Problem Sheet 2 for Oscillations and Waves

Module F12MS3 **2007-08**

1 It is found experimentally that a 1 kg mass stretches a spring $\frac{49}{320}$ m.

(a) If the mass is pulled down an additional $\frac{1}{4}$ m and then released with an upward speed of 0.5m s^{-1}, find the subsequent motion. Neglect air resistance and take $g = 9.8$ m/sec^2.

(b) Find the period and amplitude of the motion.

2 The bob of a pendulum performs small oscillations with a period of 4 seconds. Given that the pendulum is situated near the earth's surface, how long is it?

3 Complex number revision.

(a) If $z = -3 + 4i$, express z, its complex conjugate $\bar{z}$ and its reciprocal $1/z$ in modulus-argument form.

(b) Find the moduli and (principal) arguments of $z = -2 - 2i$ and $w = 1 + \sqrt{3}i$. Hence calculate the moduli and arguments of *i)* zw, *ii)* z/w. Draw z, w, zw and z/w in the Argand diagram.

(c) A quadratic equation with real coefficients has $(1 - \frac{1}{2}i)$ as one of its roots. Find the other root and find the simplest such equation with all integer coefficients.

(d) Display the relationships $\cos\theta = (e^{i\theta} + e^{-i\theta})/2$ and $i\sin\theta = (e^{i\theta} - e^{-i\theta})/2$ geometrically by drawing each of the terms in an Argand diagram (Pick a convenient value of θ).

(e) If $C = 5 - 5i$ and the complex-valued function z is given by $z(t) = e^{3it}$ find the real and imaginary parts of $Cz(t)$ and $C\dot{z}(t)$. (Hint: express C in modulus-argument form).

4 Differential equations revision. Find the general solutions of the following differential equations:

(a) $\dfrac{d^2x}{dt^2} + 4\dfrac{dx}{dt} + 4x = 0,$

(b) $\dfrac{d^2x}{dt^2} - 2\dfrac{dx}{dt} + 2x = 0,$

(c) $\dfrac{d^2x}{dt^2} + 4\dfrac{dx}{dt} + 4x = t^2,$

(d) $\dfrac{d^2x}{dt^2} - 4\dfrac{dx}{dt} + 3x = 2e^t,$

(e) $\dfrac{d^2x}{dt^2} + 2\dfrac{dx}{dt} + 5x = \cos(2t).$