Problem Sheet 4 for Oscillations and Waves

Module F12MS3 **2007-08**

1 An object of mass 1 kg is attached to a spring with spring constant $k = 4$ Nm^{-1}. The spring is immersed in a viscous liquid with damping constant $r = 1$ N sec m^{-1} and an external force $F(t) = 2\cos\omega t$ N is applied to the mass, where $\omega \geq 0$.

(a) Write down the differential equation governing the motion of the object and find its general solution.

(b) Consider now the steady state solution. Give its amplitude R as a function of ω. Find the frequency $\omega_{\max}$ at which the amplitude $R(\omega)$ attains its maximum value. Sketch the function $R(\omega)$ and mark the value of $\omega_{\max}$ in your diagram.

(c) Define the term resonance and find the value of the amplitude R at the resonance frequency. Mark the resonance frequency in the diagram you made for (b).

2 Consider the same spring and the same external force as in question 1, but now with weaker damping $r_1 = 0.5$ N sec m^{-1} and very weak damping $r_2 = 0.1$ N sec m^{-1} .

(a) For both values of the damping coefficient, find, as a function of ω, (i) the amplitude $R(\omega)$ of the steady state solution and (ii) the phase $\phi(\omega)$ of the steady state solution relative to the external force f.

(b) Using Maple or otherwise, plot R and ϕ as a function of ω in both cases.

(c) Using Maple or otherwise, plot the steady state solutions *at resonance* for both $r_1 = 0.5$ N sec m^{-1} and $r_2 = 0.1$ N sec m^{-1} as a function of time. Plot the driving force as a function of time in the same diagram.

3 An object of mass 4 kg is attached to a spring with spring constant 64 N/m and is acted on by an external force $f(t) = A\cos^3(pt)$ in the downwards direction. Ignoring air resistance, find all values of p at which resonance occurs.

4 Linear algebra revision. Find the eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} a & b \\ b & a \end{pmatrix},$$

where a and b are arbitrary real numbers.

5 Trigonometry revision. Convert the following sums into products of trigonometric functions, and, using Maple or otherwise, plot them against t.

(a) $\psi_1(t) = \cos(2t) + \sin(5t + \frac{\pi}{3})$

(b) $\psi_2(t) = 2\cos(4t + \frac{\pi}{4}) + 2\cos(5t)$