Problems for Quantum Computing: week 7

Module F14ZD1 **2007-08**

1. **Measurement using density operators**

Consider a system with Hilbert space $V = \mathbb{C}^3$, equipped with the canonical inner product. The observable

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

is measured when the system is in the state with density operator

$$\rho = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}.$$

(a) Show that the possible outcomes in the measurement of A are -1 and 1.

(b) What is the probability of obtaining the outcome -1 in a measurement of A, given that the system is in the state ρ at the time of the measurement?

(c) Find the expectation value and standard deviation of A in the state described by ρ.

(d) If the outcome 1 is measured, find the density operator of the system after the measurement. (Give your answer as a 3×3 matrix.)

2. **Entangled states**

Show, by an elementary argument, that the so-called Bell state

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

is an entangled state in $\mathbb{C}^2 \otimes \mathbb{C}^2$.

3. **Tensor products**

(a) Compute the tensor products $X \otimes Y$ and $Y \otimes X$ for the the two matrices $X = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ and $Y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(b) Show that the tensor product of two projection operators is a projection operator.

(c) Show that the tensor product of two Hermitian operators is an Hermitian operator.

(d) Show that the tensor product of two unitary operators is a unitary operator.

continued overleaf

4. **Entangled operators**

Show that the matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

is not of the form $M \otimes N$ for 2×2 matrices M and N.