FLUX INTERACTIONS ON STACKED JOSEPHSON JUNCTIONS

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Abstract

Perturbation methods are used to study the dynamics of locked fluxon modes on stacked Josephson junctions and single crystals of certain high-T_c superconductors. Two limiting cases are considered: (i) The *nonlinear diffusion* regime in which fluxon dynamics are dominated by energy exchange between the bias and loss parameters, and (ii) The *propagating* regime in which the interplay between magnetic and electric field energies governs the fluxon dynamics. Conditions for stability of locked fluxon modes are shown to be different in these two regimes.

INTRODUCTION

Recently there have been several theoretical, numerical, and experimental studies of the dynamics of fluxon interactions on "stacked" structures of long Josephson junctions. In such systems, one junction is placed directly above another with a separating layer that is thin enough (compared with the London penetration depth) to permit electromagnetic coupling between the fluxon motions on the two junctions [1, 2, 3, 4, 5, 6, 7, 8]. The dynamic description of such a system is closely related to that of certain single crystals of high-T_c superconductors. In $Bi_2Sr_2CaCu_2O_{x+8}$, for example, the superconductivity is confined to two dimensional bilayers of CuO_2 that are separated by insulating regions of Bi_2O_3 and SrO, which constitute a Josephson junction [9, 10].

The aim of this note is to indicate how perturbation methods [11] can be used to study the locking of fluxon motion on two or more junctions. The spirit of the approach is not new; it was used previously to study interactions between electrochemical voltage pulses (or "action potentials") on parallel nerve fibers [12].

GENERAL ANALYSIS

The analysis presented here is of the coupled, perturbed sine- Gordon system [1]

$$\begin{pmatrix} \phi_{1,xx} \\ \phi_{2,xx} \end{pmatrix} = \begin{pmatrix} 1 & -\Delta \\ -\Delta & 1 \end{pmatrix} \begin{pmatrix} \phi_{1,tt} + \sin\phi_1 - \gamma_1 + \alpha\phi_{1,t} - \beta\phi_{1,xxt} \\ \phi_{2,tt} + \sin\phi_2 - \gamma_2 + \alpha\phi_{2,t} - \beta\phi_{2,xxt} \end{pmatrix}, \tag{1}$$

with variables and parameters normalized as in [2] and defined as follows:

- $-\infty < x < +\infty$ and $t > 0$ are normalized longitudinal position along the junctions and time.
- $\phi_1(x,t)$ and $\phi_2(x,t)$ are the differences in superconducting phases across junctions 1 and 2, respectively. (They are identical to ϕ^A and ϕ^B in reference [2].)
- γ_1 and γ_2 are transverse bias currents.
- $\alpha\phi_{1,t}$ and $\alpha\phi_{2,t}$ are loss terms arising from normal electron flow across the junctions.

- $\beta\phi_{1,xxt}$ and $\beta\phi_{2,xxt}$ are surface loss terms [13].
- Δ is a *positive* electromagnetic coupling parameter, which is equal to $-S$ defined in reference [2].

To study synchronized (or locked) fluxon motion, we look for traveling wave solutions of the form

$$\phi_k(x,t) = \Phi_k(x-ut) = \Phi_k(\xi)\,, \tag{2}$$

where $k = 1,\,2$ and $\xi \equiv x - ut$. Such a solution can represent two pulses, one on each junction, both moving with the same speed.

In the analysis sketched here, no restrictions are placed on the magnitudes of the parameters α, β, or γ_k, but the electromagnetic coupling parameter (Δ) is selected as the small parameter for a perturbation analysis. Thus Φ_1 and Φ_2 are expressed as power series in Δ:

$$\Phi_k = \Phi_{k0} + \Delta\Phi_{k1} + \Delta^2\Phi_{k2} + \cdots, \tag{3}$$

where $k = 1,\ 2$. Provisionally assuming different fluxon velocities on the two junctions to first order in Δ permits one also to write:

$$u^{(k)} = u_0 + \Delta u_1^{(k)} + \Delta^2 u_2^{(k)} + \cdots \tag{4}$$

Substituting these power series into Equations (1) and equating terms of order Δ^0 gives the pair of zero order equations

$$(1-u_0^2)\Phi_{k0,\xi\xi} - \sin\Phi_{k0} = \gamma_k - u_0(\alpha\Phi_{k0,\xi} - \beta\Phi_{k0,\xi\xi\xi})\,. \tag{5}$$

Equating terms of order Δ gives the two first order equations

$$L_k\Phi_{k1} = u_1^{(k)}(2u_0\Phi_{k0,\xi\xi} - \alpha\Phi_{k0,\xi} + \beta\Phi_{k0,\xi\xi\xi}) - \Phi_{\bar{k}0,\xi\xi}\,, \tag{6}$$

where $\bar{k} = 2,\ 1$, and

$$L_k \equiv (1-u_0^2)\frac{d^2}{d\xi^2} - \cos\Phi_{k0} + u_0(\alpha\frac{d}{d\xi} - \beta\frac{d^3}{d\xi^3}) \tag{7}$$

are linear operators.

It is helpful to use the *Fredholm alternative theorem* to insure solvability of the perturbation equations at order Δ and higher [14, 11]. To see how this goes, define first an inner product as

$$(v,w) \equiv \int_{-\infty}^{\infty} v(\xi)w(\xi)d\xi\,,$$

and the *adjoint*—$L^\dagger$—of operator L as

$$(Lv, w) = (v, L^\dagger w) \, .$$

Thus

$$L_k^\dagger = (1 - u_0^2)\frac{d^2}{d\xi^2} - \cos\Phi_{k0} - u_0(\alpha\frac{d}{d\xi} - \beta\frac{d^3}{d\xi^3}) \, . \tag{8}$$

Then a necessary and sufficient condition for Equations (6) to be solvable is that their right-hand-sides be orthogonal to the null spaces of the corresponding adjoint operators [14]. In effect, the first order corrections to the traveling wave velocities ($u_1^{(1)}$ and $u_1^{(2)}$) are obtained by requiring that Equations (6) have solutions. Computationally, this condition is satisfied by solving

$$L_k^\dagger y_k(\xi) = 0 \tag{9}$$

and setting the inner products of the right-hand-sides with the y_k to zero.

Applying this condition to Equations (6), requires that

$$u_1^{(1)} = \frac{1}{N_1}\int_{-\infty}^{\infty} y_1(\xi)\Phi_{20,\xi\xi}(\xi)d\xi \tag{10}$$

and

$$u_1^{(2)} = \frac{1}{N_2}\int_{-\infty}^{\infty} y_2(\xi)\Phi_{10,\xi\xi}(\xi)d\xi \, , \tag{11}$$

where

$$N_1 \equiv \int_{-\infty}^{\infty} y_1(2u_0\Phi_{10,\xi\xi} - \alpha\Phi_{10,\xi} + \beta\Phi_{10,\xi\xi\xi})d\xi \tag{12}$$

and

$$N_2 \equiv \int_{-\infty}^{\infty} y_2(2u_0\Phi_{20,\xi\xi} - \alpha\Phi_{20,\xi} + \beta\Phi_{20,\xi\xi\xi})d\xi \, . \tag{13}$$

To see how Equations (10) through (13) work, suppose that $\Phi_{10}(\xi)$ and $\Phi_{20}(\xi)$ represent two pulses of the same shape traveling in the $+\xi$ direction with $\Phi_{20}(\xi)$ *ahead* of $\Phi_{10}(\xi)$ by a certain distance δ. Then

$$\Phi_{20}(\xi) = \pm\Phi_{10}(\xi - \delta) \tag{14}$$

and

$$y_2(\xi) = y_1(\xi - \delta) \, , \tag{15}$$

where the "+" and "−" signs in Equation (14) correspond respectively to solutions that are *symmetric* and *antisymmetric* with respect to the transverse direction.

From Equations (12) and (13) it is seen that $N_1 = N_2 \equiv N$ for symmetric solutions and $N_1 = -N_2 \equiv N$ for antisymmetric solutions. Since the definitions of N_1 and N_2 are independent of the pulse separation δ, Equations (10) and (11) can be written in the following forms:

- Symmetric solutions:

$$u_1^{(1)}(\delta) = u_1^{(2)}(-\delta) = \frac{1}{N}\int_{-\infty}^{\infty} y_1(\xi+\delta)\Phi_{10,\xi\xi}(\xi)d\xi\,. \tag{16}$$

- Antisymmetric solutions:

$$u_1^{(1)}(\delta) = u_1^{(2)}(-\delta) = -\frac{1}{N}\int_{-\infty}^{\infty} y_1(\xi+\delta)\Phi_{10,\xi\xi}(\xi)d\xi\,. \tag{17}$$

One necessary condition for locking the pulse motion on the two junctions is evidently $u_1^{(1)} = u_1^{(2)}$, but this requirement is not sufficient because one must also insure that the synchronized motion is *stable.* The question of stability must be approached differently in two limiting cases: the nonlinear diffusion regime in which fluxon dynamics are governed by energy exchange between the bias and loss parameters, and the propagating regime in which the interplay between magnetic and electric field energies dominates the fluxon dynamics. Let us consider these two cases separately.

(i) *The nonlinear diffusion regime*

To deal with this limiting case, we define δ_0 as a pulse spacing that satisfies

$$u_1^{(1)}(\delta_0) = u_1^{(2)}(\delta_0)\,.$$

This solution is *unstable* if

$$u_1^{(1)}(\delta_0+\epsilon) < u_1^{(2)}(\delta_0+\epsilon)\,,$$

where ϵ is an incremental increase in the spacing between the two pulses. In simple terms, the synchronized motion is unstable if the leading pulse (pulse number 2) goes faster as the pulse spacing increases. Clearly, then, a necessary condition for stable locking of fluxon motion in the nonlinear diffusion regime is

$$\frac{du_1^{(1)}}{d\delta} > 0\,, \tag{18}$$

at the value of δ for which $u_1^{(1)} = u_2^{(2)}$.

If the unperturbed magnetic flux is stationary, electric field energy is zero and one is always in the nonlinear diffusion regime. Then $u_0 = 0$ in Equations (5) and (8) so

$$y_k(\xi) = \frac{d\Phi_{k0}}{d\xi}\,,$$

and

$$N = -\int_{-\infty}^{\infty}\left[\alpha(\Phi_{10,\xi})^2 + \beta(\Phi_{10,\xi\xi})^2\right] d\xi\,.$$

Evidently $u_1^{(1)}(\delta) = u_1^{(2)}(\delta)$ for $\delta = 0$, and near this value of pulse separation

$$u_1^{(1)}(\delta) - u_1^{(2)}(\delta) = \mp 2\delta \frac{\int_{-\infty}^{\infty}(\Phi_{10,\xi\xi})^2 d\xi}{\int_{-\infty}^{\infty}\left[\alpha(\Phi_{10,\xi})^2 + \beta(\Phi_{10,\xi\xi})^2\right] d\xi}, \tag{19}$$

where the upper sign corresponds to the symmetric solution (the upper sign in Equation (14)), and the lower sign corresponds to the antisymmetric solution.

From Equation (19), we see that the symmetric static case is unstable because $u_1^{(1)} < u_1^{(2)}$ for $\delta > 0$, which implies that the leading pulse (number 2) increases speed as it moves ahead. For the antisymmetric static case, the necessary condition for stability given in Equation (18) is satisfied. These results are in agreement with recent numerical studies of Equations (1) [2].

(ii) *The propagating regime*

In this limiting case, losses are small, and the electric field energy is about equal to the magnetic field energy. The two pulses are electromagnetic waves, and the local temporal behavior has been described for single fluxons by Grønbech-Jensen, Cai and Samuelsen [3].

To investigate this limiting case, we return to Equations (1) and set α, β, γ_1, and γ_2 equal to zero. To first order in Δ, these equations have an electromagnetic energy density

$$\mathcal{H}_{\rm em} = \frac{1}{2}\left(\phi_{1,x}^2 + \phi_{1,t}^2 + \phi_{2,x}^2 + \phi_{2,t}^2\right) - \Delta\phi_{1,t}\phi_{2,t}\,. \tag{20}$$

Near $\delta = 0$, solutions of Equations (16) or (17) are of the form

$$u_1^{(1)} \doteq a + b\delta$$

and

$$u_1^{(2)} \doteq a - b\delta\,,$$

where a and b are real constants. Thus to first order in Δ, the total electromagnetic energy is

$$\int_{-\infty}^{\infty} \mathcal{H}_{\rm em}dx \doteq \text{const.} \pm \frac{1}{2}\Delta\delta^2 u_0^2 \int_{-\infty}^{\infty} \Phi_{10,\xi\xi}^2 d\xi\,, \tag{21}$$

where the upper (lower) sign corresponds to a symmetric (antisymmetric) solution.

Equation (21) gives—to order Δ—the δ dependence of the total electromagnetic energy of the two pulses in the propagating regime. We see that the symmetric case is stable because this energy increases with δ. (Think of the electromagnetic energy as an elastic force—or rubber band—that is holding the two pulses together.) In the antisymmetric case, on the other hand, the electromagnetic energy decreases with δ so the synchronized pulses are unstable.

As an experimentalist starts up the current-voltage characteristic of a stacked Josephson junction pair from zero bias currents ($\gamma_1 = 0$ and $\gamma_2 = 0$), the fluxons will at first move slowly to establish a balance between power input from the bias currents and dissipative effects. When the fluxons move slowly, the electric field that is generated by their motion is small enough to be neglected. As the bias currents are increased, however, the fluxons move faster and their magnetic fields induce ever larger electric fields. Finally, the electric field energy comes into a dynamic balance with the magnetic field energy, and the behavior becomes essentially propagatingin nature, with dissipative and quantum effects being but small perturbations. This is the propagating regime, and the conditions for the stability of coupled fluxon motion change because the qualitative nature of the dynamics has changed.

Why should we ignore the stability implications of Equations (16) and (17) in the propagating regime? Because these equations were derived to express the special case of *steady state* corrections to the pulse speeds. From Equations (1), it is evident that if δ is instantly displaced from zero in the nonlinear diffusion regime, there is a time delay before the balance between power input from the bias terms and dissipation is reestablished, and this delay is long compared with the time constants for the propagating regime [11]. In the propagating regime, Equations (1) imply that a small displacement of δ from an equilibrium value leads either to an oscillation with zero average value or to exponential growth, depending upon the sign on the last term in Equation (21) [3]. Conclusions concerning stability in the propagating regime that are drawn from Equations (1) must supersede those drawn from equations (16) and (17) because Equations (1) are not limited by the steady state approximation.

To decide whether a given solution is in the propagating regime, we look again at Equations (1). If $\phi_{1,tt}$ and $\phi_{2,tt}$ dominate the right hand sides, then one is in the propagating regime. A convenient test is to examine the ratio

$$R = \frac{u^2\sqrt{|1-u^2|}}{|1-u^2|^{3/2} + \alpha u|1-u^2| + \beta u}\,, \tag{22}$$

and note that $R \gg 1$ implies the propagating regime.

A NUMERICAL EXAMPLE

Here we present some numerical results to show that the computations sketched in the previous section are feasible. As fixed parameters we choose $\alpha = 0.1$ and $\beta = 0$ because these correspond to those of an extensive numerical investigation of Equation (1) [2]. Assuming a single fluxon on each junction and setting $\gamma = 0.35$ Equations (5) have the solution $\Phi_0(\xi)$, the derivative of which is plotted in Figure 1. The corresponding velocity is

$$u_0 = 0.941\,869\,544\,766\,475.$$

Analytically, $\Phi_0(\xi)$ is the separatrix between the oscillating and rotating solutions of Equations (5). The calculation of so many significant digits in the value of u_0 is necessary because of the strong dependence of the solution with small variations of that parameter. Note that this results also in a very sensitive dependence of the behavior of $\Phi_0(\xi)$ on the numerical roundoff errors.

From $\Phi_0(\xi)$ and u_0, Equation (9) is solved for $y(\xi)$, which is the null vector of the adjoint operator $L^\dagger$ defined in Equation (8). We also plot $y(\xi)$ in Figure 1 and note that the vertical scale of this plot is arbitrary because y is the solution of a homogeneous linear equation.

From Equations (10) through (13), we next assume a *symmetric* solution ($\Phi_{10} = \Phi_{20}$) and compute $u_1^{(1)}(\delta)$ and $u_1^{(2)}(\delta)$, which are plotted in Figure 2. Evidently near $\delta = 0$

$$u_1^{(1)}(\delta) \doteq 0.4705 - 2.75\delta \,. \tag{23}$$

For $\delta = 0$, $u_1^{(1)} = u_1^{(2)}$, the order Δ estimate of the speed of the locked pulses from Equation (4) is

$$u \doteq u_0 + 0.4705\Delta \,. \tag{24}$$

Were the operation in the nonlinear diffusion regime, Equation (23) would imply instability, but with $\Delta = 0.3$, $R = 5.4$. This indicates the propagating regime so stability is determined from Equation (21). Since the electromagnetic field energy *increases* with δ, this fluxon-fluxon solution is stable. This is in accord with detailed solutions of the pde system of Equations (1) [2].

For an *antisymmetric* solution ($\Phi_{10} = -\Phi_{20}$), a similar procedure leads to

$$u_1^{(1)}(\delta) \doteq -0.4705 + 2.75\delta$$

so

$$u \doteq u_0 - 0.4705\Delta \,, \tag{25}$$

and therefore that $R = 1.6$ at $\Delta = 0.3$. This is close to the propagating regime, where from Equation (21) the electromagnetic field energy *decreases* with increasing δ. Thus the fluxon-antifluxon solution is expected to be marginally unstable, again in agreement with detailed solutions of Equations (1) [2].

CONCLUDING COMMENTS

The above discussion is intended merely as a sketch of how the solvability condition of the Fredholm alternative theorem can be used to study locked fluxon motions on stacked Josephson transmission lines. Several variations on the theme come to mind.

1. *More interesting fluxon configurations.* In the simple example presented above, the motions of single fluxons on two infinitely long junctions were studied, but several more complex starting configurations (Φ_{10} and Φ_{20}) can be readily considered: (i) These might be periodic flux trains described by Jacobi elliptic functions on the infinite x-axis or with periodic boundary conditions appropriate for a ring oscillator [2, 11]. (ii) With a sufficiently large value for the surface loss parameter (β), (Φ_{10} and Φ_{20}) could be the "bunched-fluxon" pulses that were first observed and studied by Wayne Johnson [15, 11, 16]. (iii) Finally, the locked fluxon modes need not be either symmetric or antisymmetric. There might, for example, be a three-fluxon bunch on one junction and a five- antifluxon bunch on the other, with γ_1 and γ_2 appropriately adjusted to synchronize the wave speeds.

2. *Relation to the coupled nerve problem.* The qualitative features of the problem discussed here are close to those of the problem of coupled pulses on parallel nerve fibers, which was studied by Steve Luzader [17, 12]. This similarity is not accidental; with large losses, the Josephson junction equation becomes a nonlinear diffusion system. To appreciate this similarity, the reader should compare Equations (5) and (6) above with corresponding nerve axon equations (Equations (18) and (19) of reference [12]). Thus physical insights from the study of one of these problems may help to understand the other. With surface losses, for example, one expects (stable) locked pulses for $\delta \neq 0$.

3. *Dynamics of pulse interactions.* The above analysis assumes that the pulses on the two junctions travel at the same speed, constituting thereby a traveling wave solution of the entire system. In the perturbation expansion of Equation (4), it was only "provisionally" assumed that $u_1^{(1)} \neq u_1^{(2)}$ in order to introduce flexibility into the analysis. In the nonlinear diffusion regime, however, one can extend the perspective by defining a *velocity difference function*

$$D(\delta) \equiv u_1^{(1)}(\delta) - u_1^{(2)}(\delta)$$

and supposing that $\delta(t)$ is governed by the ordinary differential equation

$$\frac{d\delta}{dt} = -\Delta D(\delta)\,.$$

For parameters that correspond to a typical nerve impulse, this assumption leads to results that are in agreement with numerical integration of the original partial differential equations [18].

4. *More than two Josephson junctions.* In practical applications of stacked Josephson junctions and single crystals of high-T_c superconductors as high frequency oscillators, the coupling of two junctions is viewed only as a first step. N- fold stacks are being seriously considered as a means to increase both the voltage and the output impedance of oscillator structures [1, 10]. Again it is amusing to notice that a similar class of problems arises as the study of *condensed pulse states* on parallel bundles of nerve fibers, which are found throughout the vertebrate nervous systems (for example in the axon bundles of motor nerves and the optic nerve, the spinal chord, parallel fibers of the cerebellum, and in the corpus callosum) [12, 17].

In such problems, the relative displacements between the N pulses must satisfy the obvious constraint

$$\delta_{12} + \delta_{23} + \cdots + \delta_{N1} = 0$$

for any numbering of the uncoupled system. Thus there are $N-1$ independent δ's, which can be conveniently chosen by requiring that the pulse number i is not ahead of pulse number $(i+1)$. In other words, $\delta_{i(i+1)} \geq 0$ for all $N-1$ independent values of i.

Proceeding as above, one can compute $N-1$ differences of the corresponding first order velocity corrections as

$$D_i = u_1^{(i)} - u_1^{(i+1)} ; \quad i = 1, 2, \ldots, N-1 .$$

Then $\bar{D} \equiv (D_1, D_2, \ldots, D_{N-1})$ is an $N-1$ dimensional vector function that is defined on the $N-1$ dimensional "δ-space": $\bar{\delta} \equiv (\delta_{12}, \delta_{23}, \ldots, \delta_{(N-1)N})$.

With these definitions, a necessary condition for a condensed pulse state to exist is that $\bar{\delta}$ must be a zero of $\bar{D}(\bar{\delta})$. A necessary condition for such a pulse state to be stable is

$$\frac{\partial D_i}{\partial \delta_{i(i+1)}}|_{\bar{D}=\bar{0}} > 0$$

for all $N-1$ values of i. Such considerations may be useful for the design of stacked Josephson junction oscillators.

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References

[1] S. Sakai, P. Bodin, and N.F. Pedersen, *J. Appl. Phys.* **73** (1993) 2411–2418.

[2] A. Petraglia, A.V. Ustinov, N.F. Pedersen, and S. Sakai, *J. Appl. Phys.* **77** (1995) 1171–1177.

[3] N. Grønbech-Jensen, D. Cai, and M.R. Samuelsen, *Phys. Rev. B* **48** (1993) 16160–16163.

[4] P. Barbara and G. Costabile, *Physica B* **194–196** (1994) 69–70.

[5] P. Barbara, G. Costabile, J. Mygind, and N.F. Pedersen, *Physica B* **194–196** (1994) 71–72.

[6] P. Barbara, A. Ustinov, and G. Costabile, *Physics Letters A* **191** (1994) 443–448.

[7] P. Barbara, J. Mygind, N.F. Pedersen, G. Costabile, A. D'Anna, R.D. Parmentier, and B.A. Malomed, "Fluxon dynamics in stacked long Josephson junctions: Experiments and numerical simulations," Proc. Capri Conference on Superconductivity, September 1994.

[8] P. Barbara, "Fluxon dynamics in single and stacked Josephson tunnel junctions," Ph.D. Thesis, Technical University of Denmark, 1995.

[9] R. Kleiner, G. Kunkle, and P. Müller, *Phys. Rev. Lett.* **68** (1992) 2394–2397.

[10] R. Kleiner, P. Müller, H. Kohlstedt, N.F. Pedersen, and S. Sakai, *Phys. Rev. B* **50** (1994) 3942–3952.

[11] D.W. McLaughlin and A.C. Scott, *Phys. Rev. A* **18** (1978) 1652–1680.

[12] A.C. Scott and S.D. Luzader, *Physica Scripta* **20** (1979) 395–401.

[13] A.C. Scott, *Solid-State Electronics* **7** (1964) 137–146.

[14] I. Stackgold, *Green's Functions and Boundary Value Problems,* Wiley-Interscience (1979).

[15] W.J. Johnson, "Nonlinear wave propagation on superconducting tunnel junctions," Ph.D. Thesis, University of Wisconsin, 1968.

[16] P.S. Lomdahl, O.H. Sørensen, and P.L. Christiansen, *Phys. Rev. B* **25** (1982) 5737–5748.

[17] S.D. Luzader, "Neurophysics of parallel nerve fibers," Ph.D. Thesis, University of Wisconsin, 1979.

[18] J.C. Eilbeck, S.D. Luzader, and A.C. Scott, *Bulletin of Math. Biol.* **43** (1981) 389– 400.

FIGURE CAPTIONS

Figure 1. $d\Phi_0(\xi)/d\xi$ is the derivative of a single fluxon solution of Equation (5) with $\alpha = 0.1$, $\beta = 0$, and $\gamma = 0.35$. $y(\xi)$ is the corresponding solution of Equation (9). (Note that the scale of y is arbitrary.) For both curves, $u_0 = 0.941\,869\,544\,766\,475$.

Figure 2. Order Δ corrections to the steady state single fluxon speeds for symmetric solutions ($\Phi_{10} = \Phi_{20}$) as a function of the pulse spacing (δ).

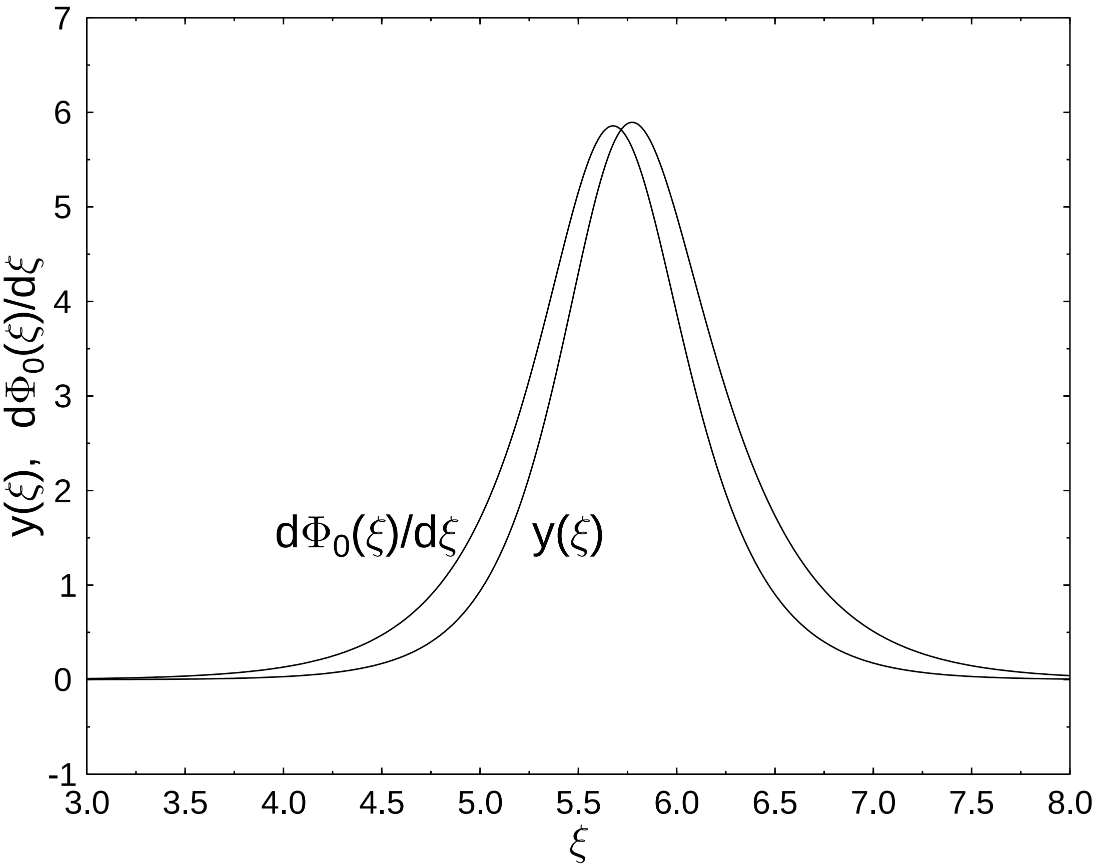

fig.1

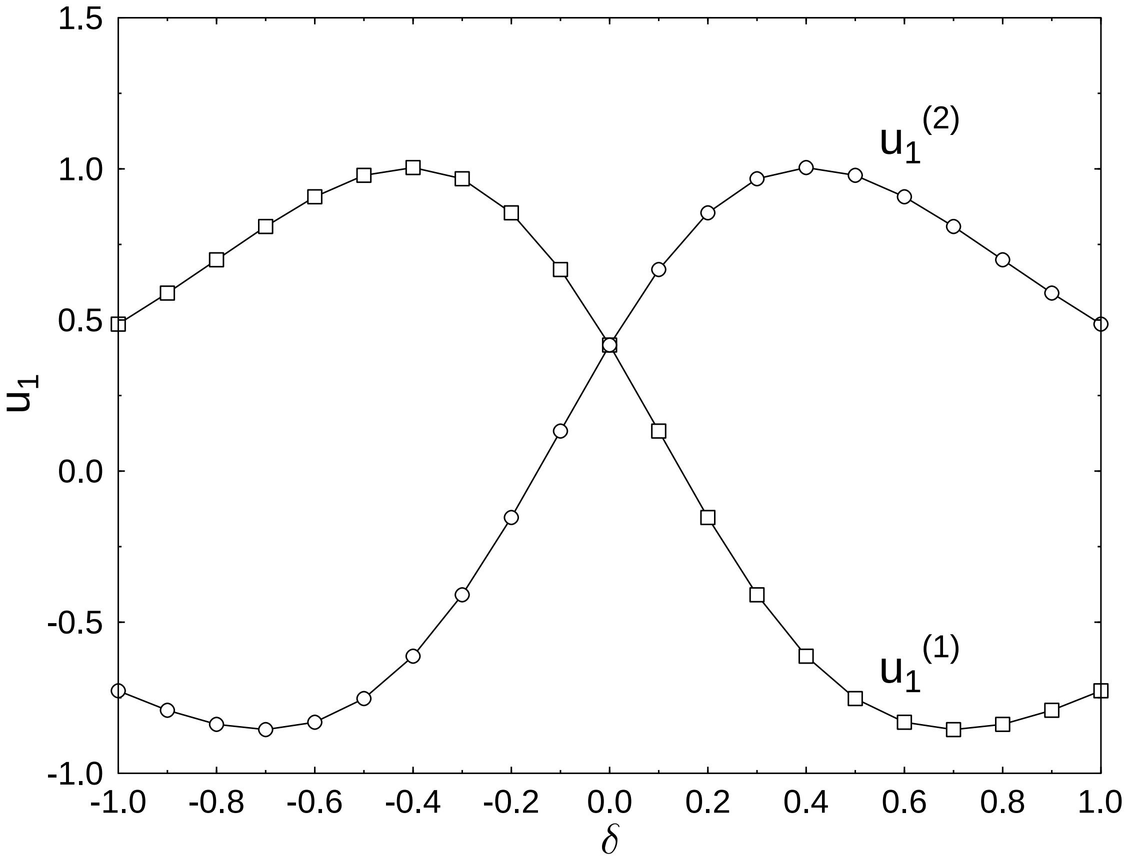

fig.2