{d\left(\left. \frac{\partial s}{\partial \gamma_m} \right|_{\gamma_m = \gamma}\right)}{df} = a'(\gamma)\beta[fI'(f) + I(f)] - qI'(f), \quad (\text{A.12})$$

483 Since $a'(\gamma) < 0$ and $I'(f) > 0$, all the terms above result in the selection gradient becoming
 484 more negative as f is increased. The result, therefore, is that a small increase to f leads to
 485 the host evolving to a lower value of recovery, or in other words, lower defence. See figure 1b.

486 In the final case we assume that defence is through lowered virulence. In this case, the
 487 fitness gradient is given by,

$$\left. \frac{\partial s}{\partial \alpha_m} \right|_{\beta_m = \beta} = a'(\alpha)[\alpha + b + \gamma + f\beta I] + [a(\alpha) - qN - b - \beta I]. \quad (\text{A.13})$$

488 Here the resulting change to the fitness gradient will be,

$$\frac{d\left(\left. \frac{\partial s}{\partial \alpha_m} \right|_{\alpha_m = \alpha}\right)}{df} = I'(f)[f\beta a'(\alpha) - q - \beta] + I(f)\beta a'(\alpha), \quad (\text{A.14})$$

489 As with avoidance, we are unable to draw a firm conclusion from this, but numerics suggest
490 that this is always negative, such that an increase in f will always lead to lower α , and
491 therefore higher tolerance. See figure 1c.

492 **A.3. Derivation of fitness gradient for clearance when $f = 0$.** If we take the Next
493 Generation Method (Hurford et al, 2010) then the fitness of a mutant host evolving its defence
494 through clearance is,

$$s = \frac{a(\gamma_m) - qN}{b + \beta I} + \frac{\beta I[f(a(\gamma_m) - qN) + \gamma_m]}{(b + \beta I)(\alpha + b + \gamma_m)} - 1. \quad (\text{A.15})$$

495 Consequently the fitness gradient is given by,

$$\left. \frac{\partial s}{\partial \gamma_m} \right|_{\gamma_m = \gamma} = \frac{a'(\gamma)}{b + \beta I} + \frac{\beta I[fa'(\gamma) + 1]}{(b + \beta I)(\alpha + b + \gamma)} - \frac{\beta I[f(a(\gamma) - qN) + \gamma]}{(b + \beta I)(\alpha + b + \gamma)^2}. \quad (\text{A.16})$$

496 If we take the extreme case that $f = 0$, this expression reduces to,

$$\left. \frac{\partial s}{\partial \gamma_m} \right|_{\gamma_m = \gamma} = \frac{a'(\gamma)}{b + \beta I} + \frac{\beta I}{(b + \beta I)(\alpha + b + \gamma)} - \frac{\beta I\gamma}{(b + \beta I)(\alpha + b + \gamma)^2}. \quad (\text{A.17})$$

497 As was the case with avoidance resistance in the main text, then, we see that the only
498 population feedback to host evolution is to the infected density - that is the exposure to
499 disease. The result for the evolution of tolerance when $f = 0$ can be derived in a similar
500 manner to be,

$$\left. \frac{\partial s}{\partial \alpha_m} \right|_{\alpha_m = \alpha} = \frac{a'(\alpha)}{b + \beta I} - \frac{\beta I\gamma}{(b + \beta I)(\alpha + b + \gamma)^2}. \quad (\text{A.18})$$

501 **A.4. Slices through plots of figure 2.** Here we present line plots of investment along
502 epidemiological gradients for (left) $f = 0$ and (right) $f = 0.5$. These are essentially slices
503 taken horizontally along the top and bottom of figure 2.

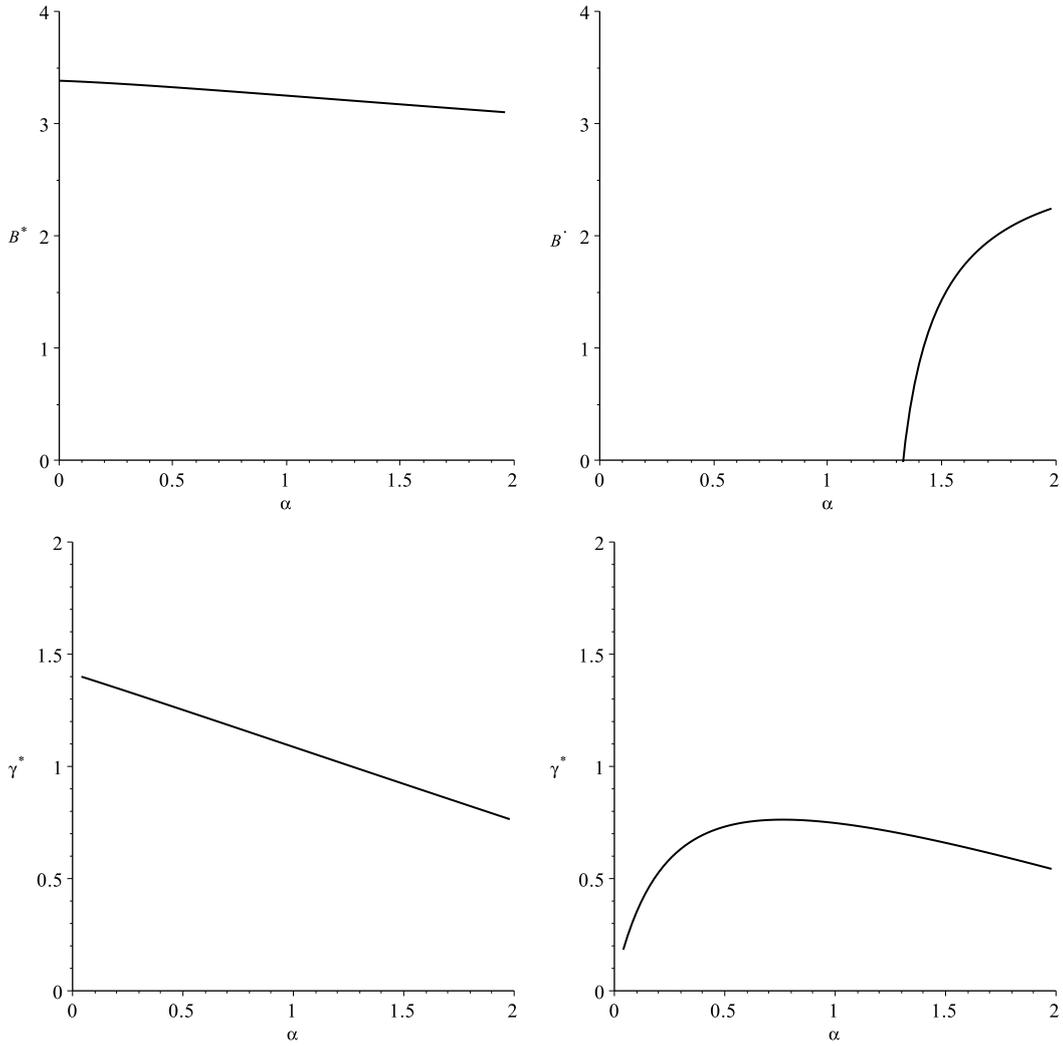


FIGURE A.1. Investment in (top-row) avoidance, and (bottom-row) clearance, as virulence is varied. In the right-hand plots $f = 0$, in the left-hand plots $f = 0.5$.

504 **A.5. Exploring parameter space of figure 4.** Here we present plots showing the mutual
505 invadability (MI), e.g. $\partial^2 s / \partial \beta_m \partial \beta$, at the singular point. This corresponds to the difference
506 between the ES and CS lines in figure 4 in the main text, with a negative MI meaning that
507 the ES and CS curves are arranged such that branching is possible. The larger the (negative)
508 value of MI, the larger range of trade-offs that allow branching. Here we show that the general
509 trend of less potential for branching as sterility, f , increases is preserved across parameter
510 space. We note that while we see a non-monotonic response for low values of α , b and γ ,

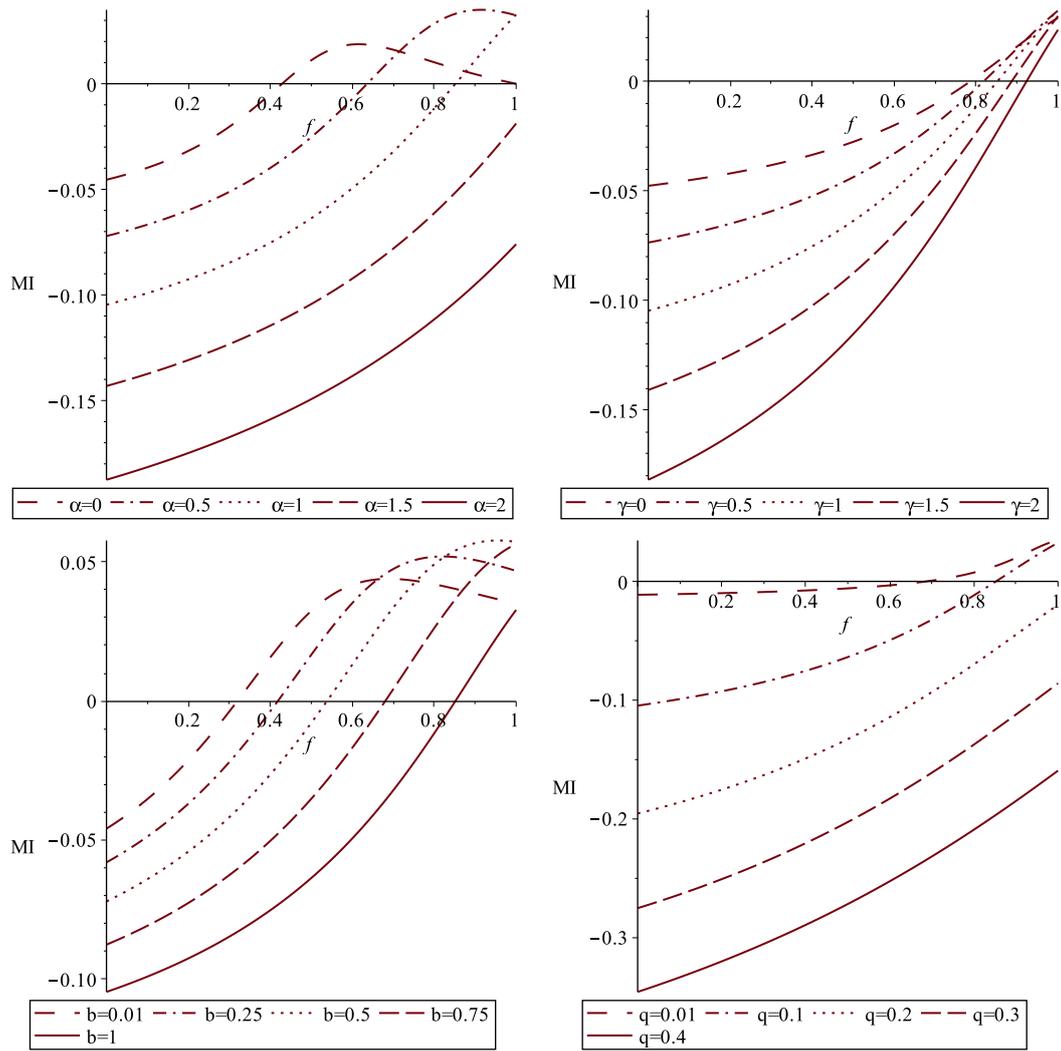


FIGURE A.2. The potential for branching in avoidance. Mutual invasibility (MI), the difference between ES and CS plotted as a function of f for a range of parameter values (as shown in the keys). Parameters are as of figure 1 with singular points chosen at $\beta^* = 2$, $a^* = 2$.

511 analysis shows that it is not possible to achieve a negative MI value for $f = 1$, and therefore
 512 branching still does not occur.

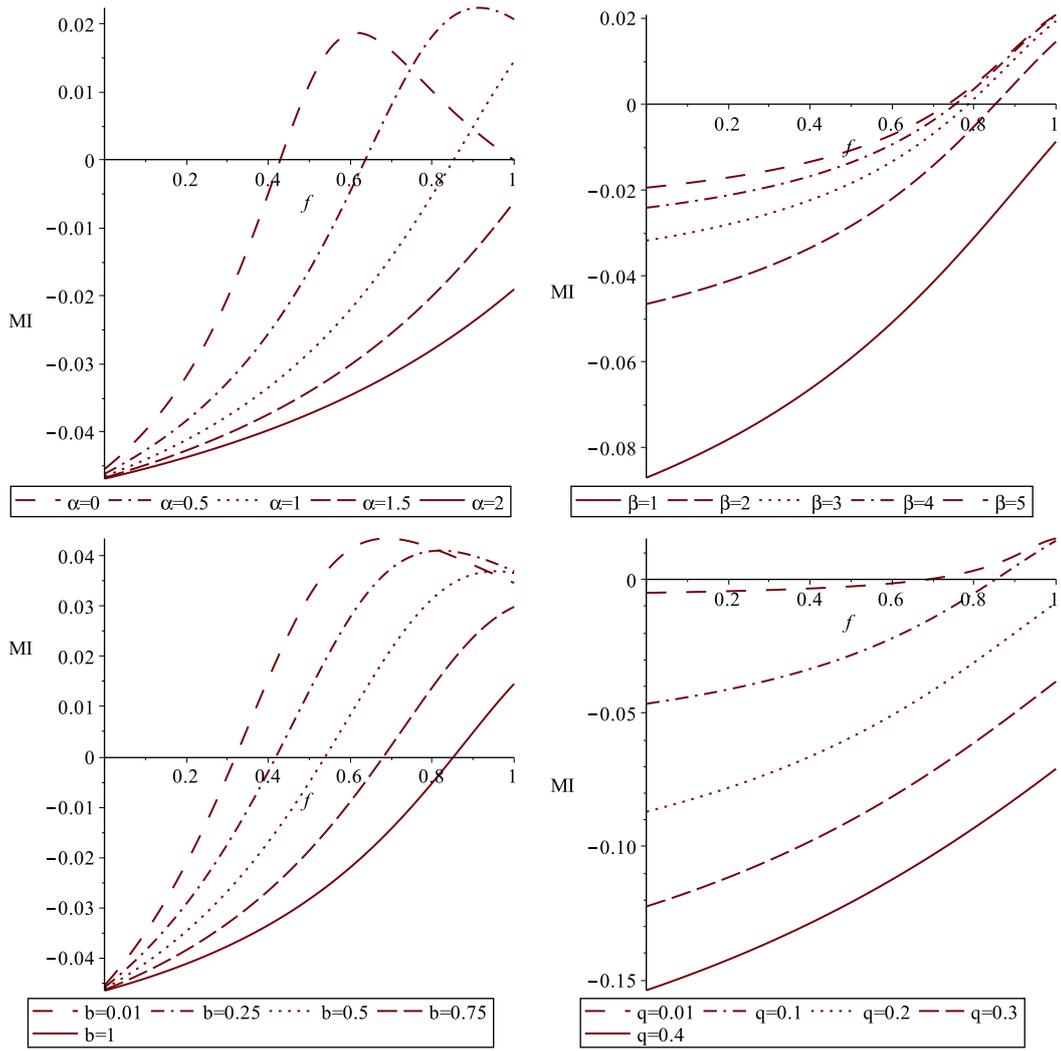


FIGURE A.3. The potential for branching in clearance. Mutual invasibility (MI), the difference between ES and CS plotted as a function of f for a range of parameter values (as shown in the keys). Parameters are as of figure 1 with singular points chosen at $\gamma^* = 2$, $a^* = 2$.