Problem Sheet 5 for Oscillations and Waves

Module F12MS3 **2007-08**

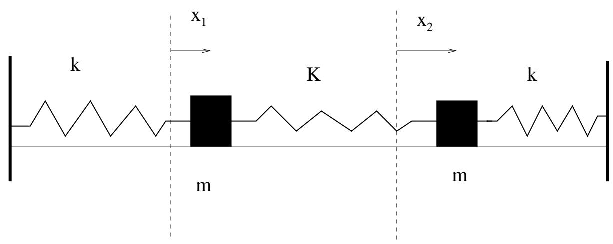

Figure 1: Coupled springs

1 Consider two particles, both of mass $m = 1$ kg , connected by a spring of spring constant $K = 8$ N m^{-1} as shown in the figure. The particles lie on a horizontal plane and are connected to walls by springs with spring constant $k = 9$ N m^{-1}. Their motion is restricted to a line containing all three springs.

(a) Give the equations of motion for the displacements x_1 and x_2 of the particles from the equilibrium position.

(b) Find the normal modes and normal angular frequencies of the system.

(c) Particle one is initially displaced 1 m to the right and particle two is at the equilibrium position. Then both particles are released from rest. Find the displacements x_1 and x_2 at time $t > 0$.

(d) Now suppose that both particles are initially at the equilibrium position. Particle two is given an initial speed 5 m/s to the right, and particle one is initially at rest. Find the displacements x_1 and x_2 at time $t > 0$ in this case.

2 Consider the two particles in the figure above, both of mass m, connected to walls by a spring of spring constant k and connected to each other with a spring of spring constant K.

(a) Write down the equations of motion for the diplacements x_1 and x_2 of the particles.

(b) Hence show that the total energy

$$E = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 + \frac{1}{2}kx_1^2 + \frac{1}{2}kx_2^2 + \frac{1}{2}K(x_1 - x_2)^2$$

is conserved during the motion of the particles.

(c) Express the total energy in terms of the (normalised) normal modes

$$y_1 = \frac{1}{\sqrt{2}}(x_1 + x_2) \quad y_2 = \frac{1}{\sqrt{2}}(x_1 - x_2)$$

and interpret the result.

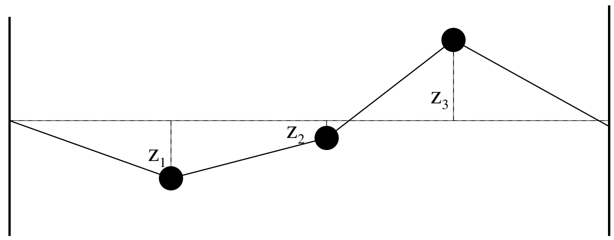

Figure 2: Three beads on a string

3 Consider three identical beads on a flexible string whose endpoints are connected to walls. Each bead has a mass of 20 grams. The tension in the string is $\tau = 1$N and the beads are separated from each other and the walls by a distance of $l = 0.5$ m.

(a) Give the equations of motion for the transverse displacements z_1, z_2 and z_3 of the beads

(b) Use the eigenvector method to find the normal modes and the normal angular frequencies of the system. Describe each mode qualitatively.

4 Consider the function

$$f(x,t) = \sin(kx)\cos(\omega t),$$

where ω and k are real constants. Show that f satisfies the equation

$$\frac{\partial^2 f}{\partial t^2} = c^2 \frac{\partial^2 f}{\partial x^2}$$

where c is related to ω and k in a way you should determine.