Beyond the Diffusion Approximation

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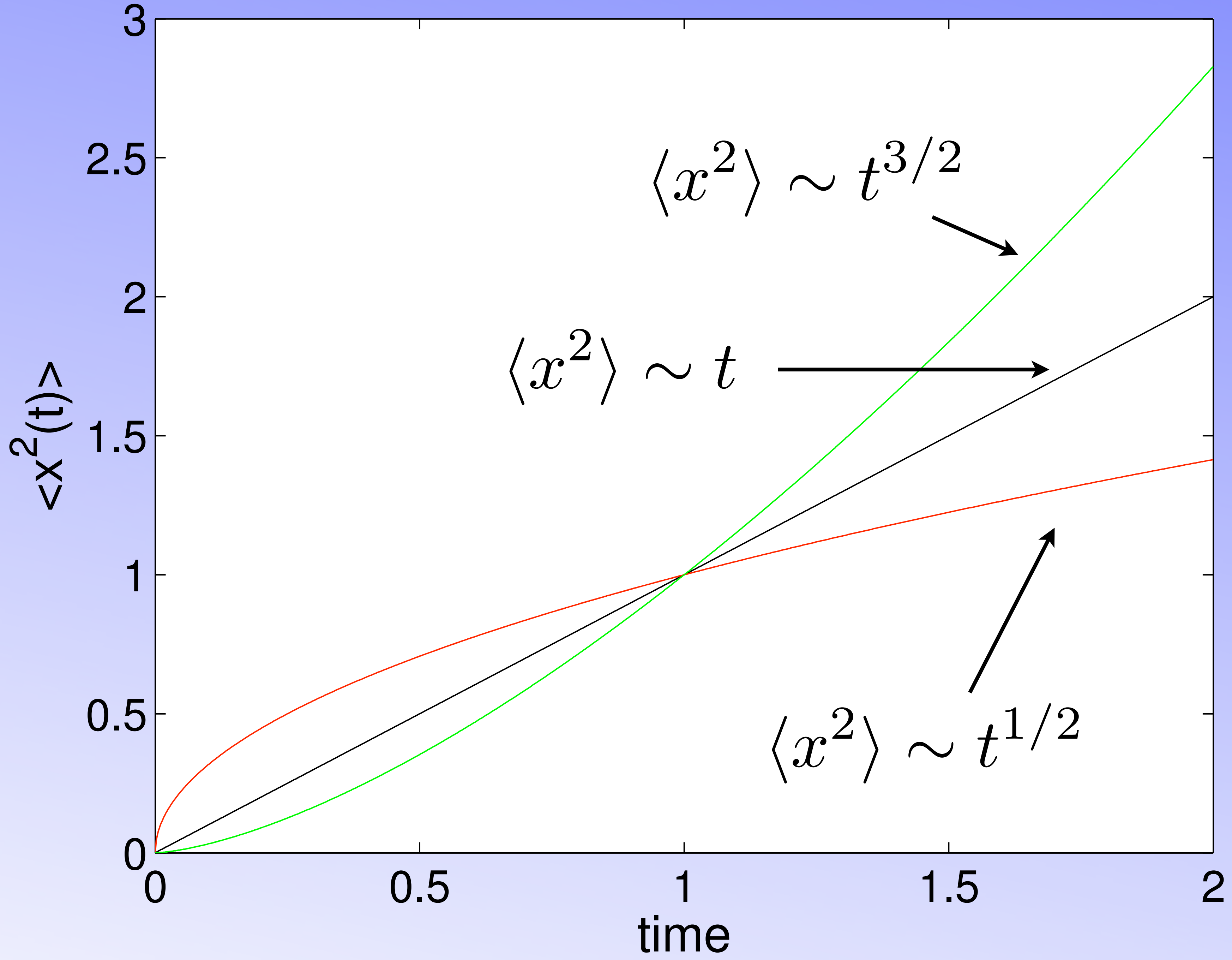
Spatio-temporal and network modelling of diseases

LUCA GIUGGIOLI
Center for Biocomplexity & Department of Ecology and Evolutionary Biology
Princeton University

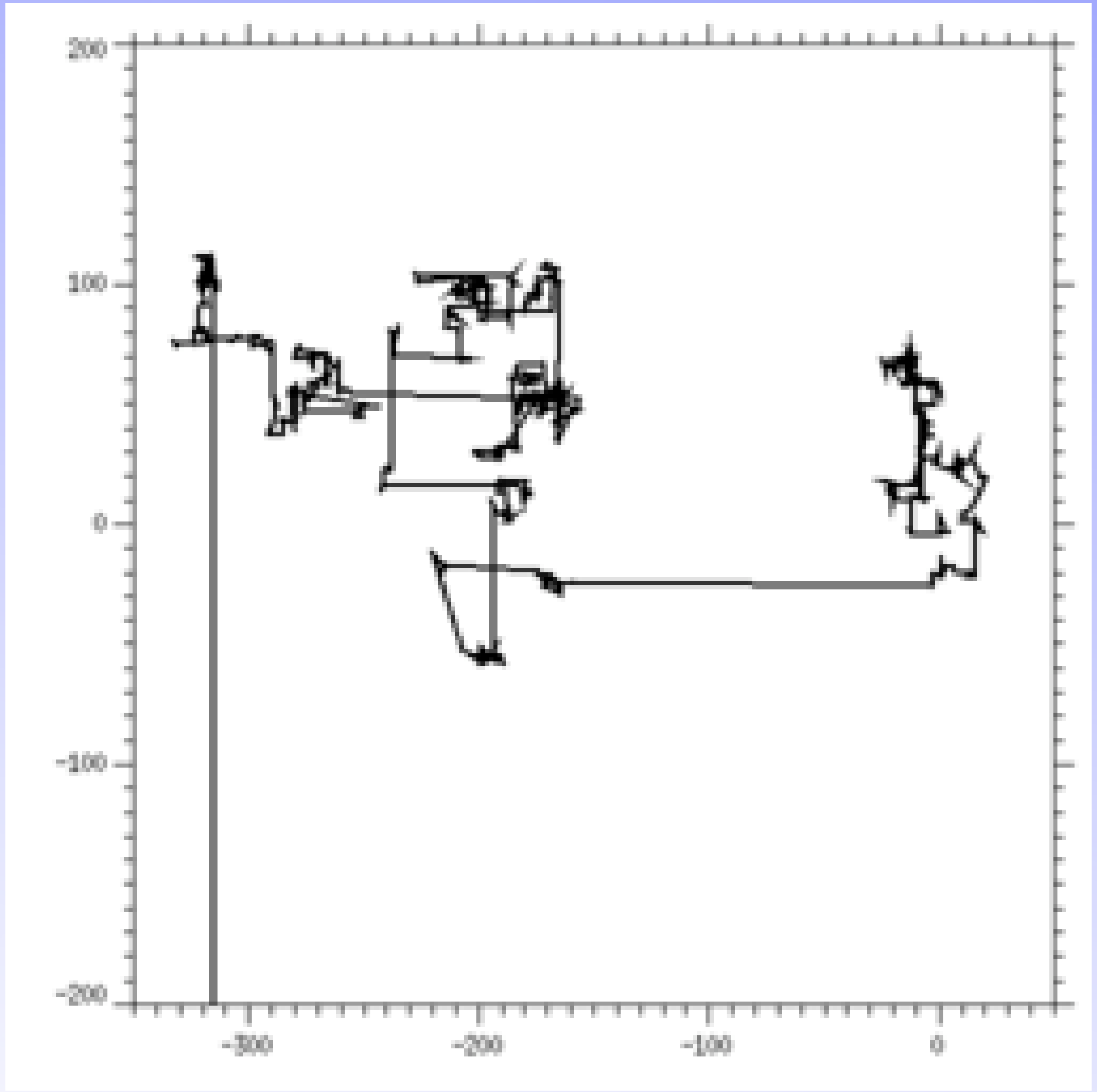
ANOMALOUS DIFFUSION

$$\langle x^2 \rangle \propto t^\delta$$

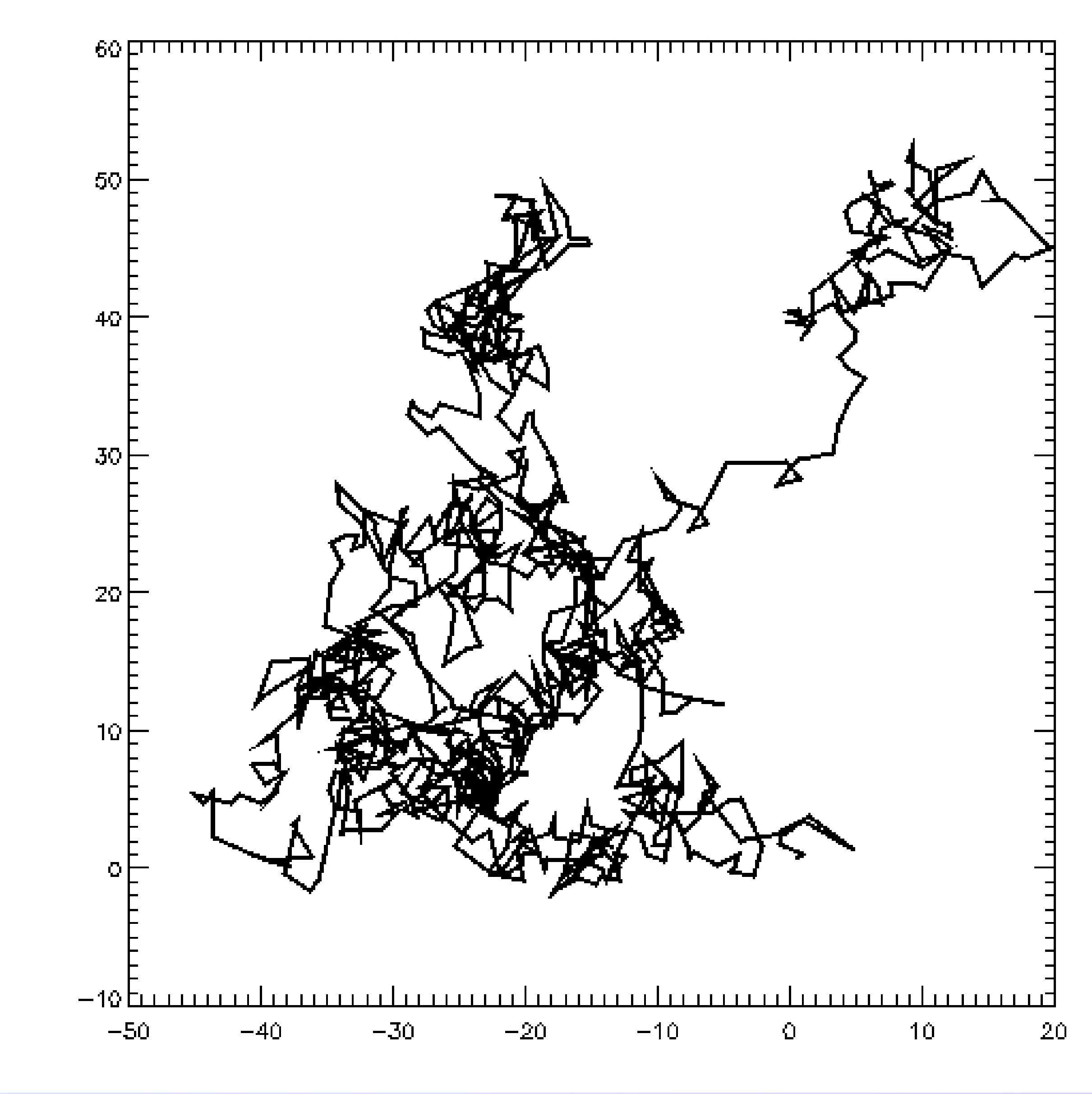
ANOMALOUS DIFFUSION



Superdiffusion



Brownian diffusion



GENERALIZATION OF THE DIFFUSION EQUATION

$$\frac{\partial P(x,t)}{\partial t} = \frac{\partial P(x,t)}{\partial x^2}$$

$$\frac{\partial P(x,t)}{\partial t} = \int_{-\infty}^{+\infty} dx' \, \mathcal{W}(x - x') \, P(x', t)$$

$$\frac{\partial P(x,t)}{\partial t} = \int_0^t ds \, \phi(t - s) \, \frac{\partial^2 P(x,s)}{\partial x^2}$$

$$\frac{\partial P(x,t)}{\partial t} = \int_{-\infty}^{+\infty} dx' \, \mathcal{W}(x - x') \int_0^t ds \, \phi(t - s) \, P(x', s)$$

GENERALIZED MASTER EQUATION APPROACH

$$\frac{\partial P(x,t)}{\partial t} = \int_0^t ds\, \phi(t-s)\, \frac{\partial^2 P(x,s)}{\partial x^2}$$

$$\frac{d\left\langle x^2 \right\rangle}{dt} = 2\int_0^t \phi(s)$$

Correlated Random Walk

$$\phi(t) = e^{-\alpha t}$$

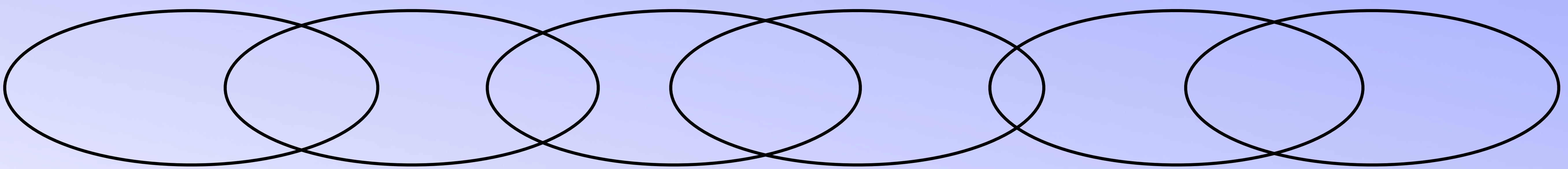
$$\frac{\partial^2 P(x,t)}{\partial t^2} = -\alpha \frac{\partial P(x,t)}{\partial t} + \lambda \frac{\partial^2 P(x,t)}{\partial x^2}$$

GENERALIZED MASTER EQUATION APPROACH

$$\frac{\partial P(x,t)}{\partial t} = \int_{-\infty}^{+\infty} dx' \, \mathcal{W}(x-x') \int_0^t ds \, \phi(t-s) \, P(x',s)$$

$$P(k,\epsilon) = \frac{1}{\epsilon + \mathcal{W}(k)\phi(\epsilon)}$$

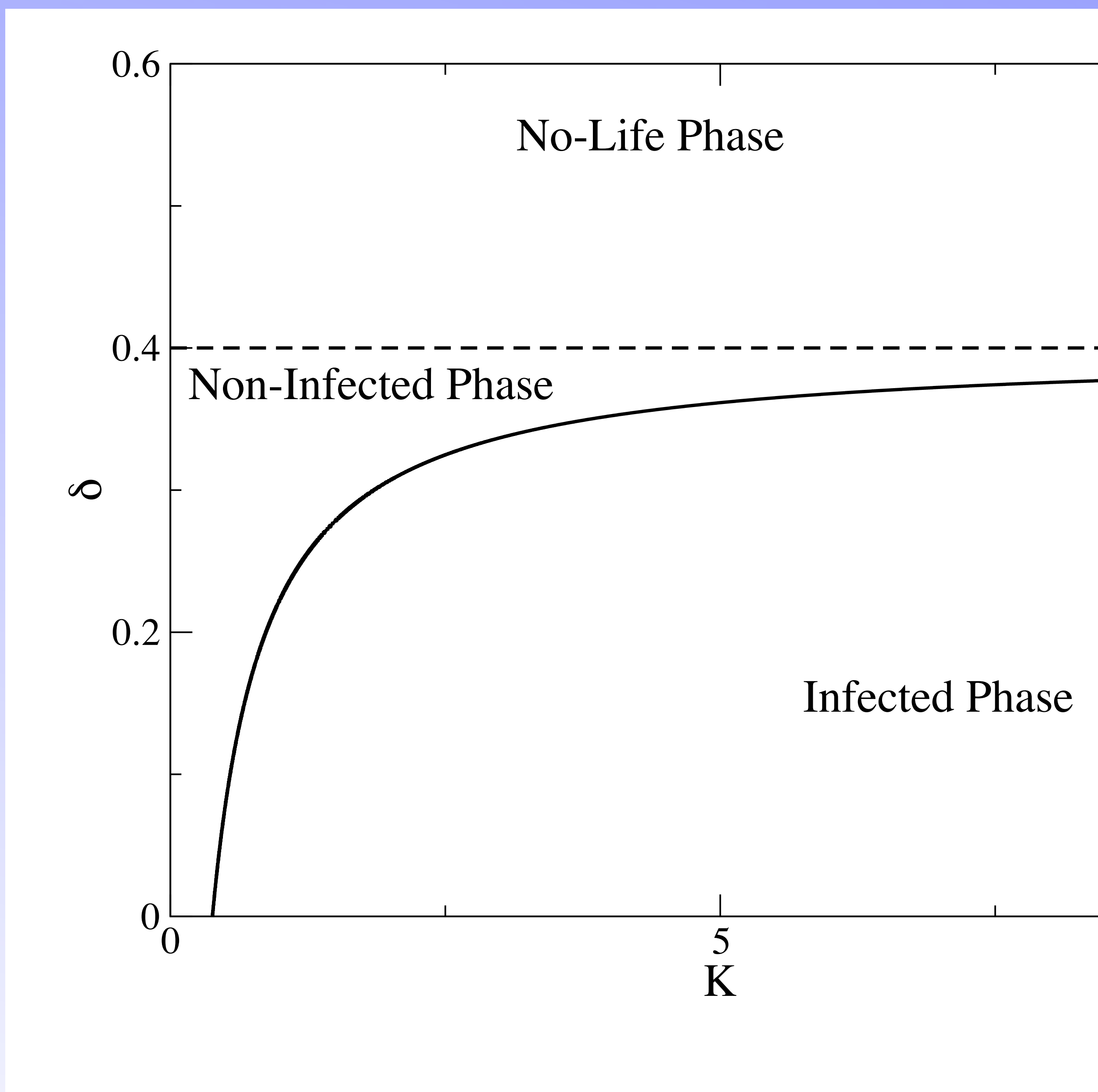
AGE-DEPENDENT MOVEMENTS

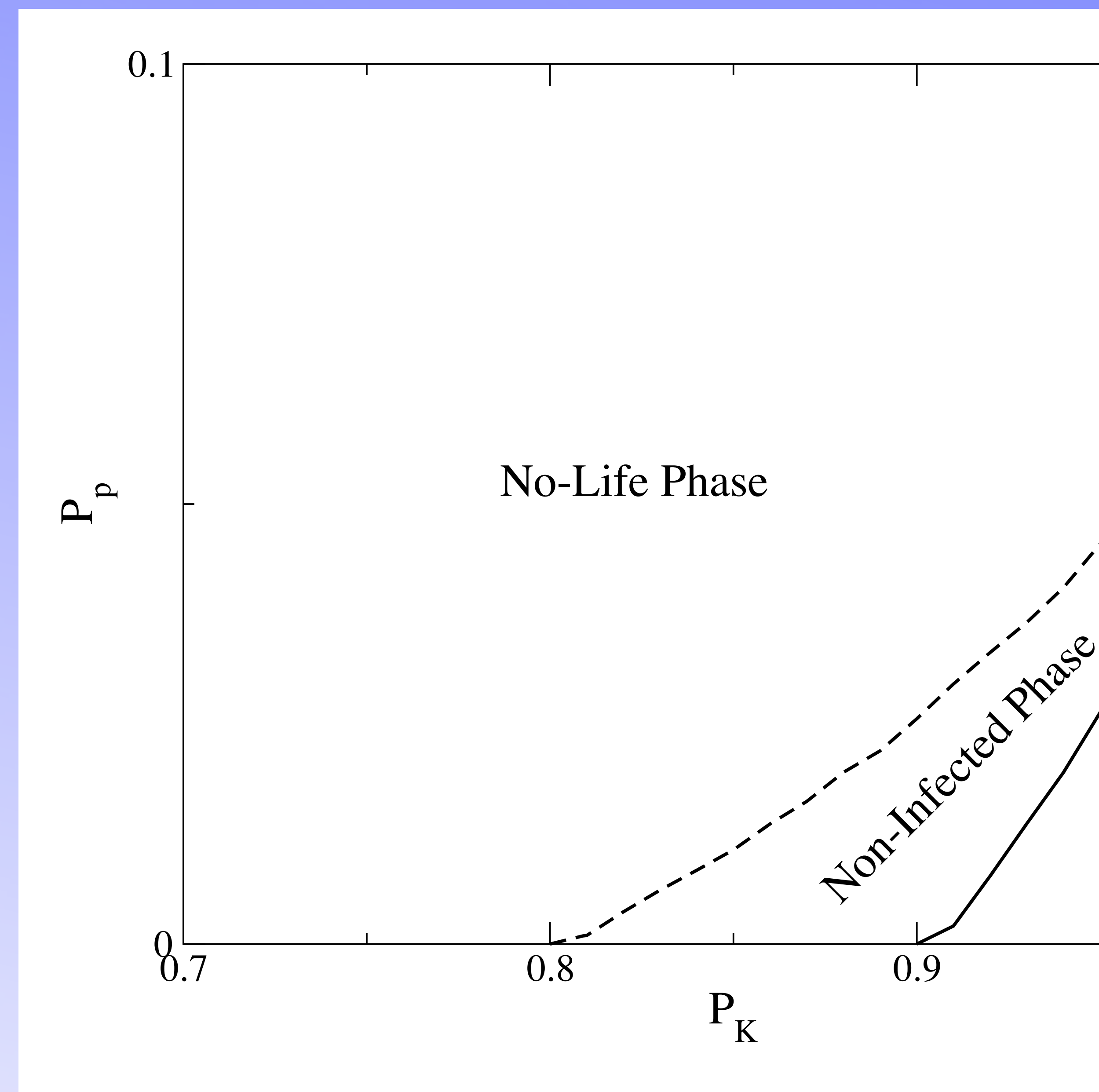


HANTA VIRUS EPIDEMIC

$$
\begin{aligned}
\frac{\partial B_i(x,t)}{\partial t} &= -c_B B_i - \frac{B_i(A+B)}{K(x,t)} + aB_s(A_i+B_i) + D\nabla^2 B_i - G(x)B_i, \\
\frac{\partial B_s(x,t)}{\partial t} &= bA - c_B B_s - \frac{B_s(A+B)}{K(x,t)} - aB_s(A_i+B_i) + D\nabla^2 B_s - G(x)B_s, \\
\frac{\partial A_i(x,t)}{\partial t} &= -cA_i - \frac{A_i(A+B)}{K(x,t)} + aA_sB_i + G(x)B_i, \\
\frac{\partial A_s(x,t)}{\partial t} &= -cA_s - \frac{A_s(A+B)}{K(x,t)} - aA_sB_i + G(x)B_s, \qquad (1)
\end{aligned}
$$

HANTA VIRUS EPIDEMIC





HANTA VIRUS EPIDEMIC