Problem Sheet 3 for Oscillations and Waves

Module F12MS3 **2007-08**

1 An object of mass 1 kg is attached to a spring with spring constant 1 N/m, is immersed in a viscous fluid with damping constant 2 Nsec/m and allowed to oscillate freely.

(a) Give the equation governing the object's motion.

(b) At time $t = 0$ the object is lowered $\frac{1}{4}$ m and given an initial velocity of 1 m/sec in the upward direction. Find its subsequent motion.

(c) Show that the object will overshoot its equilibrium position once and then return to equilibrium.

(d) Sketch the position of the object as a function of time.

2 A spring is stretched 2 m by a mass of 2 kg. The mass is immersed in a viscous fluid with damping constant 4 Nsec/m and is acted on by an external force of $4\cos(2t)$ N in the downwards direction. If the object is initially at rest at the equilibrium position find its position at time $t > 0$. (You may assume $g = 10$ m s$^{-}2$).

3 A particle of mass 1 kg is attached to a spring with spring constant 9 N/m and is acted on by an external force $\cos(3t)$ N in the downward direction at time t.

(a) Ignoring air resistance, write down the differential equation describing the motion of the particle and find its general solution.

(b) If the particle is initially in the equilibrium position and at rest show that its displacement at time $t = \dfrac{3\pi}{2}$ sec is $\dfrac{\pi}{4}$ m.

4 An object of mass 1 kg is attached to a spring with spring constant 16 Nm^{-1} and allowed to oscillate freely.

(a) Neglecting air resistance, find the angular frequency and the period of the oscillations.

(b) The object is acted upon by the external force $f_1(t) = \cos 2t$. Give the equation of motion and find its general solution.

(c) The external force is changed to $f_2(t) = \cos 4t$. Find a particular solution of the new equation of motion.

(d) Now both forces f_1 and f_2 act on the object. Using your results from (b) and (c) or otherwise, find a particular solution of the resulting equation of motion.