Problems for Quantum Computing: week 3

Module F14ZD1 **2007-08**

1. Eigenvalues and eigenvectors

Find the eigenvalues and an orthonormal basis of eigenvectors for the observable

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

used in Example 3.3.1 of the lectures.

2. Measurements and expectation values

A system with Hilbert space $V = \mathbb{C}^3$ is in the state $|\psi\rangle = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. The observable

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

is measured.

(a) Show that the possible outcomes of the measurement are 1 and -1 and compute the probability of each.

(a) Find the expectation value and standard deviation of A in the state $|\psi\rangle$.

3. Spectral decomposition

(a) Find the eigenvalues λ_1, λ_2 and normalised eigenvectors $|v_1\rangle, |v_2\rangle$ of each of the Pauli matrices σ_1, σ_2 and σ_3.

(b) For each Pauli matrix compute the projectors $P_1 = |v_1\rangle\langle v_1|$ and $P_2 = |v_2\rangle\langle v_2|$ and check that each Pauli matrix agrees with its spectral decomposition $\lambda_1 P_1 + \lambda_2 P_2$.