Problem Sheet 1 for Oscillations and Waves

Module F12MS3 **2007-08**

1 Trigonometry revision

(a) Plot the following as a function of t.

$$(i)\ x_1(t) = \sin(t+2), \quad (ii)\ x_2(t) = 3\cos(\pi t), \quad (iii)\ x_3(t) = 2\cos\left(\frac{1}{2}(t-\pi)\right).$$

In each case give the amplitude and period of the function and mark it in your plot

(b) Write the following in the form $R\cos(\omega t - \phi)$, where R, ω, ϕ are real constants which you should determine.

$$(i)\ y_1(t) = \sin(3t) + \sqrt{3}\cos(3t), \quad (ii)\ y_2(t) = \cos(\pi t) - \sin(\pi t).$$

(c) Plot the function $K(t) = \cos^2(2\pi t)$.

(d) Differentiate the functions x_1, x_2, x_3, y_1, y_2 and K given in (a)-(c) with respect to t.

(e) Compute the following indefinite and definite integrals.

$$(i)\ \int_0^{2\pi} \sin(t+2)\,dt, \quad (ii)\ \int \cos\left(\frac{1}{2}(t+\pi)\right)\,dt \quad (iii)\ \int_0^{\frac{1}{2}} \cos^2(2\pi t)\,dt.$$

2 A particle of mass m=1 kg is attached to a spring with spring constant $k = 36$ N m^{-1} and allowed to oscillate freely. The particle's displacement from the equilibrium position, measured in metres, at time t seconds is denoted $x(t)$. It satisfies the equation of motion

$$\ddot{x} + 36x = 0.$$

The initial position of the particle is $x(0) = 1$ and the initial velocity is $\dot{x}(0) = 6$.

(a) Find and plot the position $x(t)$ and the velocity $\dot{x}(t)$ as a function of time $t > 0$.

(b) What is the largest distance from the origin reached by the particle?

(c) Find and plot the kinetic energy and the potential energy as a function of time $t > 0$. Compute the total energy and check that it is constant.

(d) Compute the average kinetic energy and the average potential energy for one cycle of the motion.

3 (*Optional!*) Imagine a tunnel drilled through the centre of the earth, from Spain to New Zealand. The total gravitational force exerted by the earth's matter on a capsule travelling through the tunnel can be computed according to the following rule: *if the capsule is a distance r away from the centre of the earth, the gravitational force it experiences is as if the earth's matter inside the radius r was concentrated at the earth's centre, and the matter outside the radius r was not there at all.*

(a) Find the mass M_E and radius R_E of the earth by typing "mass of earth" and "radius of earth" into google.

(b) Assuming the earth to be a perfect ball with uniform mass density, compute the total mass of matter inside the ball of radius $r < R_E$ as as function of r.

(c) Using the rule given *in italics* above, compute the gravitational force experienced by the capsule when it is a distance r away from the centre of the earth.

(d) Derive the equation governing the fall of the capsule to the centre of the earth, neglecting air resistance and treating the capsule as a point particle.

(e) If the capsule is dropped into the tunnel in Spain with zero initial speed, find a formula for its position at time t seconds later.

(d) Using the google calculator, or otherwise, compute the time it take the capsule to reach the centre of the earth (in minutes), and the capsule's speed (in km/h) at that point? How long does it take til the capsule is in New Zealand, what is the speed when it gets there, and what is its average speed for the journey from Spain to New Zealand?