The Council of Elrond: A data-based commentary on R_0 on social networks

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Basic Reproduction Ratio – R_0

Standard Theory:

- In a **<u>homogeneous, well-mixed system</u>**,

$$R_0 = \langle \text{infectious contacts} \rangle$$

caused by introduction of a single infected individual in a wholly susceptible equilibrium population

- $R_0 < 1$ implies an epidemic cannot occur
- $R_0 > 1$ implies a pathogen may be successful

R_0 in terms of the transmission matrix (T)

- $R_0=\rho(T)$ (with some philosophical caveats) but
  - account for overlapping infectious periods
    - Loops in the contact network
    - Rate of link generaton/removal
  - A single snapshot of the epidemic captures the dynamics of the network (Ergodic Hypothesis?)

(works for directed networks with low correlation between source partners and destination partners – other systems?)

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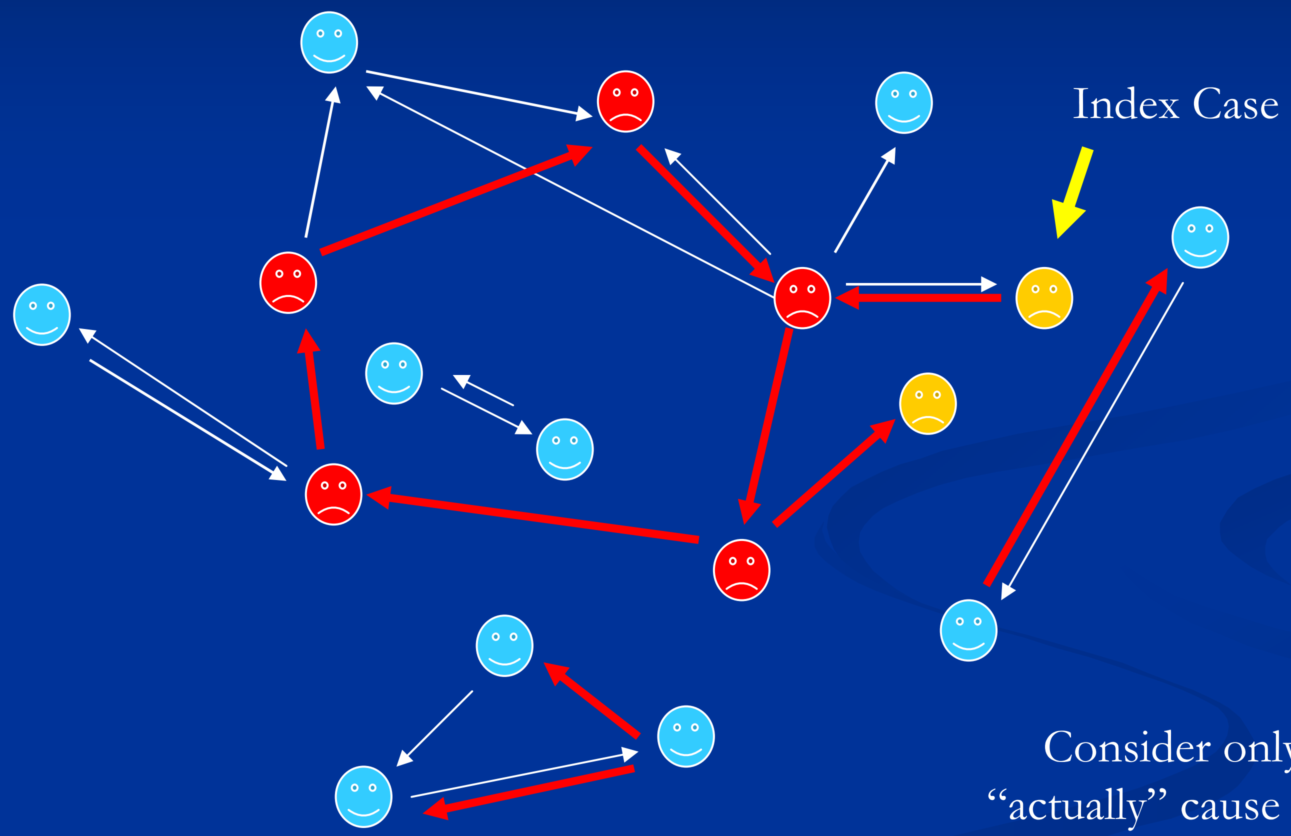
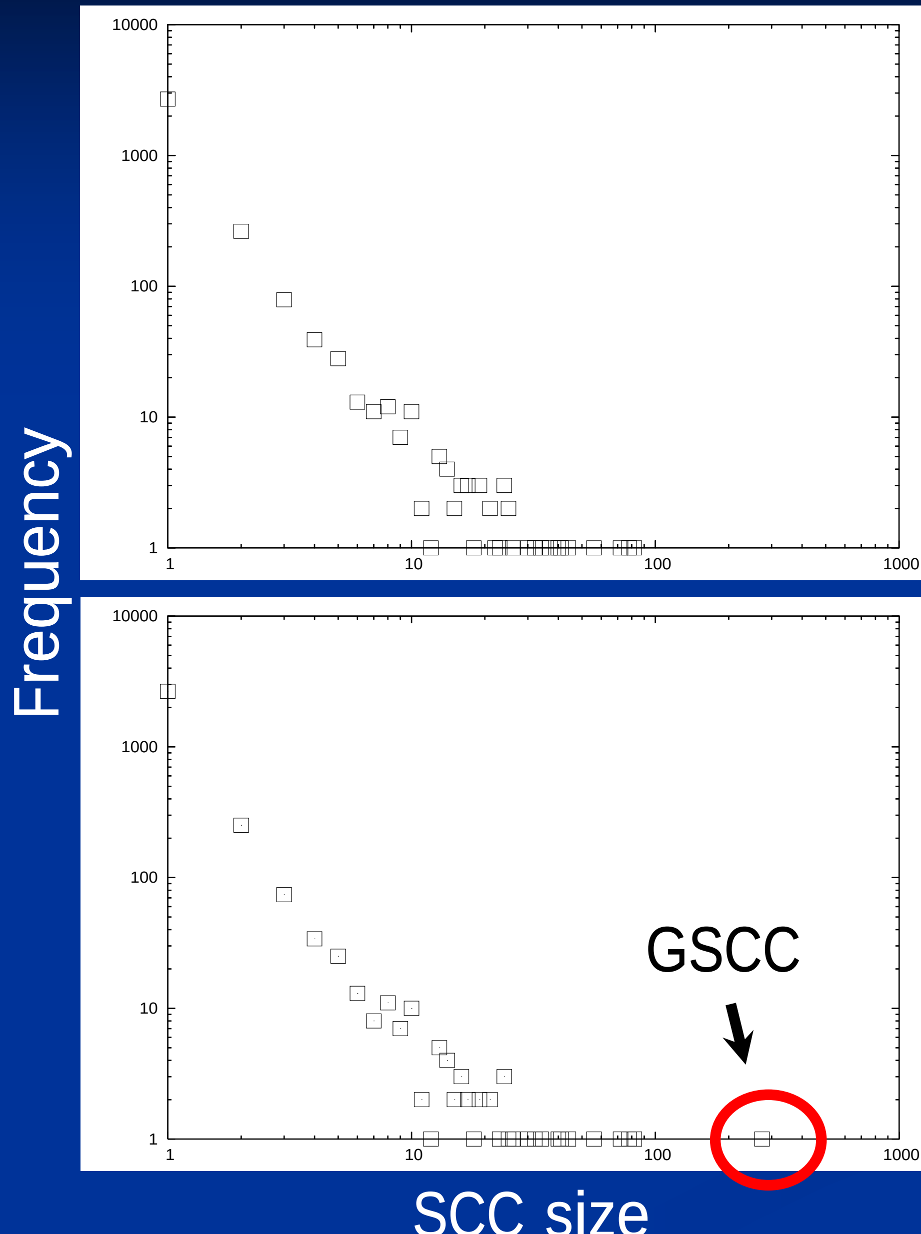
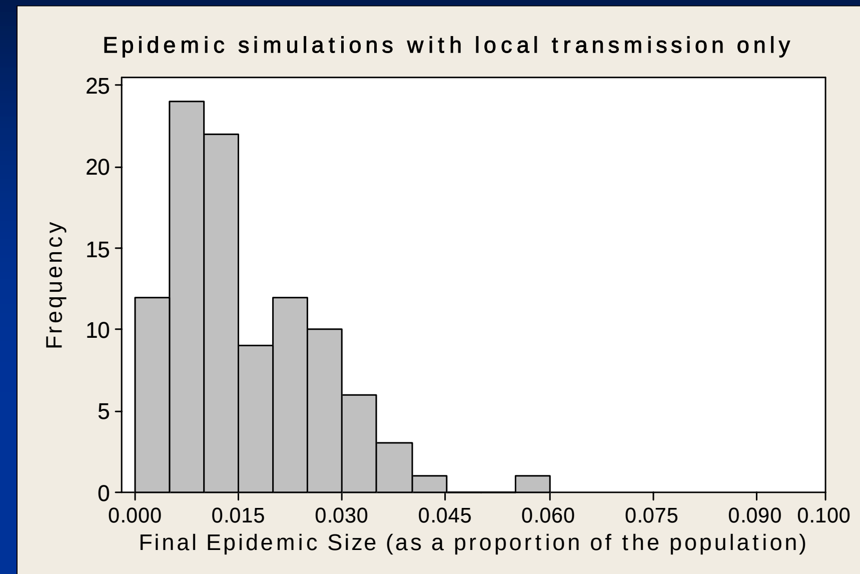
Giant Strongly Connected Component (GSCC) in Epidemiological or Transmission Networks

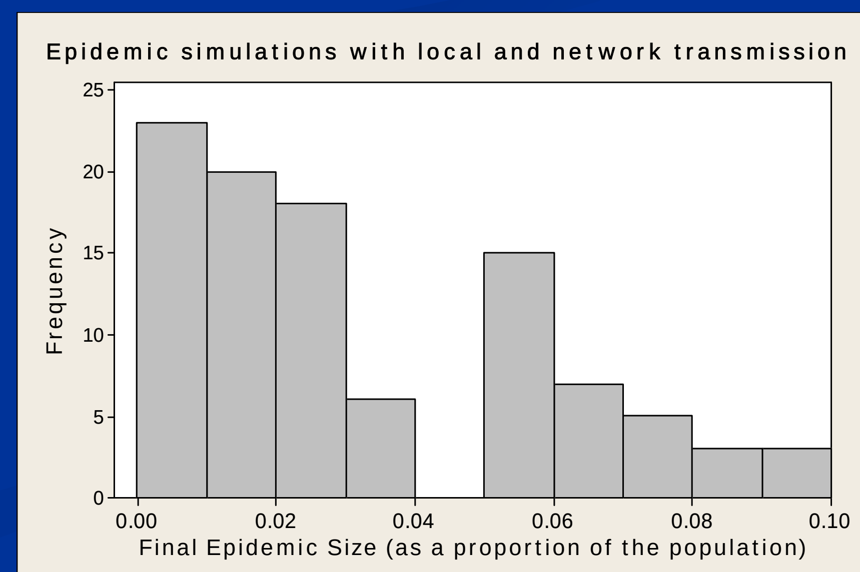


Consider only contacts that "actually" cause infection over one infectious period

Comparing SCC size to epidemics

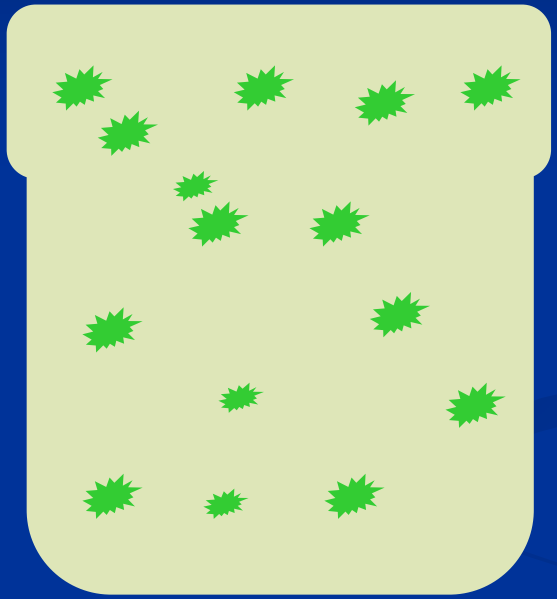






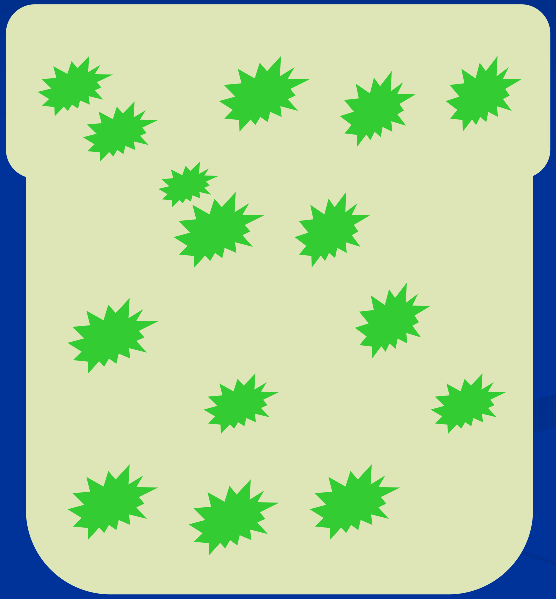
Percolation

Day 1



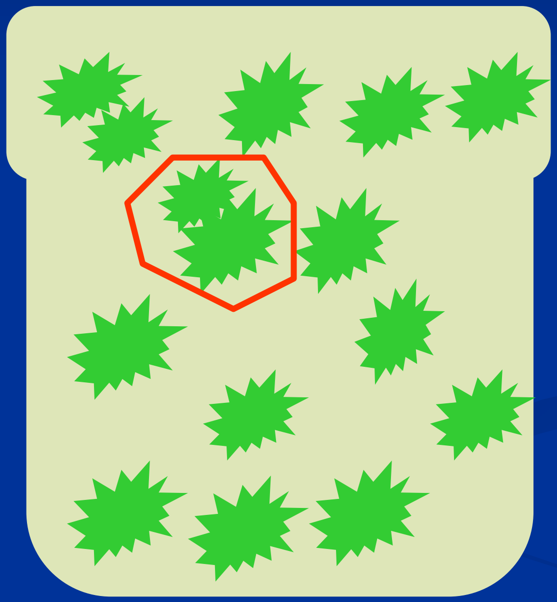
Percolation

Day 2



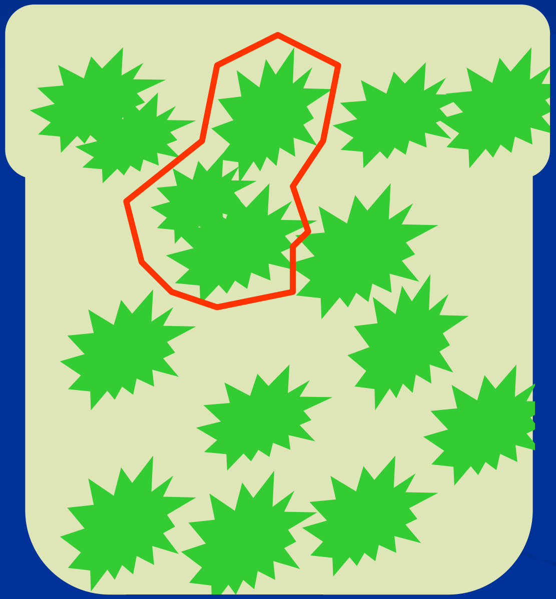
Percolation

Day 3



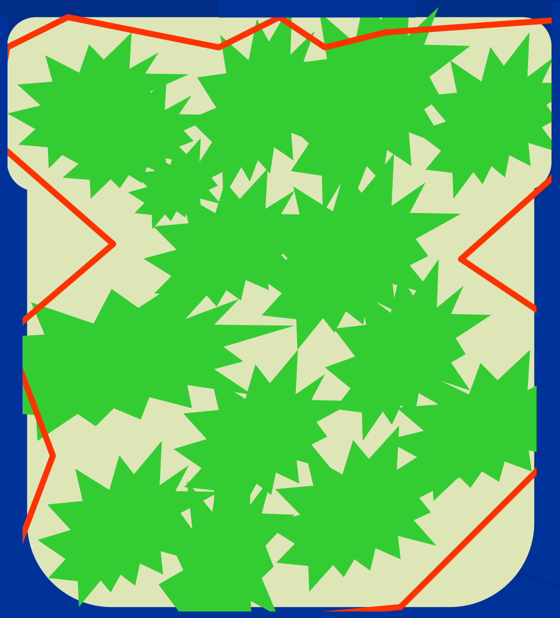
Percolation

Day 4



Percolation

Day 5



The Largest Component (Patch of Mould) Spans the Popn.

Percolation Interpretation of R_0

- Below percolation threshold, GSCC size (N_{GSCC}) fixed w.r.t. total population size (N_{pop}), i.e.

$$\lim_{N_{pop}\to\infty}\left(\frac{N_{GSCC}}{N_{pop}}\right) = 0$$

- Above percolation threshold

$$\lim_{N_{pop}\to\infty}\left(\frac{N_{GSCC}}{N_{pop}}\right) = f > 0$$

Large Epidemic ⟷ **Percolation Threshold in the Epi. Network** ⟷ $R_0 = 1$

R_0 on Randomly mixed Undirected Networks

- "infinite" network $R_0 = \lim_{g\to\infty} I_{g+1} / I_g$

- **degree-dependent mixing:** probability of any node of degree *i* being connected to a node of degree *j* is given by *P(j|i)*.

$$I_{g+1} = \sum_{i,j} \tau\left(i-1\right) P\left(j\,|\,i\right) I_{g,i}$$

$$R_0 = \tau\left(\frac{\langle k^2 \rangle}{\langle k \rangle} - 1\right)$$

Similar to Hethcote et al., for STDs in 1970s/1980s, Anderson & May for HIV

R_0 in Randomly Mixed Directed Networks

transmissibility

Infectious out links

$$R_0 = \frac{\langle \sigma k^{in} \tau k^{out} \rangle}{\langle \sigma k^{in} \rangle}$$

Infectious in links

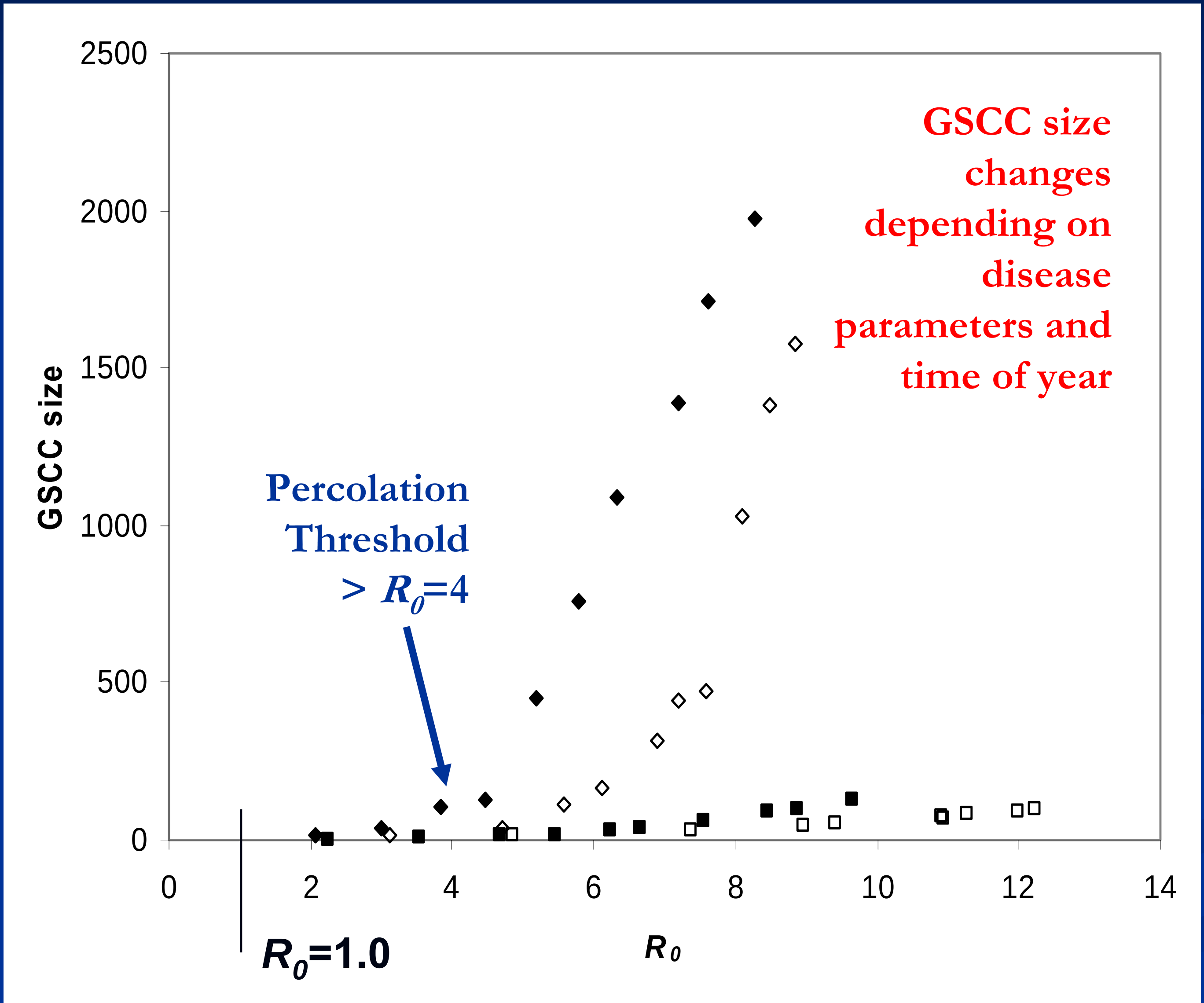
susceptibility

If k^{in} **and** k^{out} **uncorrelated:** $R_0 = \langle \sigma k^{in} \rangle = \langle \tau k^{out} \rangle$

Percolation Threshold
Schwartz et al, 2003,
Kao et al, 2006

$$\sigma k^{in} = \tau k^{out} = \tau k \Rightarrow R_0 = \tau \langle k^2 \rangle / \langle k \rangle$$

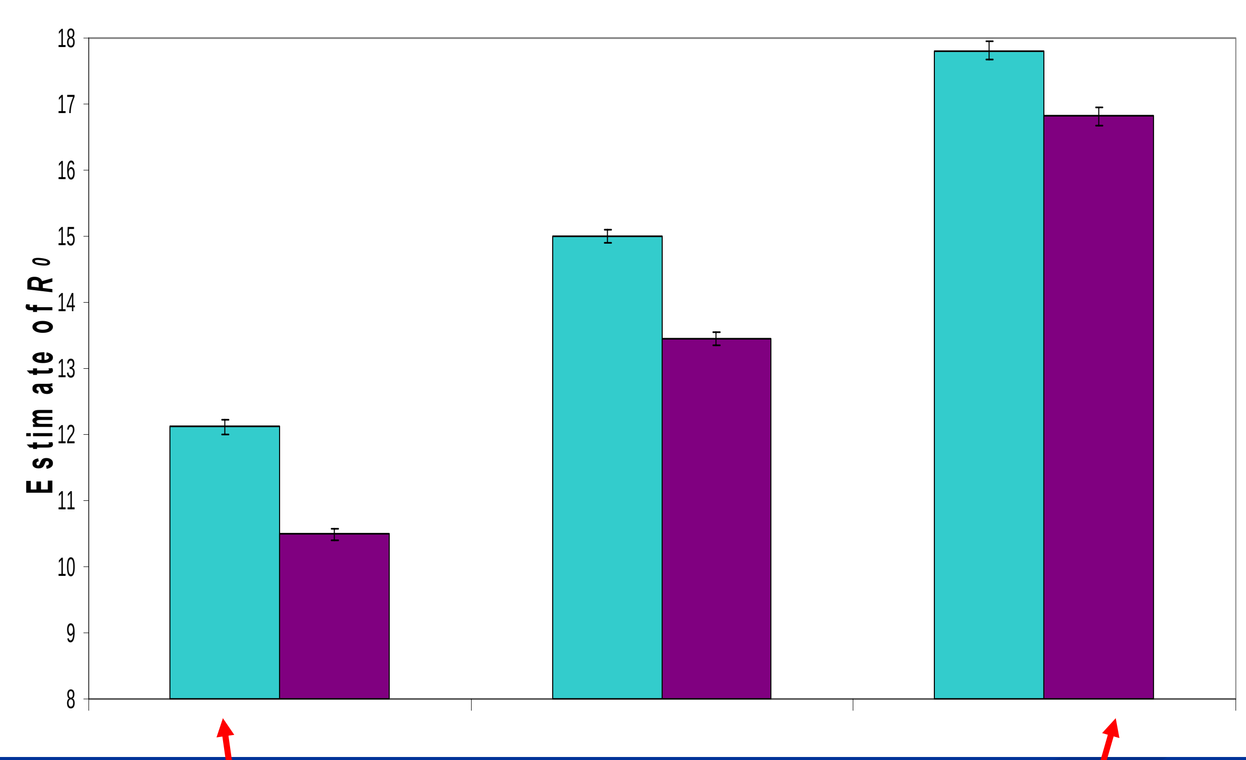
Growth of the GSCC vs. R_0



(♦) - 28 days infectious period from 19/05/04
(◊) - 28 days infectious period from 05/11/03
(■) - 7 days infectious period from 19/05/04
(□) - 7 days infectious period from 05/11/03

Markets have fixed one day infectious period

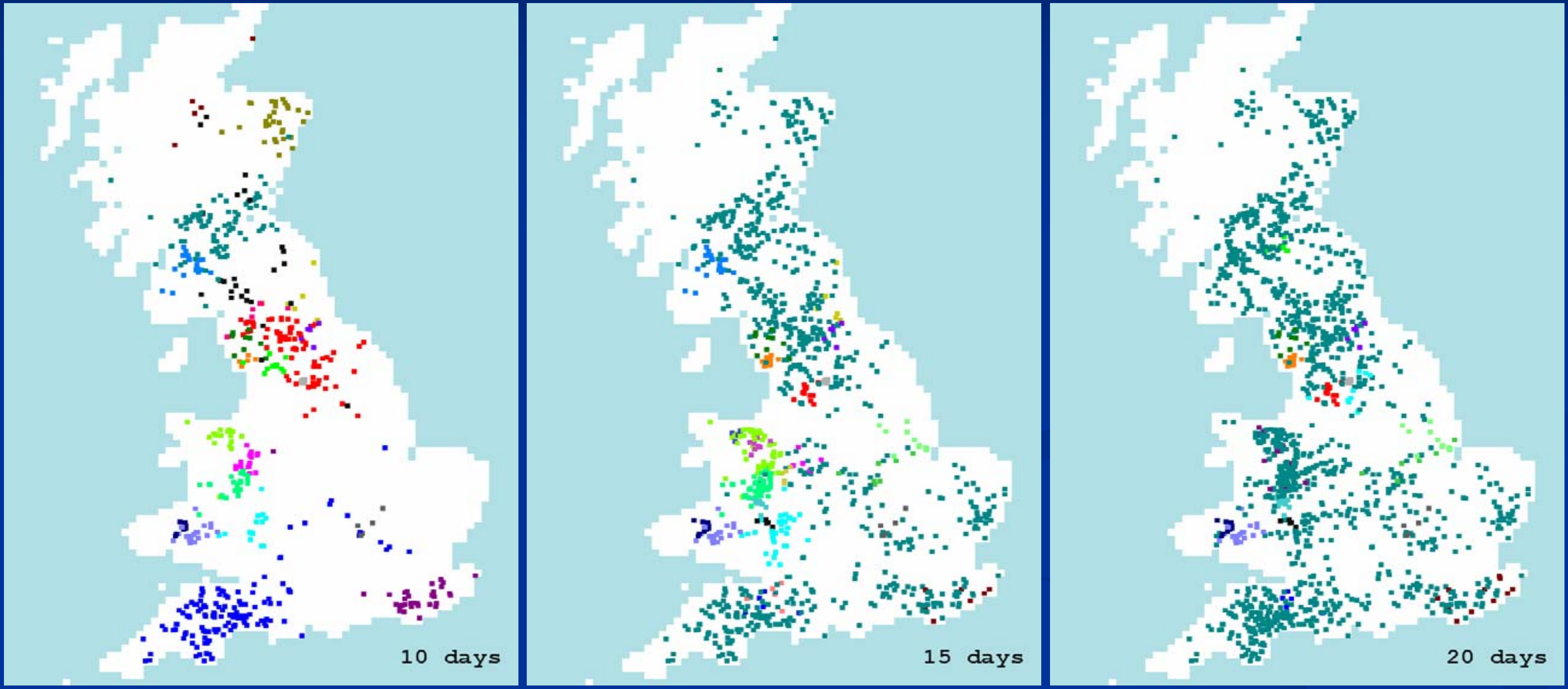
Estimating R_0 two ways



Via Contact Matrix

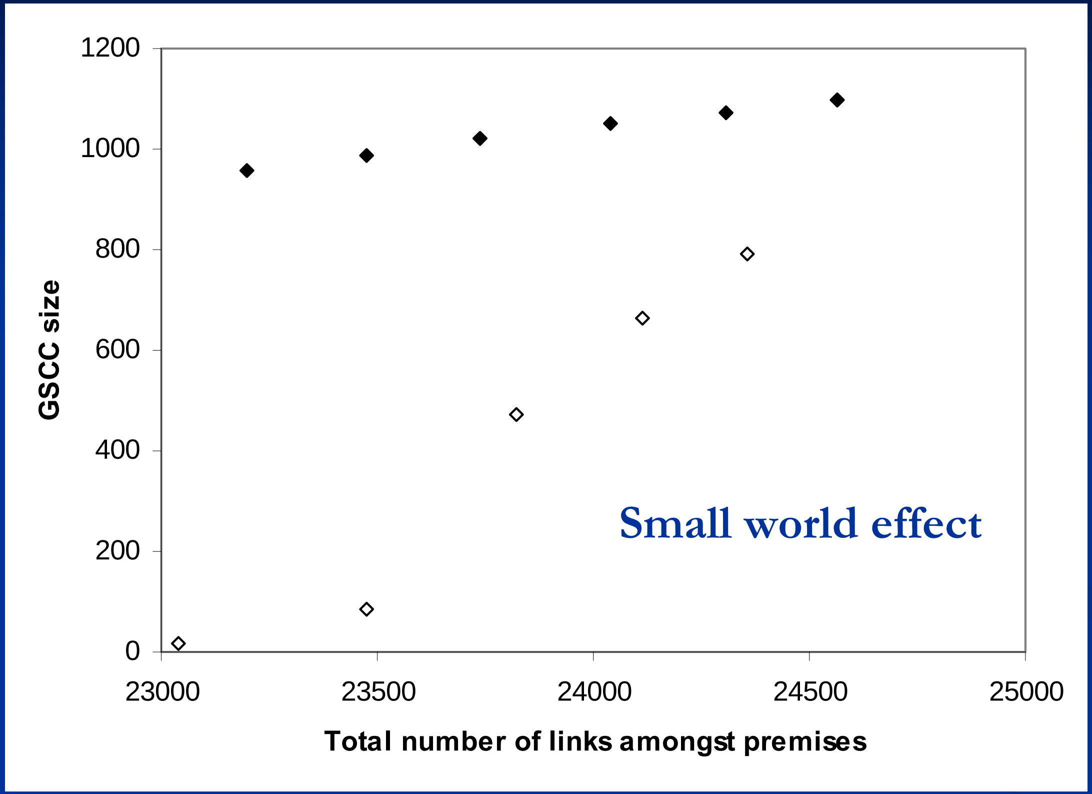
Via Equation for Random Mixing

Percolation Threshold



Growth of strongly connected components by increasing infectious period of farms

Implementation of Targeted Control



Sheep Movements from 19/05/04 to 16/06/04

Difficult to target most highly connected nodes (markets)

◊ Targeted surveillance/biosecurity

Market → movement → Farm → movement → Market

♦ Random removal

Some thoughts

- Can a general formulation be established?
- Even if there is a general description, is it the most appropriate measure?
- is there a way of capturing multiple levels of mixing in a single summary measure?

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